FLUID

MECHANICS

Syllabus

- Introduction + Fluid properties
- Fluid statics
  - → Pressure measurement
  - → Hydrostatic forces
  - → Buoyancy forces
- Fluid Kinematics
- Fluid Dynamics
- Laminar flow
- flow through Pipes
- Turbulent flow
- Boundary layer theory
- Dimensional Analysis

Books

1) Modi & Seth [ESE]
2) R.K. Bansal [ESE conventional Questions, Solved Examples]

GATE ⇒ only class notes

Fluid mechanics

it is the study of Application of loads and its effect on m/c, structure, fluids, etc.

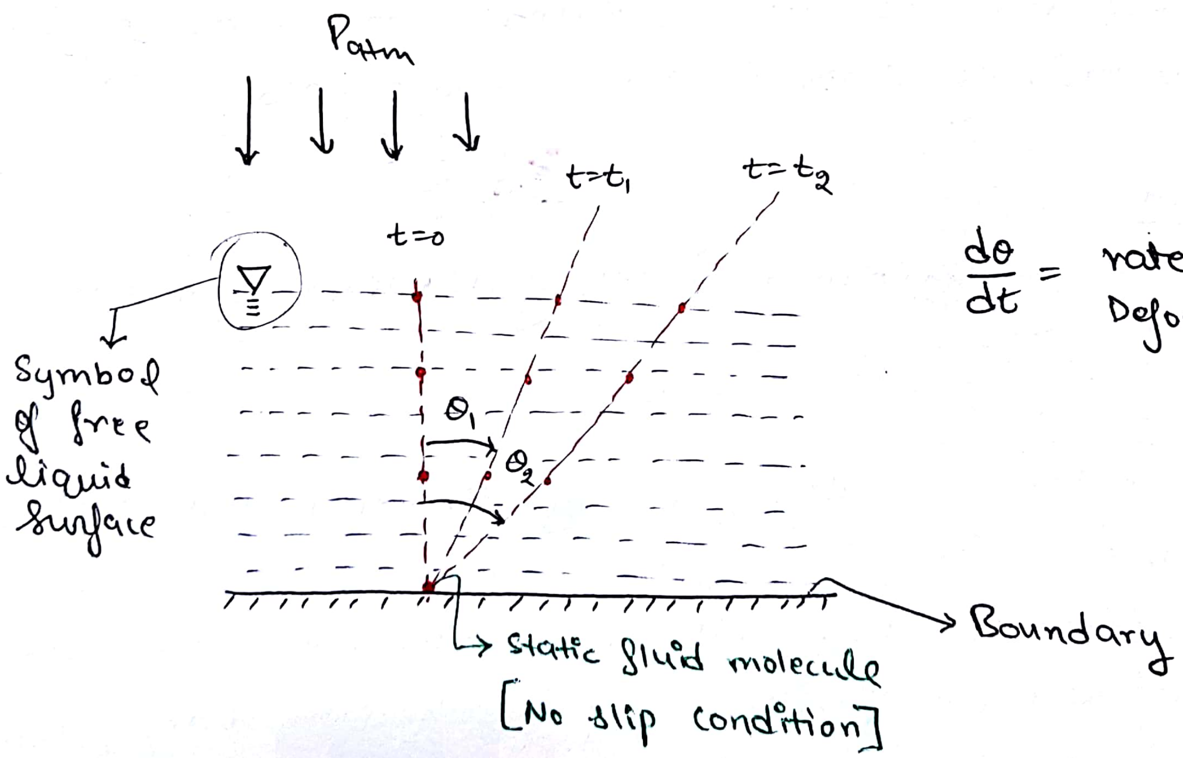
- Engineering mechanics
  - Rigid bodies
  - Static
    - Rest
  - Dynamic
    - motion
- SOM
  - deformable bodies
- Fluid mechanics
  - Kinematics
    - L ✗ E ✓
  - Kinetics
    - L ✓ E ✓

CHAPTER - 1

INTRODUCTION & FLUID PROPERTIES

- Fluids are the substances that are having the ability to flow or deform continously under the action of shear force
- For a static fluid shear force is zero
- examples of fluid
  - water
  - mercury
  - Air
  - steam
  - etc

Note: All types of liquids and gases are known as fluids



$\frac{d\theta}{dt}$ = rate of Deformation

Difference b/w Solid & fluids

- In case of solids on application of load the deformation is constant w.r.t. time whereas in case of fluids on application of force, the deformation is continous and hence in solids deformation is important whereas in fluids $\frac{d\theta}{dt}$ [rate of deformation] is more important.

- In case of solids on removal of loads the solids will try to regain their original position whereas in case of fluids they will never try to regain their origin position on removal of load.

- Solids will show resistance to all types of loads whereas fluid will only show resistance to compressive loads.
- In case of solids the deformation can be linear as well as angular whereas in case of fluids deformation is only angular.

Note: Substances are differentiated into solids or fluids on the basis of their behaviour under load.

Cohession

it is the intermolecular forces of attraction b/w molecules of same nature.

Adhesion

it is the intermolecular forces of attraction b/w molecules of different nature

ex:- water on glass will show adhesion more whereas mercury on glass will show cohesion more, water on plastic sheet will show cohesion more.

Note: Cohesion and adhesion depends upon the nature of surfaces in contact.

Fluid Properties

- Properties are certain measurable characteristics that can be quantified.
- With the help of Properties we can identify the fluid.

1. Density or mass density (ρ)

- Density is defined as the ratio of mass per unit volume and its SI units is Kg/m^3.
- Density basically represents the number of molecules of a fluid in a given volume.
- more the no. of molecules more is the mass & heavier is the fluid and hence density can also be written as representative of heavyness of of a fluid.

$\boxed{\text{Density}}$

- $\rightarrow \dfrac{\text{mass}}{\text{vol.}} \quad \dfrac{Kg}{m^3}$
- $\rightarrow \rho_{solid} > \rho_{liquid} > \rho_{gases}$
- $\rightarrow$ heaviness of a fluid

$\rho_{H_2O} = 10^3 \ Kg/m^3$

$\rho_{Hg} = 13.6 \times 10^3 \ Kg/m^3$

$\rho_{Air} = 1.2 \ Kg/m^3$

ρ
- $\rightarrow P \uparrow \Rightarrow \rho \uparrow$
- $\rightarrow T \uparrow \Rightarrow \rho \downarrow$

2. Specific weight or weight density (w)

$\rightarrow$ $w = \frac{\text{weight}}{\text{Volume}} = \frac{mg}{V} = \rho g$ $\frac{N}{m^3}$

$\rightarrow$ $w_{H_2O} = 10^3 \times 9.81 = 9810$ N/m3

$\rightarrow$ $w_{Hg} = 13.6 \times 10^3 \times 9.81$ N/m3

weight = Specific weight x volume

★ $\boxed{wt. = \rho g V}$ ★ $\Rightarrow$ $w \rightarrow P, T, g$

- Specific weight is defined as weight of fluid Per unit volume
- It Basically represents the force exerted by the fluids due to gravity in a given volume

Note :- density is an absolute quantity w.r.t. location whereas specific wt. is a variable quantity w.r.t. location

3. Specific gravity (S)

- Specific gravity is defined as the ratio of the density of the fluid to the density of the standard fluid.
- Standard fluids in case liquid is taken as water whereas standard fluids in case of gases is taken as Air.
- Specific gravity basically shows which fluids are heavier than water & which fluids are lighter than water.

$$\boxed{S = \frac{\rho_{fluid}}{\rho_{\text{standard fluid}}}}$$ Dimensionless

(i) $S = 0.750$

$\rho_{fluid} = 0.750 \times 10^3 = 750$ kg/m3

$S = 13.6$

(ii) $\rho_{fluid} = 13.6 \times 1000 = 13600 \ kg/m^3$

(iii) $S = 1$

$\rho_{fluid} = 1000 \ kg/m^3 \longrightarrow$ water

$S > 1 \Rightarrow$ fluid is heavier than water

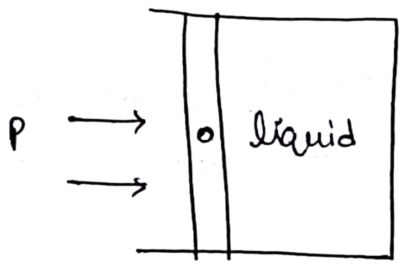
$S < 1 \Rightarrow$ fluid is lighter than water

$S = 1 \Rightarrow$ fluid is water itself.

Note: All specific gravities are relative densities but all relative densities are not specific gravities.

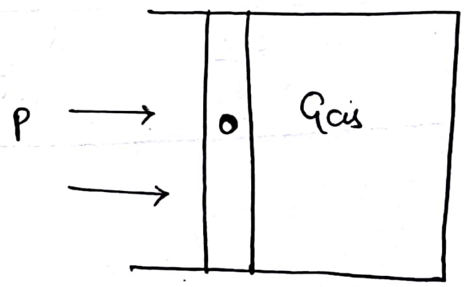
4. Compressibility (β)

If there is a change in V or ρ w.r.t. P of fluid, it is compressible.



$P \uparrow$, V ×

Incompressible



$P\uparrow$, $V\downarrow$

Compressible

$1 \ atm \longrightarrow \rho_{H_2O} = 998 \ kg/m^3$

$100 \ atm \longrightarrow \rho_{H_2O} = 1003 \ kg/m^3$

$\hookrightarrow \Delta P = 5$

$\% \text{ change} = \frac{5}{998} \times 100 \simeq .5\%$

- liquids are generally incompressible fluids whereas gases are generally compressible fluids
- with increase in Pressure, if variation of volume is large then the fluid is highly compressible
- Mathematically compressibility is defined as reciprocal of bulk modulus of elasticity.

$$\rho = \frac{m}{V} \Rightarrow m = \rho V \Rightarrow 0 = \rho\, dV + V\, d\rho$$

$$\Rightarrow \boxed{-\frac{dV}{V} = \frac{d\rho}{\rho}}$$

$$\beta = \frac{1}{K}$$

$$K = \frac{\text{Hydrostatic stress}}{\text{Volumetric Strain}} = \frac{\text{Direct stress}}{\text{Vol. Strain}} = \frac{-dP}{dV/V}$$

$$\boxed{K = -\frac{V\,dP}{dV} = \frac{\rho\,dP}{d\rho}}$$

$$\boxed{\beta = \frac{1}{K} = -\frac{dV}{V\,dP} = \frac{d\rho}{\rho\,dP}} \Rightarrow \text{if } \rho = \text{const w.r.t. } P$$

$\Rightarrow$ fluid is incompressible

- $\beta \propto \frac{1}{K}$; $K_{water} = 2 \times 10^6$ KPa

$K_{Air} = 100$ KPa

Note: 1) All those fluids are said to be incompressible fluids whose density is constant w.r.t. Pressure.

2) Air is generally 20,000 times more compressible than water.

3) if the mach no. of fluid is less than 0.3 it is treated as incompressible.

Isothermal Bulk modulus

ideal gas

$PV = nRT$

$P = \rho RT$

$T = \text{const}$

$\dfrac{dP}{d\rho} = RT$

$K_T = \rho \dfrac{dP}{d\rho} = \rho RT = P$

$\boxed{K_T = P}$

Adiabatic bulk modulus

ideal gas, adiabatic, reversible

$PV^{\gamma} = \text{const.} = c$

$P = cV^{-\gamma} = c\left(\dfrac{m}{\rho}\right)^{-\gamma} = \dfrac{c}{m^{\gamma}}\rho^{\gamma}$

$\dfrac{dP}{d\rho} = \gamma \dfrac{c\,\rho^{\gamma-1}}{m^{\gamma}}$

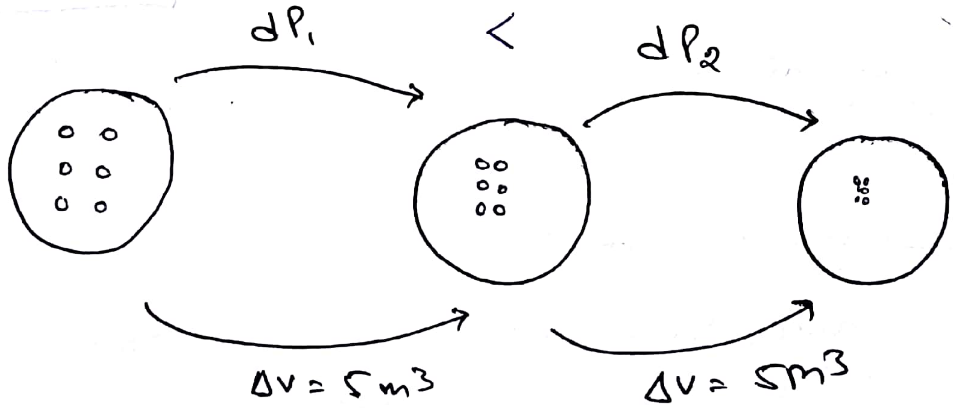
$K_a = \rho \dfrac{dP}{d\rho} = \gamma \dfrac{c\,\rho^{\gamma}}{m^{\gamma}} = \gamma P$

$\boxed{K_a = \gamma P}$

- As γ is always greater than 1 for an ideal gas

$\boxed{K_a > K_T}$

$\Rightarrow \boxed{P\uparrow \Rightarrow K\uparrow \Rightarrow \beta\downarrow}$

- 

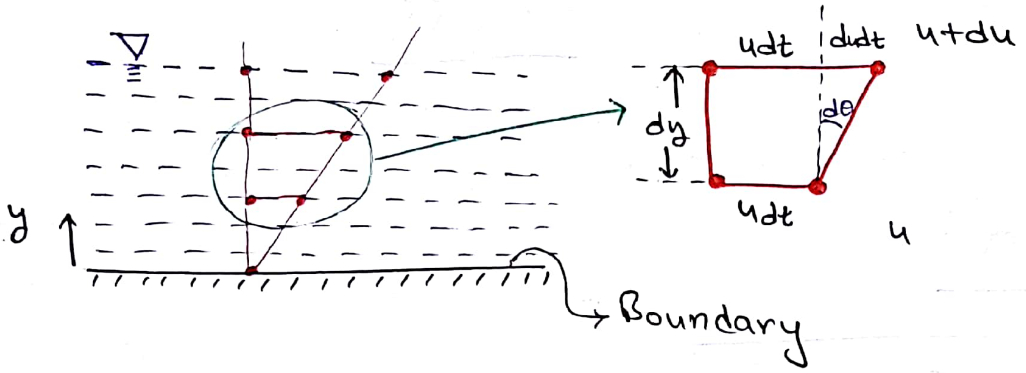
$K_1 < K_2$

Note: 1) Compressing a gas adiabatically is more difficult than compressing a gas isothermally bcz while adiabatic compression due to increase in tempr the randomness of molecules increases and hence it provides resistance to compression.

2) with increase in pressure, the Bulk modulus of a gas increases bcz due to increase in pressure molecules will come closure to each other and provide resistance to further compression & hence compressibility decreases.

5) Viscosity

viscosity is defined as the internal resistance offered by one layer of the fluid to the adjecent layer.



$$\tan(d\theta) = \frac{du\,dt}{dy}$$

$$d\theta = \frac{du\,dt}{dy}$$

$$\boxed{\frac{d\theta}{dt} = \frac{du}{dy}}$$

$\frac{d\theta}{dt}$ = Rate of deformation

$\frac{du}{dy}$ = Velocity gradient

- Solids $\longrightarrow$ Stress $\propto$ Strain.

  fluids $\longrightarrow$ Shear stress $\propto$ Rate of shear strain

  $\Downarrow$

  Rate of deformation

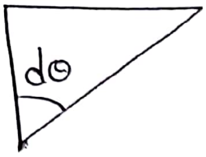
$$\tau \propto \frac{d\theta}{dt}$$

$$\boxed{\tau = \mu \frac{du}{dy}}$$

μ = viscosity

= Dynamic viscosity

= Coefficient of viscosity

A	τ	B
		
• $\frac{d\theta}{dt}$ is more		• $\frac{d\theta}{dt}$ is less
• μ is less		• μ is more
• Resistance to flow is less		• Resistance to flow is more
• Flow is easy		• Flow is difficult

Note: 1) The main reason of viscosity in case of liquid is the molecular bonding whereas in case of gases it is the molecular collision.

2) Viscosity basically shows resistance to motion & hence if the viscosity is less the flow is easy.

Newton's Law of viscosity

All those fluids are said to be newtonian fluids for whom shear stress is directly proportional to rate of shear strain.

Shear stress $\propto$ Rate of shear strain

$$\tau \propto \frac{d\theta}{dt}$$

slope = const.

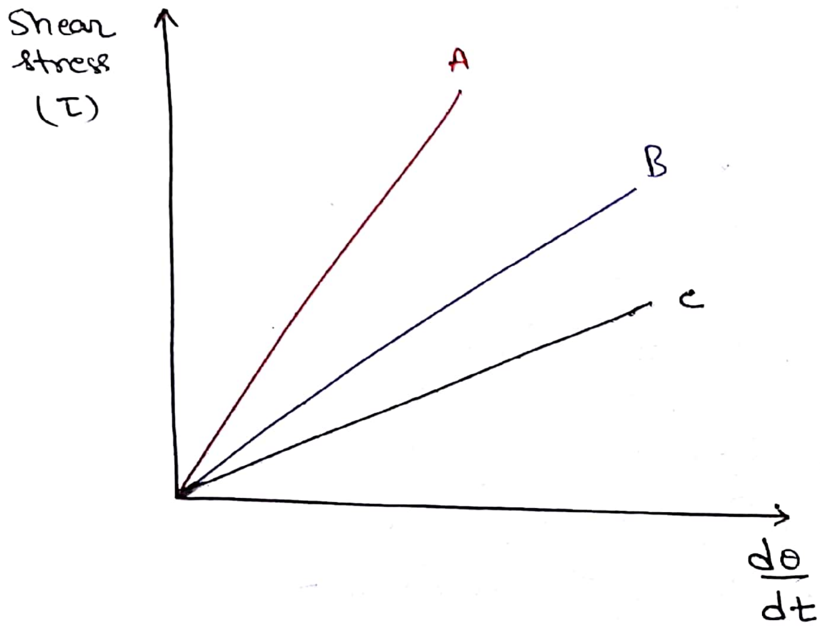
$$\boxed{\tau = \mu \frac{du}{dy}}$$

⇒ Newton Law of viscosity

⇒ Newtonian fluids

ex.:- Water
- Air
- Hg
- Petrol
- Diesel

μ = constant w.r.t. $\frac{d\theta}{dt}$ = slope of τ v/s $\frac{d\theta}{dt}$ graph.



A, B, C all are newtonian fluids.

$$\boxed{\mu_A > \mu_B > \mu_C}$$

Note: 1) For a newtonian fluid viscosity is constant w.r.t. deformation.

2) For a newtonian fluid, the relation b/w shear stress and rate of deformation is ~~constant~~ linear.

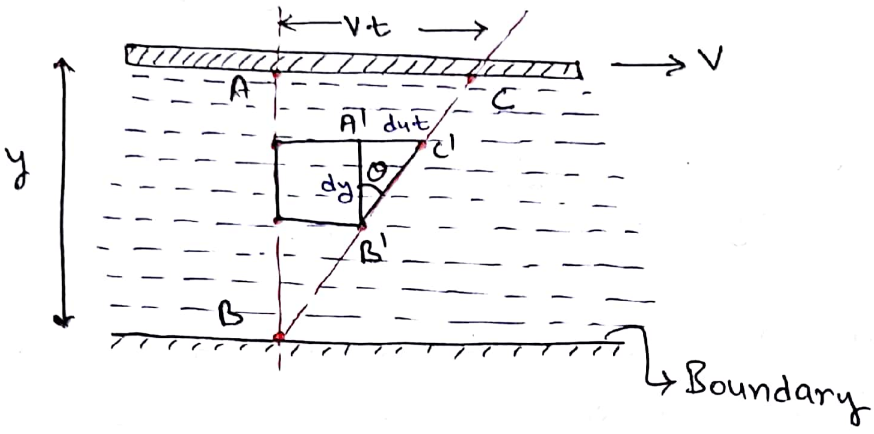
3) Liquid → T↑ ⇒ μ↓

gases → T↑ ⇒ μ↓

Variation of viscosity(μ) w.r.t. temperature:

- In case of liquids with increase in tempr the viscosity decreases bcz the main reason of viscosity in case of liquids is molecular bonding and with increase in tempr molecular breaks down & viscosity decreases.
- In case of gases the main reason of viscosity is molecular collission & due to this with increase in tempr the molecular collission increases and hence viscosity increases.

Shear force required for linear velocity Profile



$\tan\theta = \frac{Vt}{y}$

$\tan\theta = \frac{du.t}{dy}$

$\Downarrow$

compare

$\frac{V.t}{y} = \frac{du.t}{dy}$

$$\boxed{\frac{V}{y} = \frac{du}{dy}}$$

$\tau = \mu \frac{du}{dy}$

$\Rightarrow \quad \tau = \frac{\mu V}{y}$

$$\boxed{F = \frac{\mu A V}{y}}$$

F = shear force required
μ = Dynamic viscosity
A = Surface area of plate in contact of fluid
V = Velocity of plate
y = thickness of fluid film.

Units of viscosity (μ)

$F = \frac{\mu A V}{y} \Rightarrow \mu = \frac{Fy}{AV} = \frac{N\,\cancel{m}\,s}{m^2 . \cancel{m}} = \frac{N-s}{m^2}$

$\mu \Rightarrow$ Pa-s

$\mu \Rightarrow \frac{kg\,m}{s^2} . \frac{s}{m^2} \Rightarrow \frac{kg}{m-s}$

Kinematic viscosity (ν)

- Kinematic viscosity is defined as the ratio of dynamic viscosity to density.

$$\boxed{\nu = \frac{\mu}{\rho}} \qquad \frac{kg}{m\text{-}s} \times \frac{m^3}{kg} \Rightarrow \frac{m^2}{s}$$

- Dynamic viscosity shows resistance to motion whereas Kinematic viscosity shows resistance to molecular momentum transfer [molecular collision]
[only valid for liquids]

Note

Dynamic viscosity (μ) $\Rightarrow \frac{kg}{m\text{-}s} \Rightarrow \frac{1000\ gm}{100\ cm\text{-}sec} \Rightarrow 10 \frac{gm}{cm\text{-}sec} \rightarrow$ Poise

$$\Rightarrow \boxed{1 \frac{kg}{m\text{-}s} = 10 \text{ Poise}}$$

Kinematic viscosity (ν) $\Rightarrow \frac{m^2}{s} \Rightarrow 10000 \frac{cm^2}{sec} \Rightarrow \boxed{1 \frac{m^2}{s} = 10^4 \text{ Stokes}}$

$\frac{cm^2}{sec}$ → Stokes

as temp ↑

Liquids $\Rightarrow \mu \downarrow$

$$\nu = \frac{\mu \downarrow}{\rho_{=const}} \Rightarrow \nu \downarrow$$

Gas $\Rightarrow \mu \uparrow$

$$\nu = \frac{\mu \uparrow}{\rho \downarrow} \Rightarrow \nu \uparrow \text{ faster rate}$$

$T \uparrow$

	μ	ν
Liquid	$\downarrow$	$\downarrow$
Gas	$\uparrow$	$\uparrow$ faster rate

for ideal gas

$PV = mRT$

$P = \rho RT \Rightarrow \rho \propto \frac{1}{T}$

- Sutherland's eqn

$$\boxed{\mu_{gas} \propto \sqrt{T}}$$

$$\nu_{gas} = \frac{\mu_{gas}}{\rho_{gas}} \Rightarrow \frac{\sqrt{T}}{1/T} \Rightarrow T^{3/2}$$

$$\Rightarrow \boxed{\nu_{gas} \propto T^{3/2}}$$

Non-Newtonian fluids [ESE objective only]

- non-newtonian fluids are the one whose viscosity is going to vary with rate of deformation
- In non-newtonian fluids the variation of shear stress with $\frac{d\theta}{dt}$ is non-linear

- The study of non-newtonian fluids is known as Rhiology

$$\boxed{\tau = A\left(\frac{du}{dy}\right)^n + B}$$ General equation

- For newtonian fluid

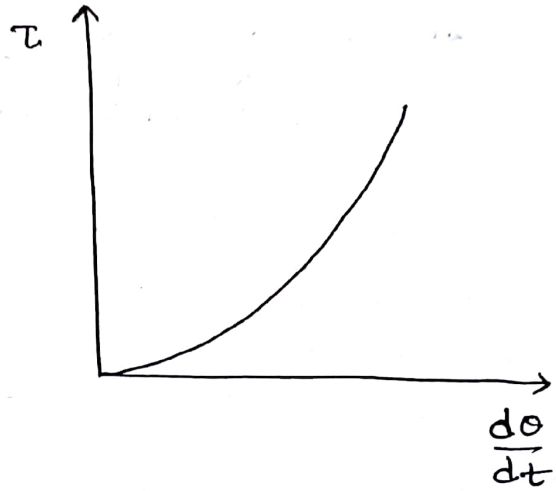
$B = 0$

$n = 1$

A) μ varries with $\frac{d\theta}{dt}$

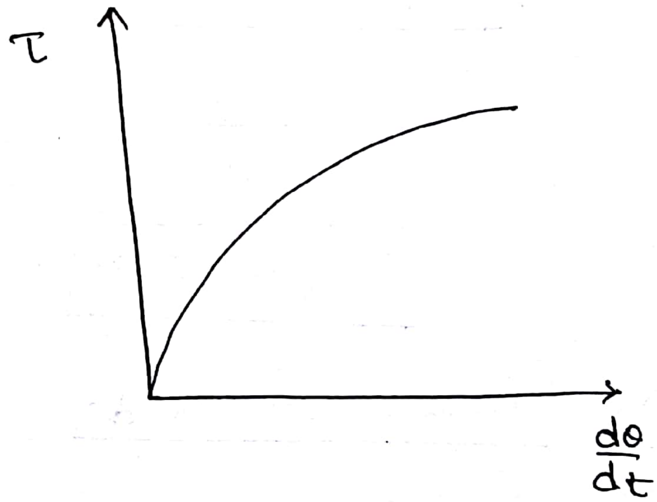
1. Dilatant fluid ($n > 1$, $B = 0$)

- $\mu \uparrow$ with $\frac{d\theta}{dt}$
- Shear thickening fluids
- ex:- Sugar in water solution
  - Rice starch



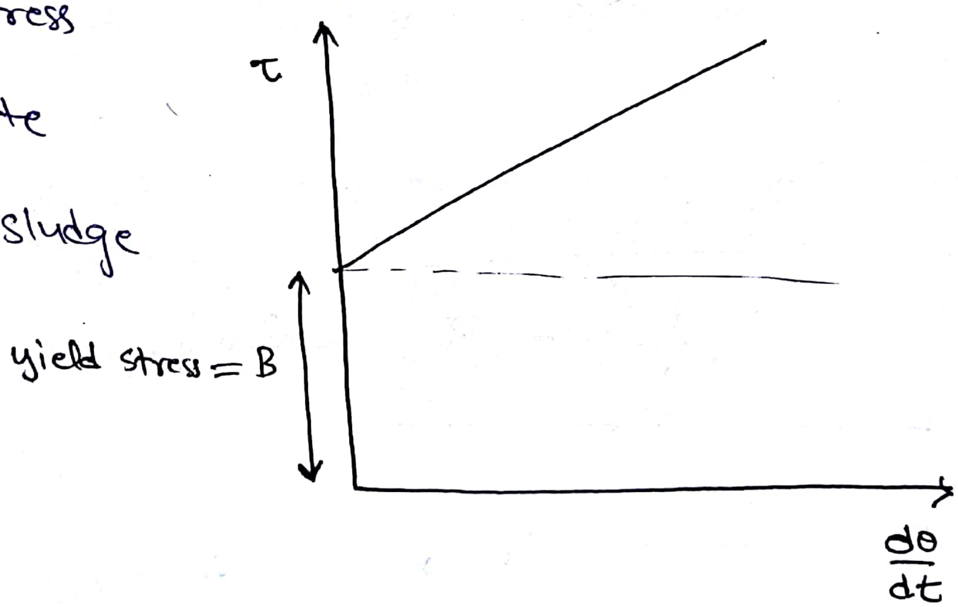
2. Pseudoplastic ($n < 1$, $B = 0$)

- $\mu \downarrow$ with $\frac{d\theta}{dt}$
- Shear thining fluids
- ex:- Paint
  - blood
  - lipstick



3. Bingham Plastic ($n = 1$, $B \neq 0$)

- B = yield stress
- ex:- tooth paste
  - Gel
  - Sewage sludge



B) μ varry with time

1. <u>Rheopectic</u> $\mu \uparrow$ with time

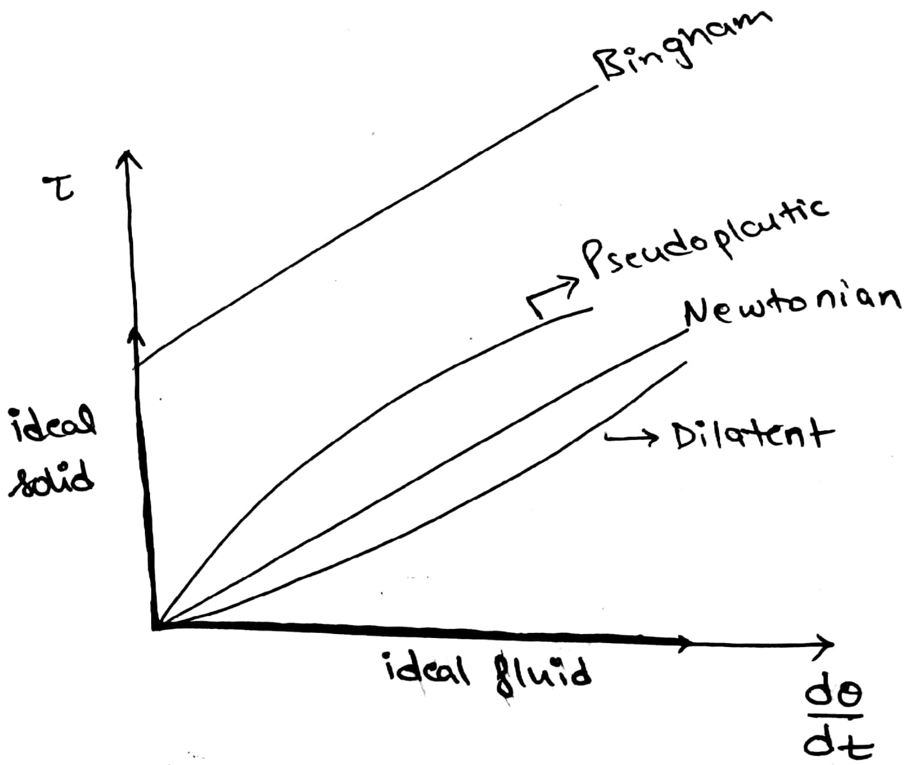
ex:- Gypsum in H_2O
- Printer ink

2. <u>Thixotropic</u> $\mu \downarrow$ with time

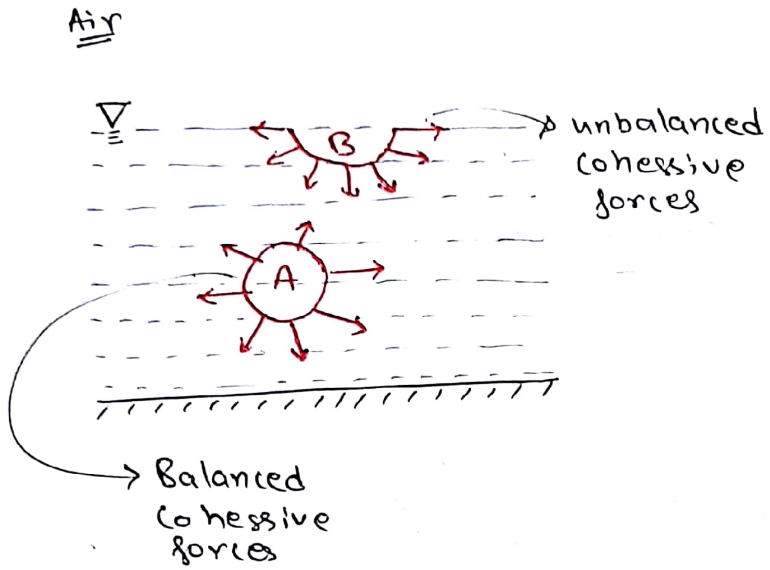
ex:- ice-cream

<u>Ideal fluid</u>

- Non-Viscous
- Incompressible



6. Surface tension (σ)



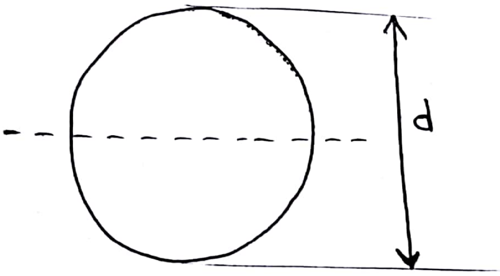
- Let us consider a molecule under ~~over~~ the surface of the fluid. This molecule "A" is under the action of balanced cohessive forces from all sides and Thus This molecule is under equilibrium.
- Let us consider a molecule "B" which is on the surface of the fluid & this molecule is under the action of net downward pull and hence this molecule is under streched condition.
- There are large number of molecules on the surface and all the molecules are under a pull and due to this there appears to be a membrane over the surface of the fluid which can bear small loads. And this property is known as surface tension.
- Surface tension is also defined as a line force with forces acting ⊥ar to this line in the plane of the surface
- Surface tension is also given as force acting per unit length and its SI unit is N/m

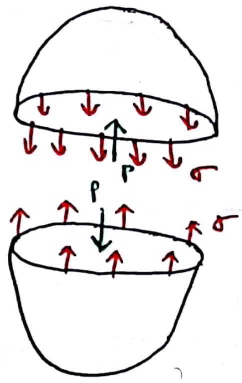
$$\boxed{\sigma = \frac{F}{L}} \quad N/m$$

F = Surface tension force
L = length over which surface tension is acting [Perimeter]

- with increase in tempr surface tension decreases. due to breaking of cohesive bonds
- At critical point as there is no interface, surface tension is zero.
- Detergents are used while washing cloths to reduce surface tension and breake the fluid film so that dirt particles can come out easily
- liquid droplet takes up the shape of a sphere due to surface tension.

⇒ Excess Pressure inside a liquid droplet

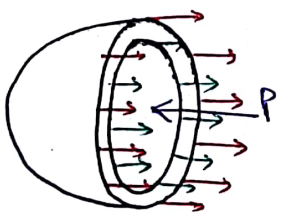




$$F_{pressure} = F_{surface\ tension}$$

$$P \times \frac{\pi}{4} d^2 = \sigma \times \pi d \Rightarrow \boxed{P_{gauge} = \frac{4\sigma}{d}}$$

⇒ Excess Pressure inside a bubble



$$F_{pressure} = F_{surface\ tension}$$

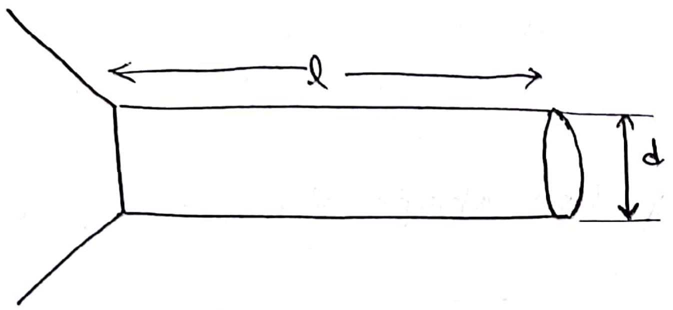
$$P \times \frac{\pi}{4} d_i^2 = \sigma \times (\pi d_i + \pi d_o)$$

$$d_i \simeq d_o \simeq d$$

$$P \times \frac{\pi}{4} d^2 = \sigma \times 2\pi d$$

$$\Rightarrow \boxed{P_{gauge} = \frac{8\sigma}{d}}$$

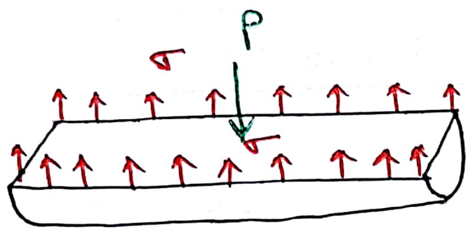
⇒ Excess pressure inside liquid jet



$F_{pressure} = F_{surface\ tension}$

$P \times D \times l = \sigma \times 2l$

$\Rightarrow \boxed{P_{gauge} = \frac{2\sigma}{d}}$

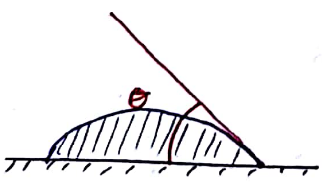


7. Capillarity

- Capillarity is defined as the rise or fall of liquid when a small diameter glass tube is inserted in it is known as known capillarity.
- The main reason of capillary rise is Adhesion.
  ex:- H_2O in glass tube
- The main reason of capillary fall is cohesion
  ex:- Hg in glass tube.

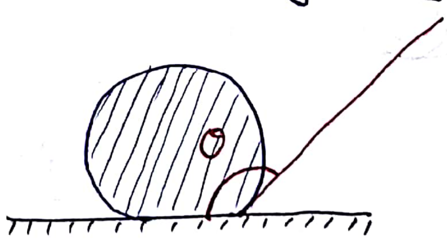
Angle of contact (θ)

wetting fluids



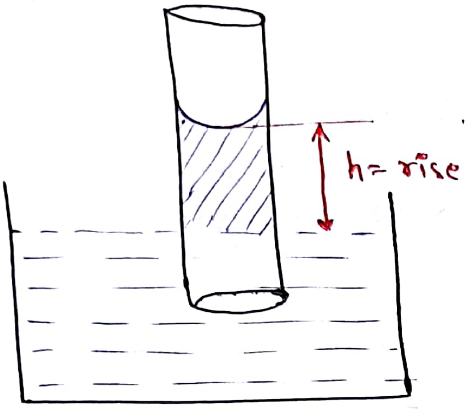
- Adhesion is more
- θ < 90°
- ex:- glass & water

non-wetting fluids



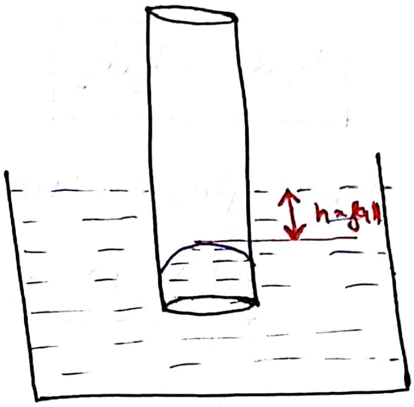
- Cohesion is more
- θ > 90°
- eg:- glass & mercury

Capillary rise



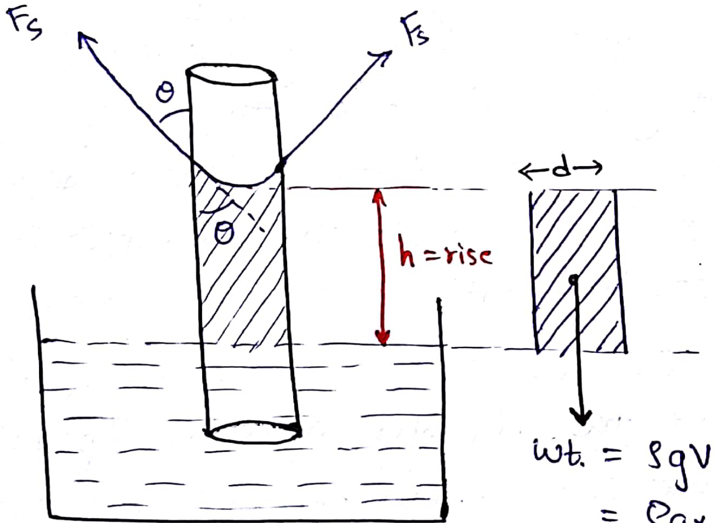
- Adhesion
- H_2O & glass tube

capillary fall



- cohesion
- Hg & glass tube

⇒ Expression for capillary rise in tube



$$wt. = \rho g V$$
$$= \rho g \times \frac{\pi}{4} d^2 h$$

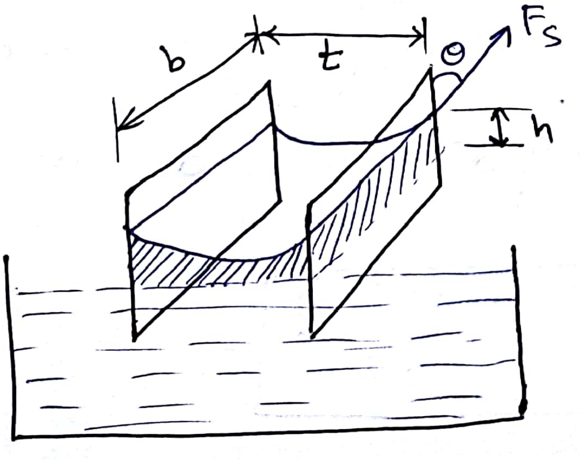
$$F_s \cos\theta = \text{wt. of fluid}$$

$$\sigma \times l \cos\theta = \rho g \times \frac{\pi}{4} \times d^2 h$$

$$\sigma \times \pi d \cos\theta = \rho g \frac{\pi}{4} d^2 h$$

$$\Rightarrow \boxed{h = \frac{4\sigma \cos\theta}{\rho g d}}$$

⇒ Expression for capillary rise b/w flat parallel plates



$$F_s \cos\theta = \text{wt. of fluid}$$

$$\sigma \times 2b \cos\theta = \rho g \times b t h$$

$$\Rightarrow \boxed{h = \frac{2\sigma \cos\theta}{\rho g t}}$$

Note: 1) $h \propto \frac{1}{d}$

so for same conditions if diameter is changed

$$\boxed{h_1 d_1 = h_2 d_2}$$

2) To neglect the capillary rise or fall

$$\boxed{d > 10 \text{ mm}}$$

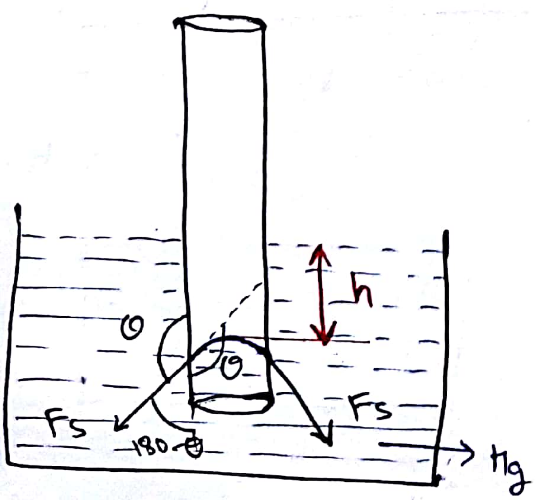
From the above we can conclude that capillary rise is inversely proportional to diameter of the tube and to neglect the effect of capillary,

ex:- in case of pressure measurement the diameter of glass tube must be taken greater than 10 mm

4) Liquid droplets takes up the shape of sphere bcz surface tension will try to minimise the surface area whereas pressure will resist this : Due to decrease in surface area the pressure inside the droplet increase and it keeps on increasing until both come in equilibrium

5) Pressure & surface tension force will come in equilibrium in such a manner that the droplet assume spherical shape which is having minimum surface area according to mathematics.

⇒ Expression for cappillary fall in glass tube

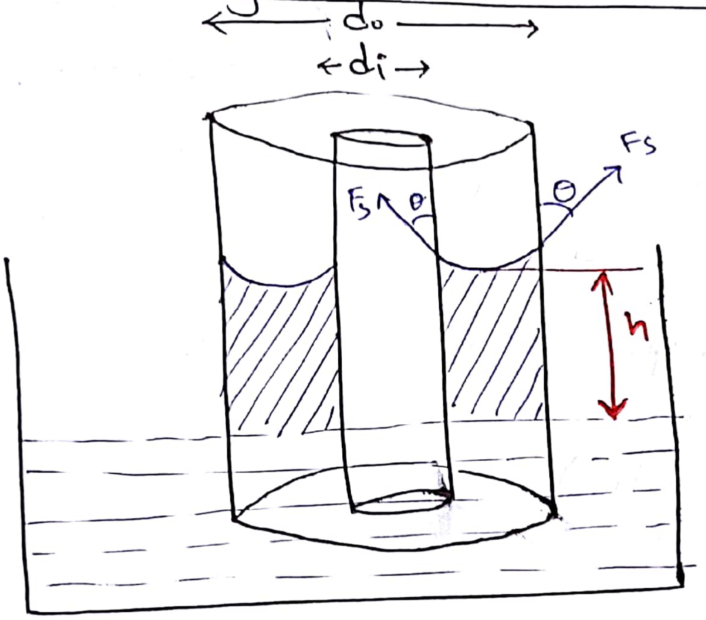


$$F_{pr.} = F_{surface\ tension} \times \cos(180-\theta)$$

$$\rho g h \times \frac{\pi}{4} d^2 = -\sigma \times \pi d \times \cos\theta$$

$$\boxed{h = \frac{-4\sigma \cos\theta}{\rho g d}}$$

⇒ Capillary rise in an annular space



wt. of fluid $= \rho g V$

$$= \rho g \times \frac{\pi}{4}(d_o^2 - d_i^2)h$$

$$F_s = \sigma \times (\pi d_o + \pi d_i) \times \cos\theta$$

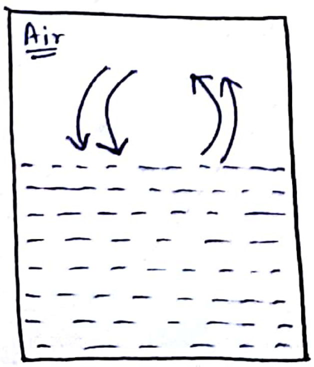
$$\cancel{\pi} \times \sigma \,\cancel{(d_o + d_i)} \cos\theta = \rho g \times \frac{\cancel{\pi}}{4} \cancel{(d_o + d_i)} (d_o - d_i) \times h$$

$$\Rightarrow \boxed{h = \frac{4\sigma \cos\theta}{\rho g (d_o - d_i)}}$$

Note :-

value of θ	combination
32°	glass & water
132°	glass & Hg
0°	pure water & glass

8. Vapour Pressure



- Let us consider a closed container that is partially filled with fluid.
- The molecules on the surface of the fluid are in highly excited ~~state~~ and by taking energy from the molecules beneath it these molecules evaporate.

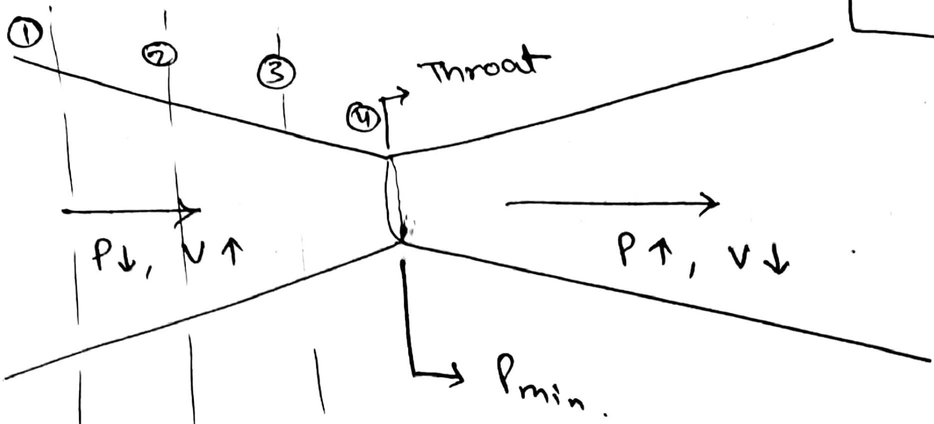
- The air above the fluid has some absorption capacity and this absorption capacity is known as saturation.
- Under saturation conditions the number of molecules leaving the surface of the fluid on evaporation becomes equal to number of molecules rejoining the surface on condensation
- The force exerted by the vapours under saturation condition over the surface of the fluid is known as saturated vapour pressure or vapour Pressure (P_{vs}).

Cavitation

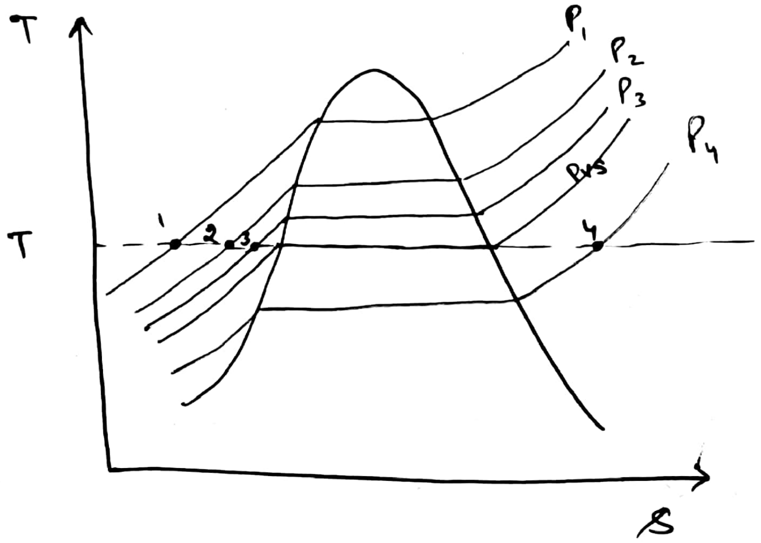
- if the pressure of the flowing fluid at any Point in the fluid flow becomes less than the saturated vapour pressure at that given tempr boiling will start and bubbles are formed. when these bubbles move to the high pressure region, the high pressure fluids collapses these bubbles and strikes with the wall and this leads to high order wear & tear and noise and this phenomenon is known as cavitation.

**
- Cavitation is a low pressure phenomenon.
- with increase in tempr vapour pressure increases
- Highly volatile fluids have higher vapour pressure at a given tempr.
- Hg is having Lowest vapour pressure because its cohesive bonds are strong and evaporation is less.
- Due to the above reason Hg is used as a manometric fluid in pressure measurements.

monday ⇒ 8 to 4
FM



- if $P_{min} > P_{vs}$ ⇒ flow is normal
- if $P_{min} < P_{vs}$ ⇒ Boiling ⇒ Bubbles ⇒ Cavitation



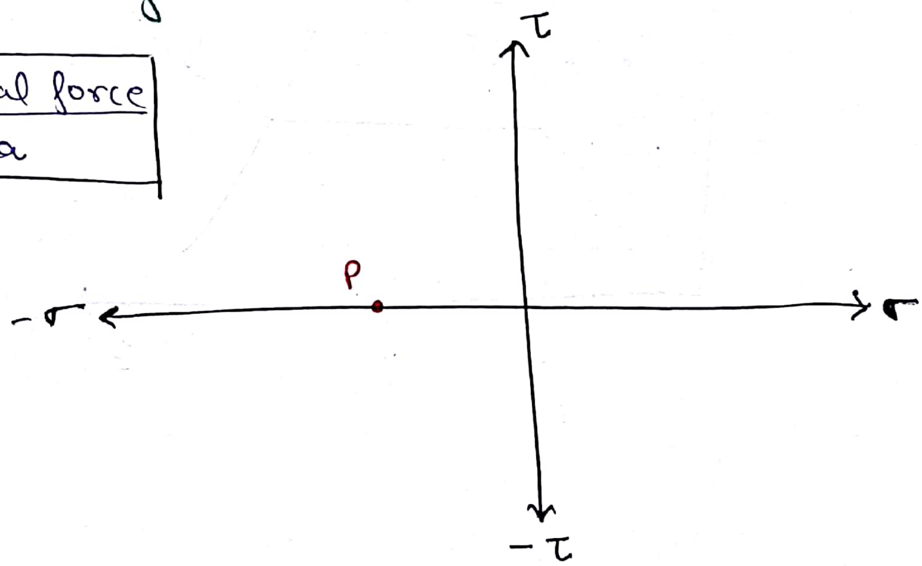
CHAPTER – 2

FLUID STATICS

A) PRESSURE MEASUREMENT

- Pressure is defined as normal force per unit area and its SI unit is N/m^2 or Pa.
- Pressure basically represents the number of molecules striking a surface, if there are no molecules than there will be no pressure and hence pressure is a representetive of molecules.
- Pressure is always compressive in nature and it is shown by a point on mohr's diagram.

$$\boxed{\text{Pressure} = \frac{\text{Normal force}}{\text{Area}}}$$



Note: Due to striking of molecules there is a change in momentum and due to that force is created.

Types of Pressure

1. Atmospheric Pressure (P_{atm})

Atmospheric Pressure is the pressure excerted by environ-mental air. Atmospheric Pressure is measure by a device known as barometer. It was invented by Toricelli.

2. Gauge Pressure (P_{gauge})

- Gauge Pressure are the pressures that are measured w.r.t. at-mospheric Pressure.
- While measuring gauge pressure atmospheric pressure is taken as zero.
- Gauge Pressure can be +ve or -ve.
- Vaccum pressures or Pressures less than atmospheric are known as -ve gauge pressures

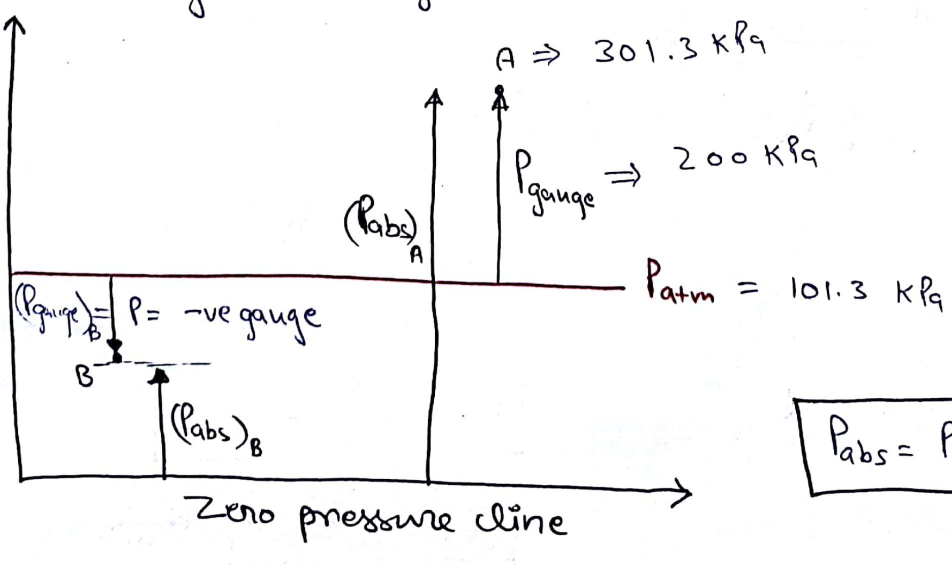
3. Absolute Pressure (P_{abs})

- These are the pressures that are measured w.r.t. zero pressure line.
- Absolute pressure are always positive.

4) Vaccum Pressures (P_{vaccum})

- All those pressures that are less than atmospheric pressures are known as vaccum pressures.
- Vaccum Pressures are also known as negative gauge pressures.

Note:
- For all numerical Problems we have to find gauge Pressure untill and unless absolute pressure is asked.
- All the negative gauged Pressures are taken and considered with a negative sign.

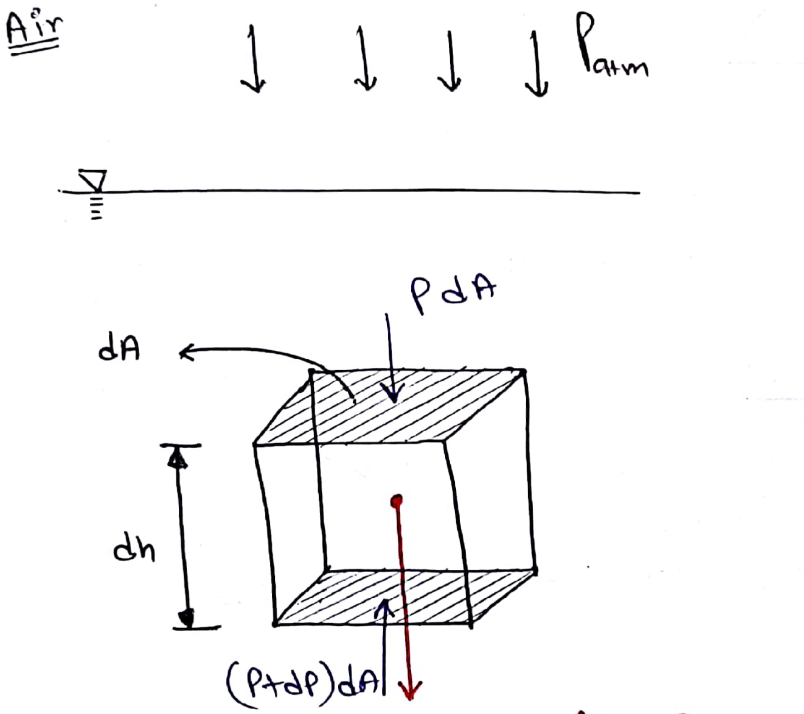


$$\boxed{P_{abs} = P_{atm} + P_{gauge}}$$

1 atm = 1.013 bar
= 101.3 KPa
= 760 mm Hg
= 10.3 m of water column

Hydrostatic Law

The variation of Pressure in a fluid in vertical dirn is directly proportional to specific weight



Wt. of fluid = $\rho.g.dA.h$

$$\Sigma F = \cancel{ma}$$

$$\Sigma F = 0$$

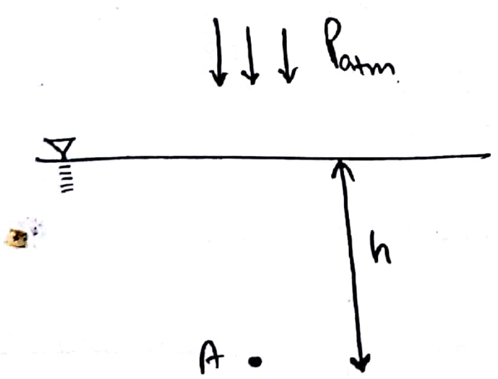
$$\cancel{PdA} + \rho g\, dA\, dh = \cancel{PdA} + dP\, dA$$

$$\rho g \cancel{dA}\, dh = dP \cancel{dA}$$

$$\frac{dP}{dh} = \rho g$$

$$\boxed{\frac{dP}{dh} = w}$$

Pressure at a depth "h"



$$\int dP = \int \rho g\, dh$$

$$P = \rho g h + C$$

at $h=0$; $P = P_{atm} \Rightarrow C = P_{atm}$

$$\boxed{P = \rho g h + P_{atm}}$$

$$\boxed{P_{gauge} = \rho g h} \quad N/m^2$$

Note:
- As we move vertically down in a fluid the pressure increase as $+\rho g h$. As we move vertically up in a fluid the pressure decreases as $-\rho g h$.
- There is no change of Pressure in horizontally same level.

Conversion of one fluid column to another fluid column.

$$\boxed{\rho_1 g h_1 = \rho_2 g h_2}$$

↳ valid for all fluids

$$\boxed{S_1 h_1 = S_2 h_2}$$

↳ valid for fluids of same nature
(i.e. liquid – liquid
or
gas – gas)

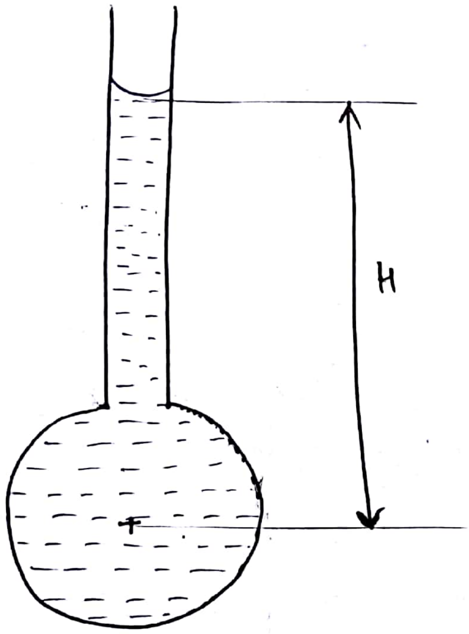
Pressure measurement

Pressure is measured by Principle of manometry and the devices used for measuring pressure are known as manometers and the branch of science dealing with Pressure measurement is known as manometry.

- Types of Pressure measuring devices
  - Simple
    - Piezometers
    - Simple U-tube manometers
    - Inclined tube manometers
  - Differential
    - U tube differential manometer
    - Inverted U-tube manometer
  - Mechanical gauges
    - Bourdon tube
    - Diaphragm
    - Bellow tube

1. Piezometers

- A Piezometer is a simple glass tube that is open at both the ends.
- One end of the Piezometer is connected to the pipeline whose Pressure needs to be measured and the other end is kept open to the atmospher.
- Piezometers cann't be used to measure very high pressure and gas Pressures.

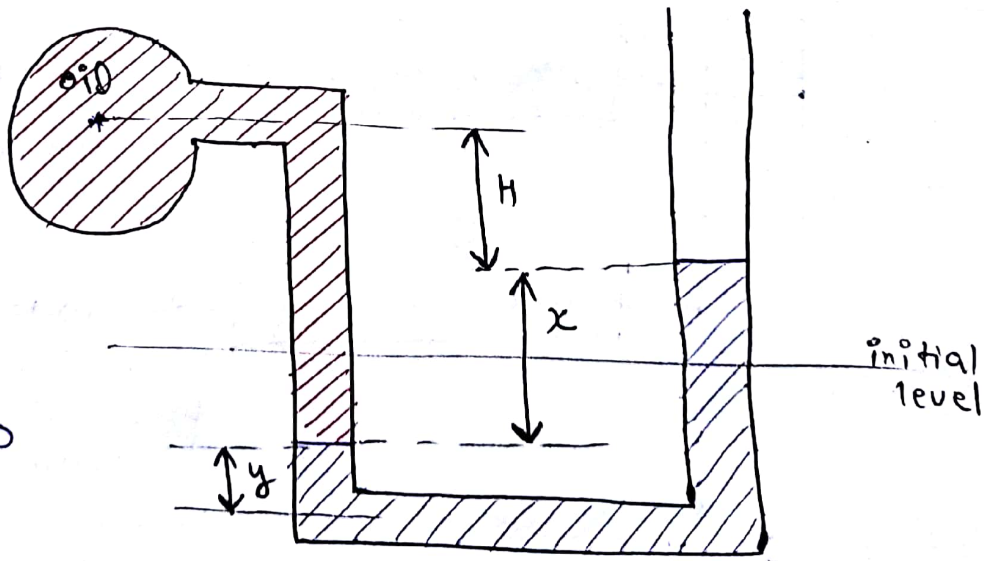


$$\boxed{P = \rho g H}$$

2. Simple U-tube manometer

Jumping Jack method

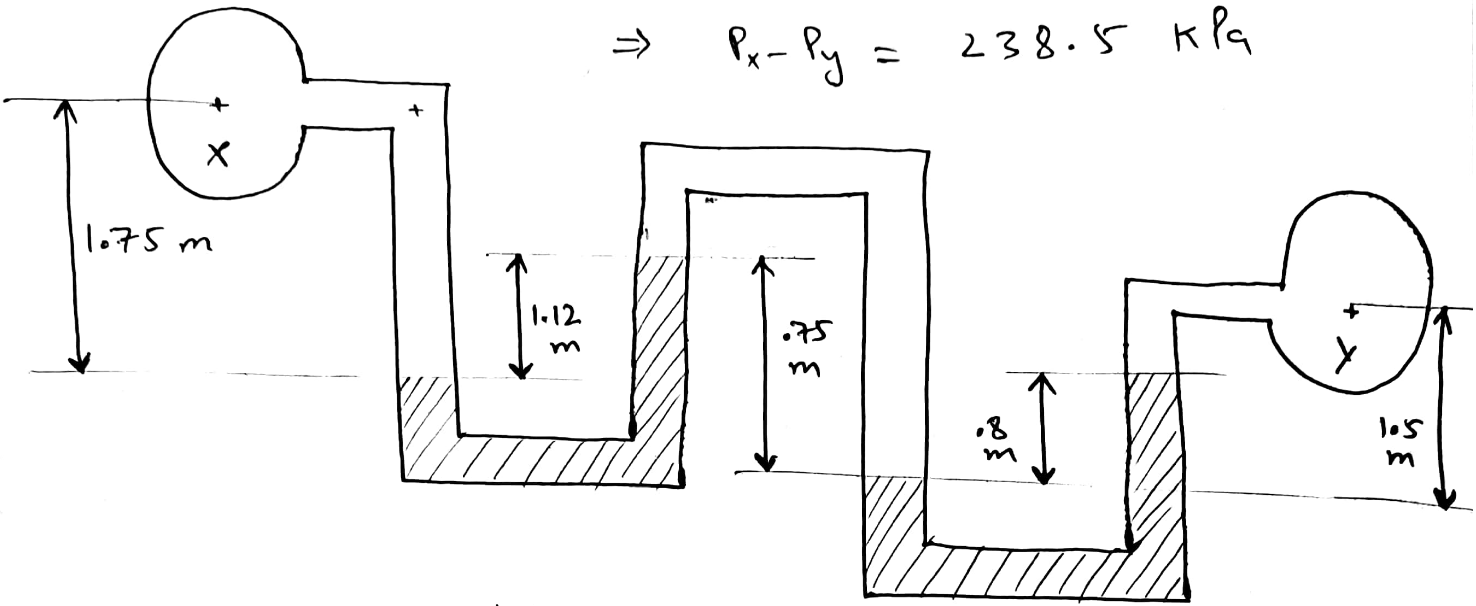
$$P_o + \rho_o g(H+x) + \cancel{\rho_m g y} - \cancel{\rho_m g y} - \rho_m g x = 0$$



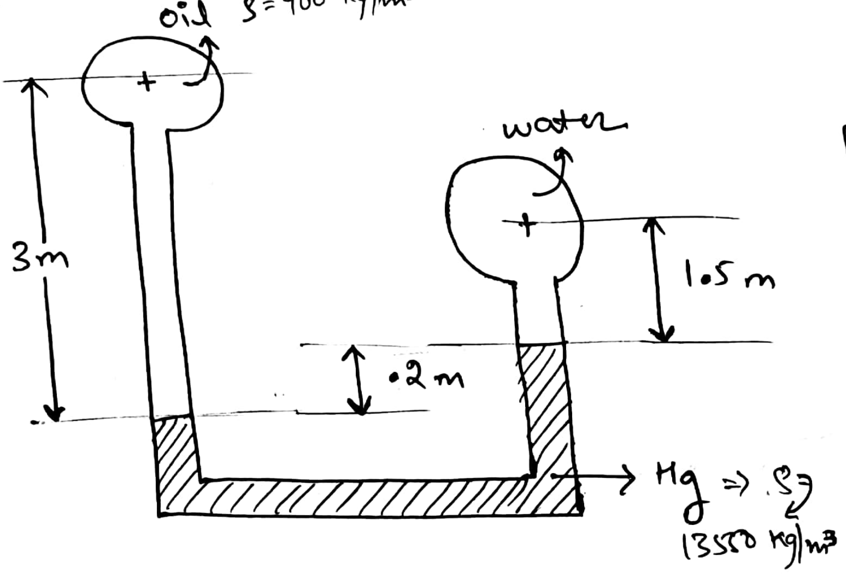
P-8
Q1

$$P_x + 10^3 \times 9.81 \times 1.75 - 13.6 \times 10^3 \times 9.81 \times 1.12 + 10^3 \times 9.81 \times .75 - 13.6 \times 10^3 \times 9.81 \times .8 - 10^3 \times 9.81 \times .7 = P_y$$

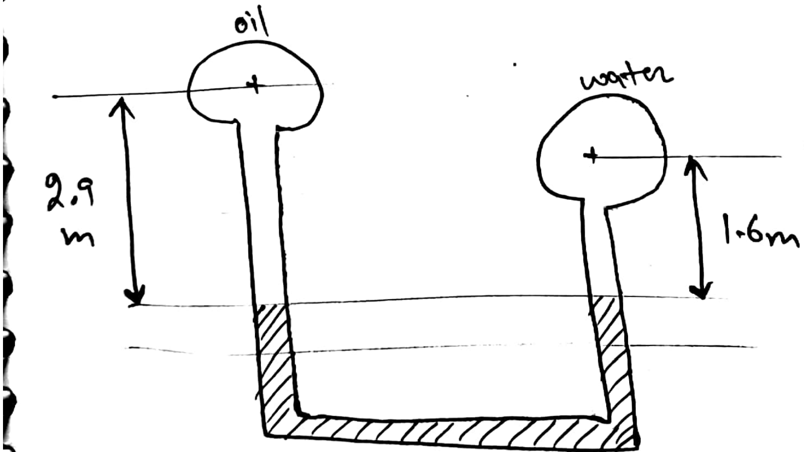
$$\Rightarrow \quad P_x - P_y = 238.5 \text{ kPa}$$



P-8
Q2



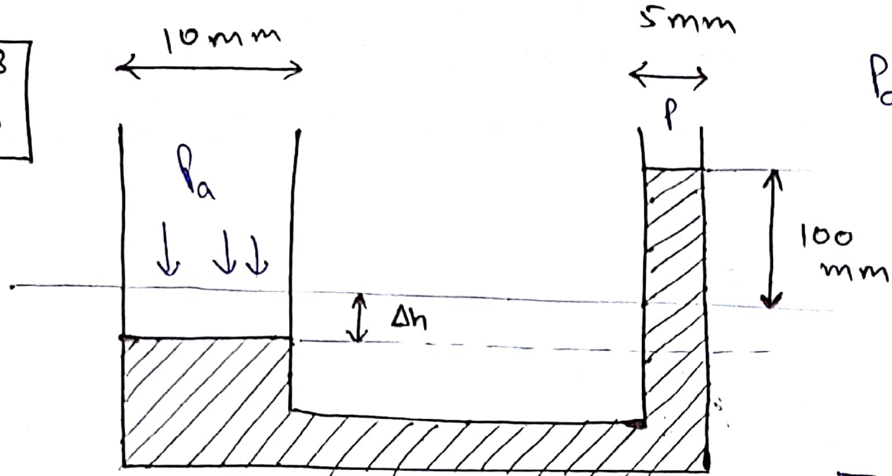
$$P_o + 900 \times 9.81 \times 3 - 13550 \times 9.81 \times .2 - 1000 \times 9.81 \times 1.5 = P_w$$



$$P_o + 900 \times 9.81 \times 2.9 - 1000 \times 9.81 \times 1.6 = P_w + \Delta P$$

$$\Delta P = 24.721 \text{ kPa}$$

P-8 Q3



$$P_a - 10^3 \times 9.81 \times \frac{(100+\Delta h)}{1000} = P$$

$$\Rightarrow P - P_a = \frac{-10^3 \times 9.81 (100+\Delta h)}{1000}$$

$$V_{rise} = V_{fall}$$

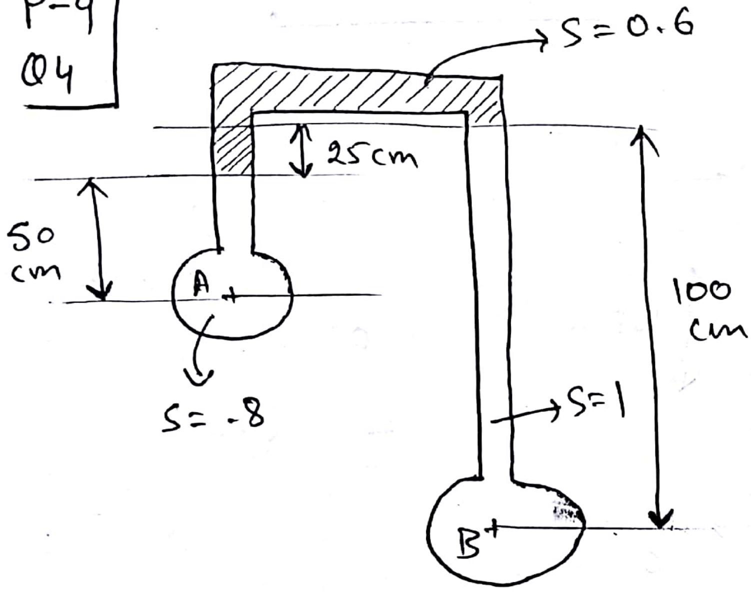
$$\frac{\pi}{4} \times (5)^2 \times 100 = \frac{\pi}{4}\, 100 \times \Delta h$$

$$\Rightarrow \Delta h = 25\,mm$$

$$P - P_a = -9.81 \times 125$$

$$= -1226.25\ Pa$$

P-9 Q4



$$P_A - 800 \times 9.81 \times 0.5$$
$$- 650 \times 9.81 \times .25$$
$$+ 10^3 \times 9.81 \times 1 = P_B$$

$$P_B - P_A = 4291.875\ Pa$$

$$4291.875 = 10^3 \times 9.81 \times h$$

$$\Rightarrow h = 43.75\ cm$$

P-9 Q5

$$h_{AW} + \overset{12}{\cancel{17 \times 1}} - 2 \times 13.6 - \cancel{5 \times 1} = \underline{\underline{1032.8}}\ cm$$

$$h_{AW} = \overset{1048.07}{\cancel{118.2}}\ cm\ water$$

$$h_1 S_1 = h_2 S_2$$

$$1 \times h_{AW} = 13.6 \times h_{AHg}$$

$$h_{AHg} = 770.64\ \underline{\underline{mm\,Hg}}$$

~~$h_{AHg} = 86.91$ cm~~
~~$= 86.91 \times 10$ mm~~
~~$= 869.1$ mm~~

P-9
Q1

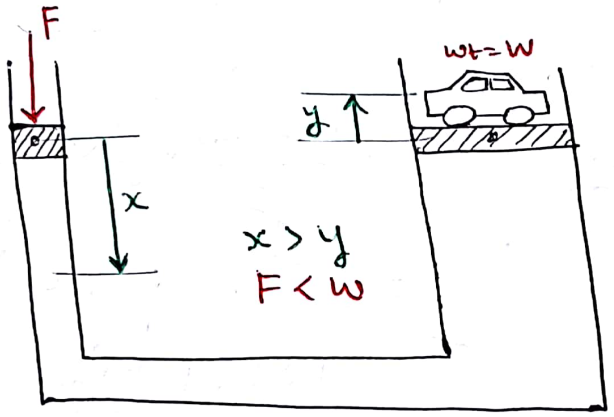
$$\cancel{\rho} \times 3h + 2\cancel{\rho} \times 1.5h + 3\cancel{\rho} h = 3\cancel{\rho} H$$

$$9h = 3H \Rightarrow \frac{H}{h} = 3$$

Pascal's Law

According to Pascal's Law in a static fluid, the pressure at a point is equally distributed in all the dirn. This is known as Pascals Law.

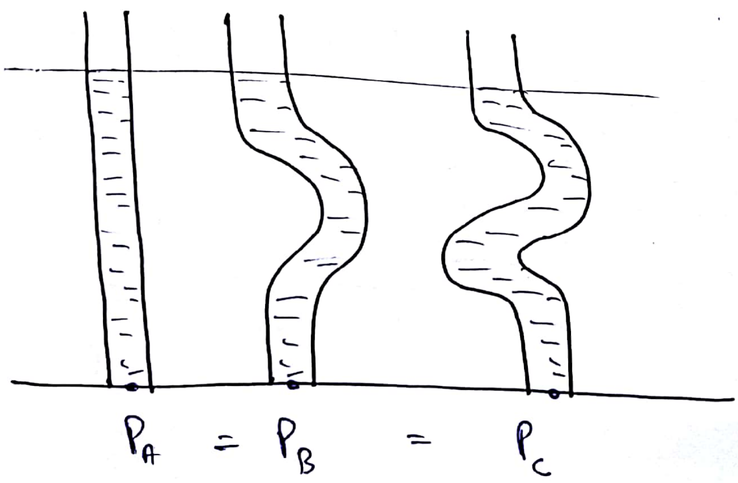
ex:- hydraulic lift
- hydraulic break



In a hydraulic lift a smaller force is required to lift a larger weight but still the conservation of energy is not voilated bcz the smaller force moves by a larger distance wherea larger wt. moves by smaller distance.

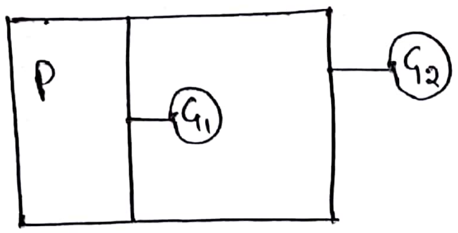
And hence ~~of conservation of~~ work done in both the cases are same and hence conservation of energy is followed.

Hydrostatic Paradox



Pressure only depends on vertical height of fluid and not on the shape of container or tube.

P-10
Q4



$P_{G_1} = 5$ bar
$P_{G_2} = 1$ bar

P-10
Q5

$P = 225 + 100 + 1.05 \times \cancel{9.81}^{10} \times 200$

$= 2425$ kPa

P-10
Q6

$\frac{mg}{V} = 12000$

$\rho g = 12000$

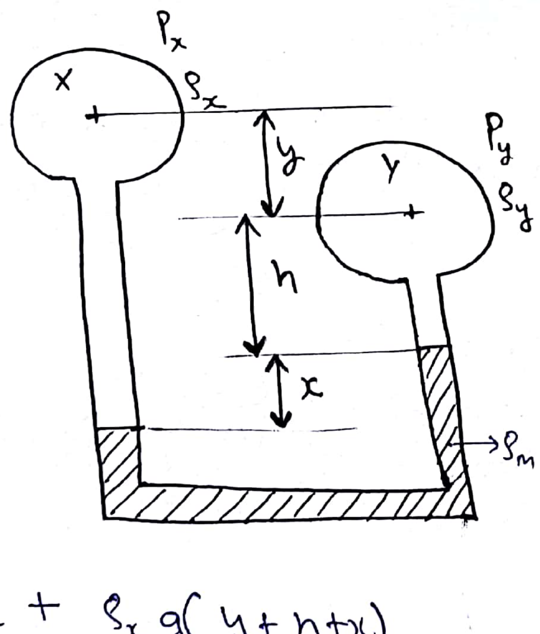
$\rho = \frac{12000}{g}$

$P = 10 \times \cancel{10^3} \times 9.81 + \frac{12000}{\cancel{g}} \times \cancel{g} \times 2$

$= 98.1 + 24$

$= 122.1$ kPa

3. Differential U-tube manometer

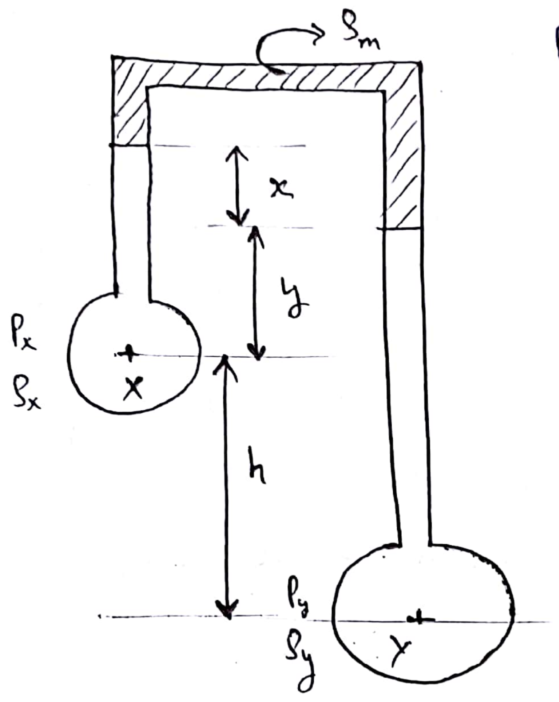


$P_x + \rho_x g(y + h + x)$

$- \rho_m g x$

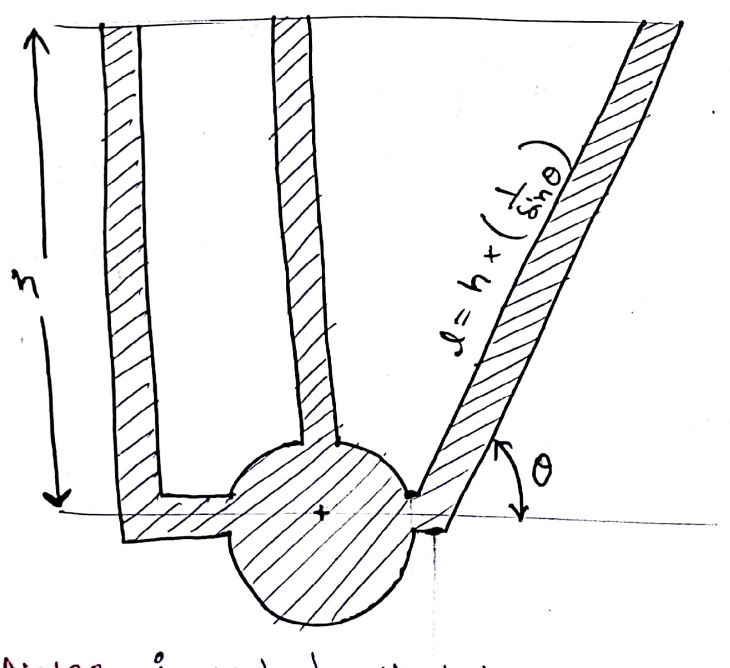
$- \rho_y g h = P_y$

4. Differential inverted U-tube manometer



$$P_x - \rho_x g(y+x) + \rho_m g x + \rho_y g(h+y) = P_y$$

5. Inclined tube manometer



$\frac{1}{\sin\theta}$ = Sensitivity of inclined manometer.

$\theta \uparrow \Rightarrow$ Sensitivity $\downarrow$

Note: • inverted U-tube manometers are used when the pipelines are underground & in these manometer the density of manometric fluid is less than the density of flowing fluid ($\rho_m < \rho$)

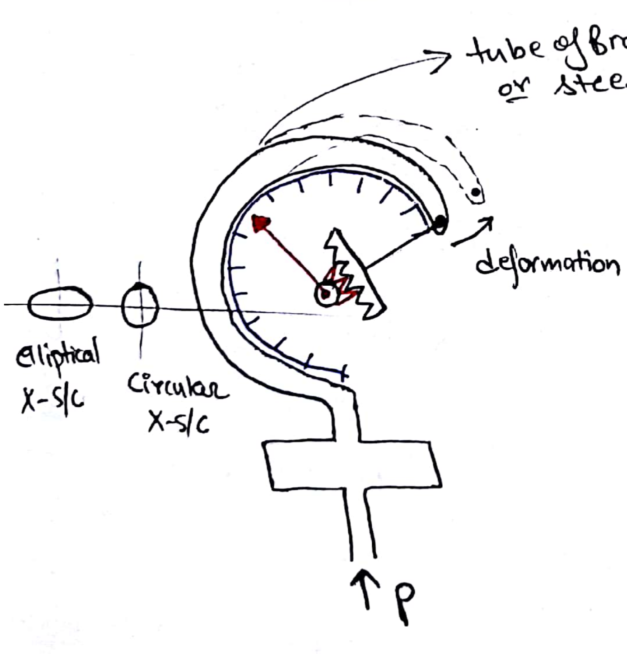
• multi U-tube manometers are used for measuring high pressures.

- The limb of the manometer is inclined to increase the sensitivity of a manometer.
- Sensitivity is defined as the smallest pressure diff. that a manometer can measure.
- By inclining the tube the sensitivity increases by a factor of $\frac{1}{\sin\theta}$
- $\theta \downarrow \Rightarrow$ Sensitivity $\uparrow$
- inclined tube manometers are used for measuring very small pressure differences in which diflection is very small.

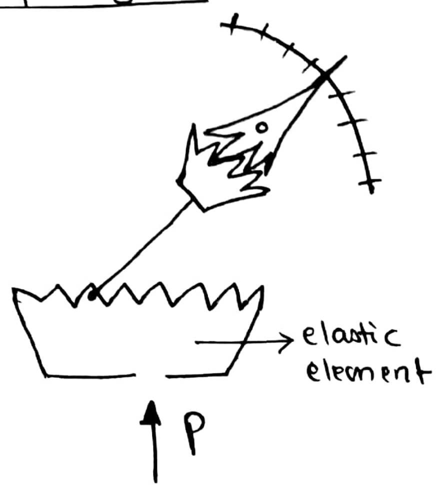
6. Mechanical Gauges

- mechanical Gauges are used to measure pressure that are very high with less accuracy
- All the mechanical gauges consist of an elastic element which gets deformed on application of pressure.
- This deformation is further magnefied by gear & pinion arrangement to a pointer scale. & pressure is measured

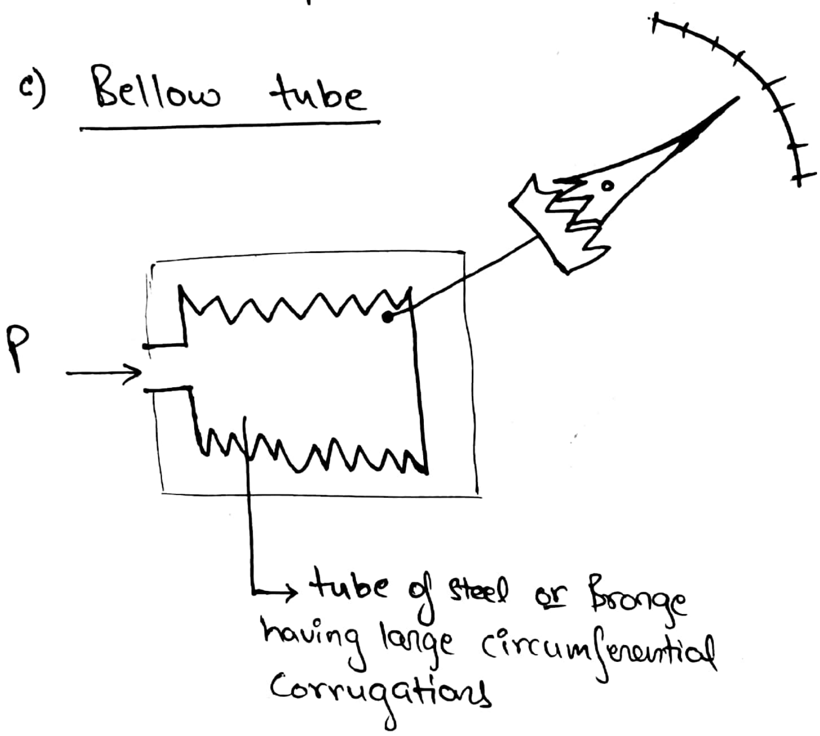
a) Bourdon tube



b) Diaphragm



c) Bellow tube

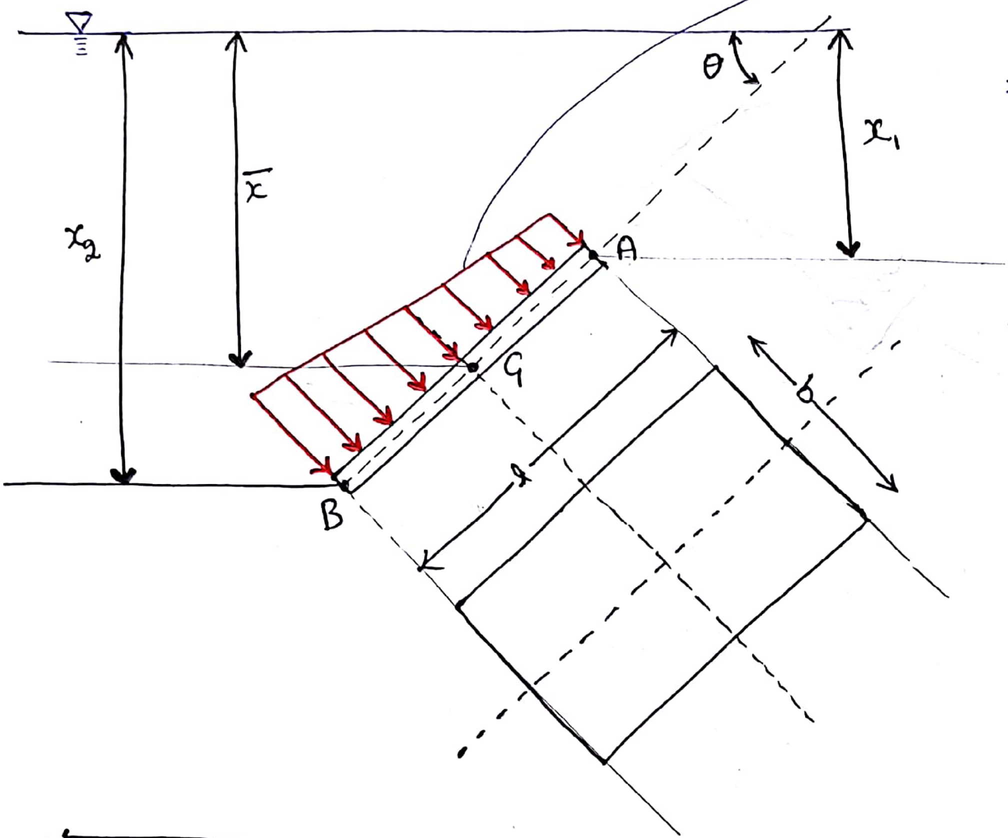


B) HYDROSTATIC FORCES

a) hydrostatic forces on plane surfaces

1. Plane inclined Surface (θ)

→ variation of Pressure on surface of plate



$\bar{x}$ = Vertical distance of Center of Gravity (cg) from free surface

$$\boxed{F = \omega A \bar{x}}$$

F = hydrostatic force

$\omega = \rho g$ = Specific ~~gravity~~ weight

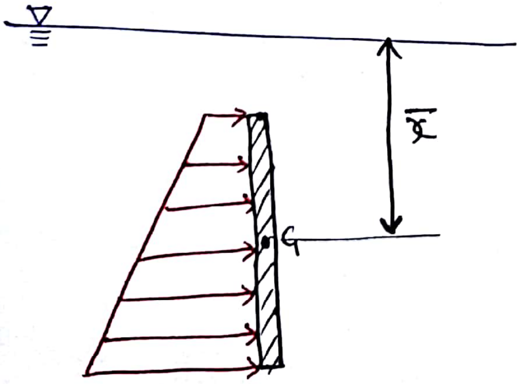
A = Surface Area in contact of fluid

$$\boxed{h = \frac{I_g}{A\bar{x}} \sin^2\theta + \bar{x}}$$

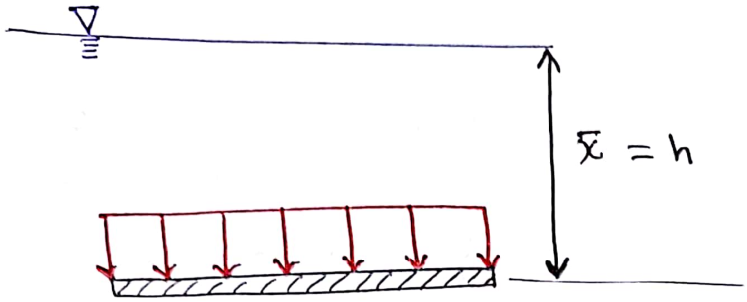
h = Vertical distance of center of Pressure from free surface

I_g = Moment of inertia @ centroidal Axis.

2) Plane vertical Surface (θ = 90°)



3) Plane horizontal Surface ($\theta = 0°$)

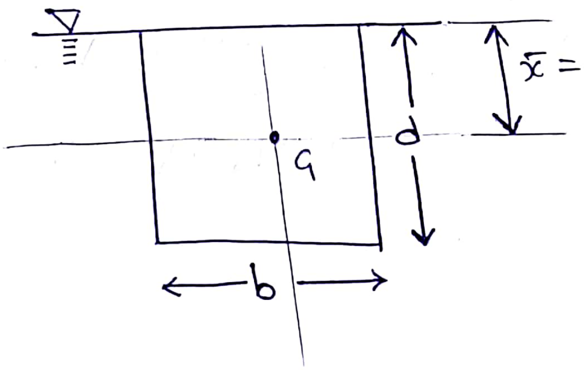


Note:

- The magnitude of hydrostatic force remains unchanged with angle of inclination.
- The center of Pressure which is the Point of application of hydrostatic force always lies either below center of gravity or coincides with center of gravity.
- Center of Pressure (C.P.) can never lie above center of gravity.

P-13
Q1

A)



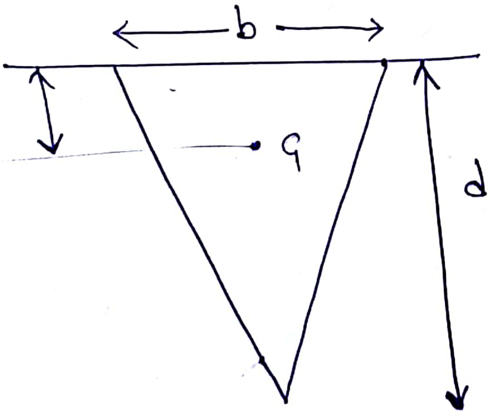
$$h = \bar{x} + \frac{I_g}{A\bar{x}} \sin^2\theta$$

$$h = \frac{d}{2} + \frac{bd^3}{12 \times bd \times d} \cdot 2$$

$$h = \frac{d}{2} + \frac{d}{6} = \frac{2d}{3}$$

B)

$\bar{x} = \frac{d}{3}$



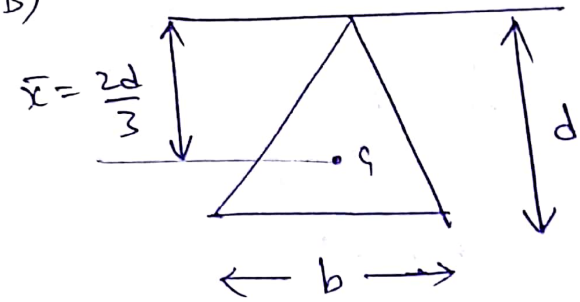
$$h = \bar{x} + \frac{I_g}{A\bar{x}} \sin^2\theta$$

$$h = \frac{d}{3} + \frac{bd^3}{36} \cdot \frac{2}{bd} \times \frac{3}{d}$$

$$h = \frac{d}{3} + \frac{d}{6}$$

$$h = \frac{d}{2}$$

B)



$$h = \bar{x} + \frac{I_G}{A\bar{x}} \sin^2\theta$$

$$= \frac{2d}{3} + \frac{bd^3}{36 \times bd \times \cancel{2}d} \cancel{2}\ \cancel{3}$$

$$= \frac{4 \times 2}{4} \frac{d}{3} + \frac{d}{12}$$

$$= \frac{9d}{12} = \frac{3d}{4}$$

P-13
Q3

F = ?

b = 3 m
d = 6 m

$$I = \frac{bd^3}{12} = \frac{3 \times \cancel{6} \times \cancel{6} \times 6}{\cancel{12}} = 54$$

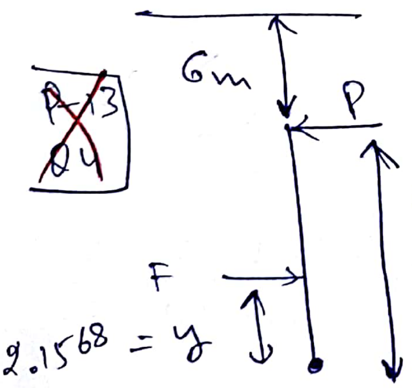
$$\bar{x} = \frac{5}{2} = 2.5 \text{ m}$$

$$F = w \times 3 \times 6 \times \bar{x} = \rho g \times 18 \times 2.5$$
$$= 441.45 \text{ kN}$$

$$h = \bar{x} + \frac{I_g}{A\bar{x}} \sin^2\theta$$

$$= 2.5 + \frac{54}{18 \times 2.5} \times \frac{1}{4} = 2.5 + .3 = 2.8 \text{ m}$$

~~P-13~~
~~Q4~~



$$\bar{x} = 6 + \frac{4 \times 4}{3\pi} = 7.697 \text{ m}$$

$$h = \bar{x} + \frac{.035\pi \times 16 \times 16}{\frac{\pi}{2} \times 16 \times 7.697} \rightarrow .1455$$

$$= 7.843$$

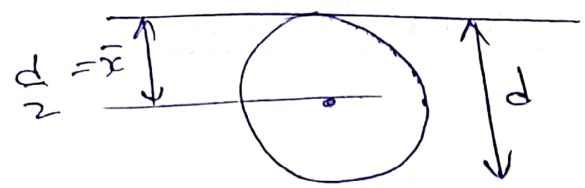
$$Fy = P \times 4$$

$$F = wA\bar{x}$$

$$P = 104.3088\ w \text{ N}$$

P-13 Q1

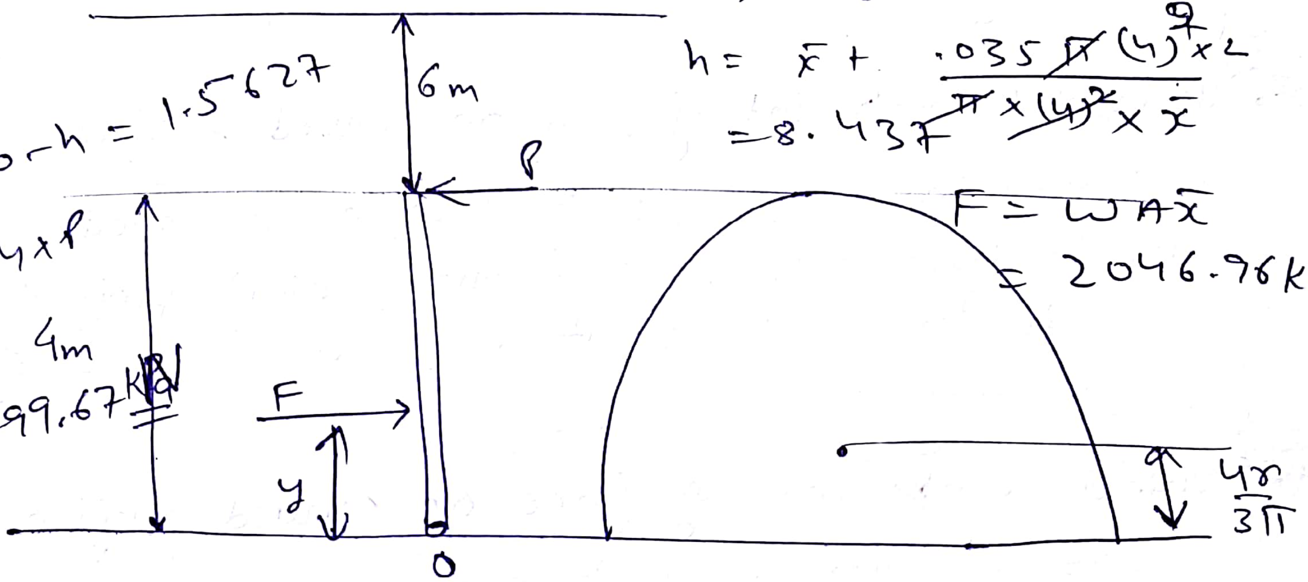
D)



$$h = \bar{x} + \frac{I_g}{A\bar{x}} \sin^2\theta = \frac{d}{2} + \frac{\frac{\pi d^4}{64} \times 4 \times 2}{\pi d^2 \times d}$$

$$= \frac{4 \times d}{4 \times 2} + \frac{d}{8} = \frac{5d}{8}$$

P-13 Q4



$\bar{x} = 8.3023$

$h = \bar{x} + \frac{.035\pi (4)^4 \times 2}{\pi \times (4)^2 \times \bar{x}}$

$= 8.437$

$F = wA\bar{x}$

$= 2046.96\,kN$

$y = 10 - h = 1.5627$

$F_y = 4 \times P$

$\Rightarrow P = 799.67\,kN$

$P \simeq 800\,kN$

P-13 Q5

$F = mg = 500 \times 9.81$ downward

$F = wA\bar{x} = 10^3 \times 9.81 \times 2 \times 2.5 \times 1 \times \mu = 500 \times 9.81$

$\mu = 0.1$

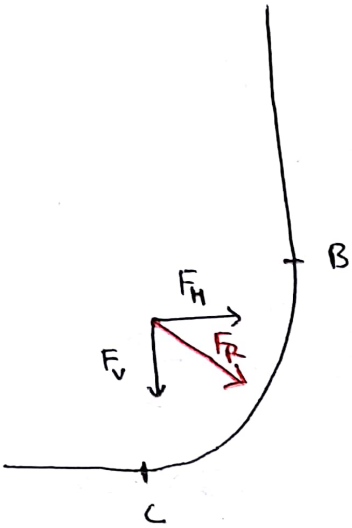
P-14 Q5

$$w(h \times y) \times \frac{h}{2} \times \left(\frac{h}{3}\right) = w \times h \times (b\,y) \times \frac{b}{2}$$

$h^2 = 3b^2$

$\Rightarrow h = \sqrt{3}\,b$

b) Hydrostatic forces on curved surfaces



BC = curved surface

F_V = Vertical component of F_R

F_H = Horizontal component of F_R

F_R = Resultant force on curved surface.

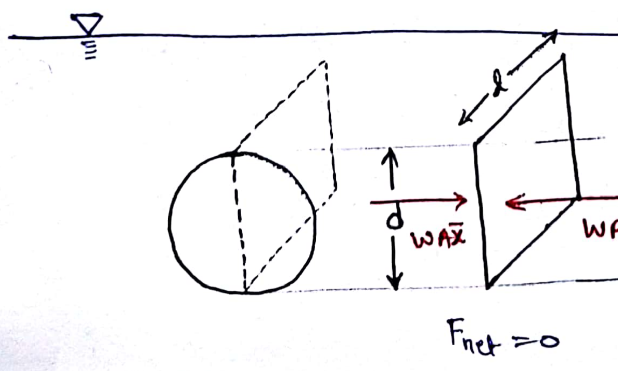
☆☆☆ • Horizontal component of force on curved surface

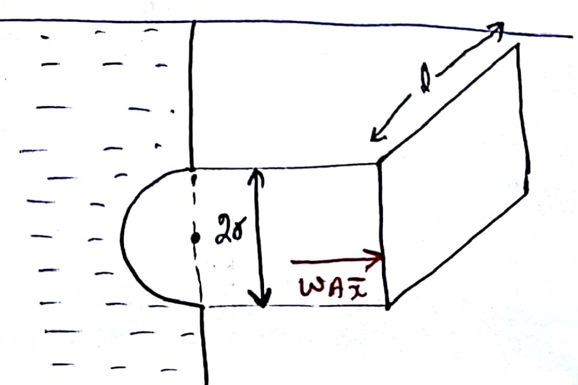
The horizontal component of force on a curved surface is equal to hydrostatic force on the vertical projection area and this force will act at center of Pressure of the corresponding area

☆☆☆ • Vertical component of force on curved surface

The vertical component of force on a curved surface is equal to weight of the fluid contained by the curved surface till the free surface and this force will act at the center of gravity of the corresponding weight.

Horizontal

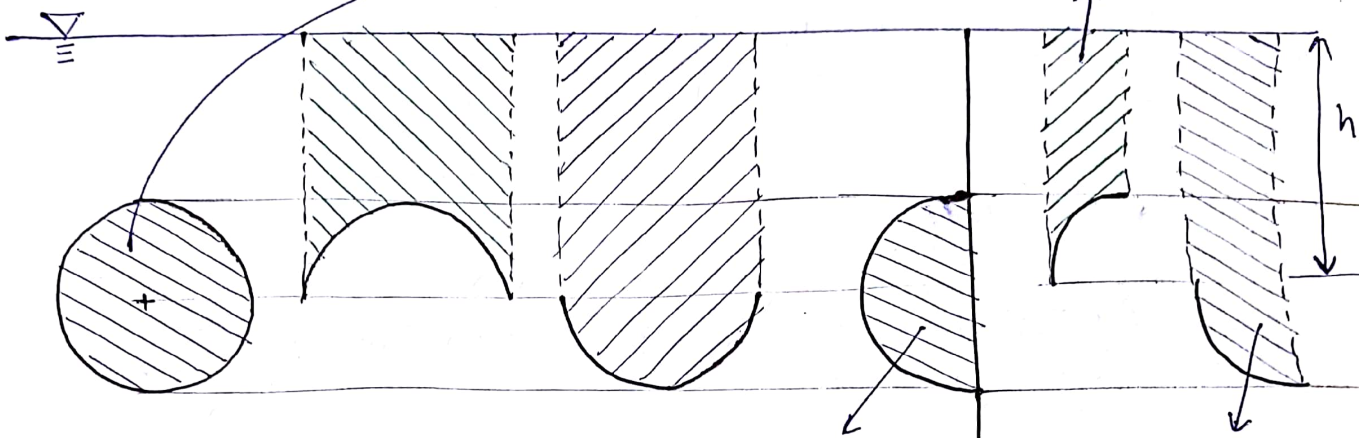




Vertical

$$F_{net} = \rho g \left(\frac{\pi}{4} \times d^2 l\right)$$

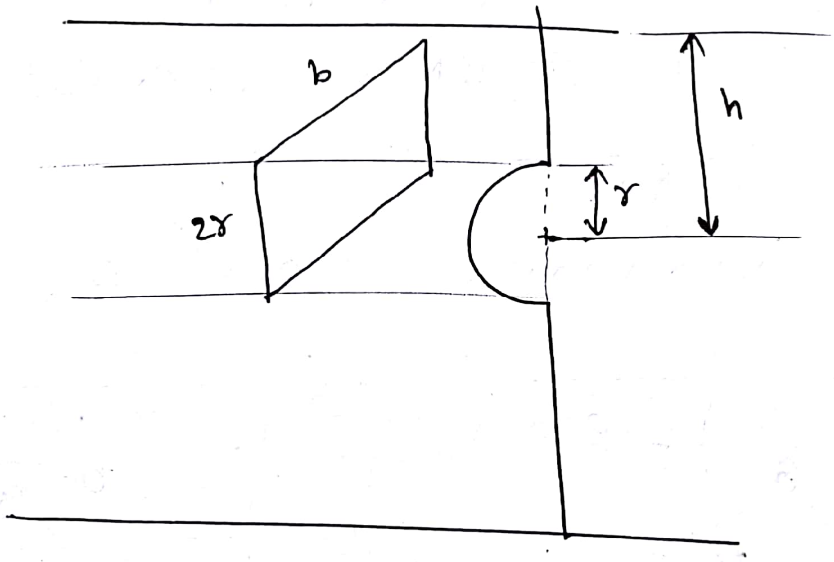
$$\rho g\left[hrb - \frac{\pi r^2 b}{4}\right]$$



$$F_{net} = \rho g\left[\frac{\pi r^2 b}{2}\right]$$

$$F_v = \rho g\left[hrb + \frac{\pi r^2 b}{4}\right]$$

P-14
Q6

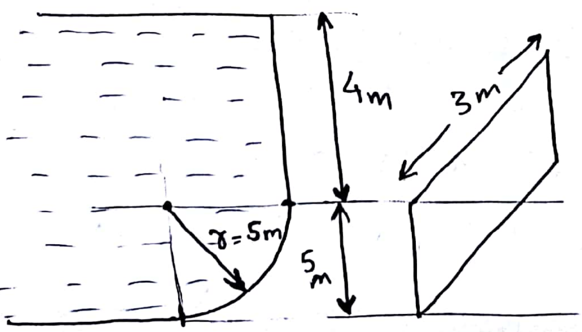


$$F_H = wA\bar{x}$$
$$= w \times 2r \times b \times h$$
$$= 2wrbh$$

$$F_v = \rho g \times \left[\frac{\pi r^2 b}{2}\right]$$

Note: The horizontal component on a cylinder completely submerged in fluid is zero whereas there will be a net vertical force on the cylinder.

P-14
Q7



$$F_H = wA\bar{x} = w \times 5 \times 3 \times 6.5$$
$$= 956.475 \text{ KN}$$

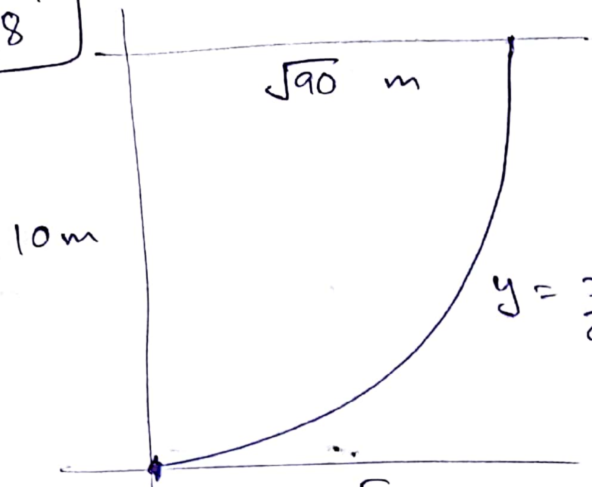
$$F_v = \rho g \times \left[4 \times 5 \times 3 + \frac{\pi}{4}(5)^2 \times 3\right]$$
$$= 1166.458 \text{ KN}$$
$$= 1.166 \text{ MN}$$

P-14
Q8

$b = 1\ m$

$x^2 = 90$

~~$x = 3a$~~



$x^2 = 9y$

$\boxed{x^2 = 90}$

$$A_r = \int_0^{\sqrt{90}} \left(10 - \frac{x^2}{9}\right) dx = 10 \times \sqrt{90} - \frac{(\sqrt{90})^3}{9 \times 3}$$

$$\boxed{\sqrt{90}\,x - \frac{x^3}{9 \times 3}}$$

$= \cancel{58.377}\ m^2$

$63.245 \quad 63.245$

$F_v = \cancel{58.377} \times 1 \times 9810$

$= \cancel{572.68\ KN} = 620.438\ KW$

$F_H = w A \bar{x}$

$= 9810 \times 10 \times 5 = 490.5\ KN$

P15
Q9

$$\cancel{700 g\left(\frac{\pi D^2}{8}\right) + 800 g\left(\frac{\pi D^2}{8}\right) = \rho_c g\left(\frac{\pi D^2}{8}\right) 2}$$

$\cancel{1500 = 2\rho_c}$ $\quad \cancel{\rho_c = 750\ kg/m^3}$

$$\rho_c g \left(\frac{\pi D^2}{4}\right) = 700 g \times \left(\frac{\pi D^2}{4}\right) \times \frac{1}{4} + \frac{800 g}{2} \times \left(\frac{\pi D^2}{4}\right)$$

$\rho_c = \frac{700}{4} + \frac{800}{2}$

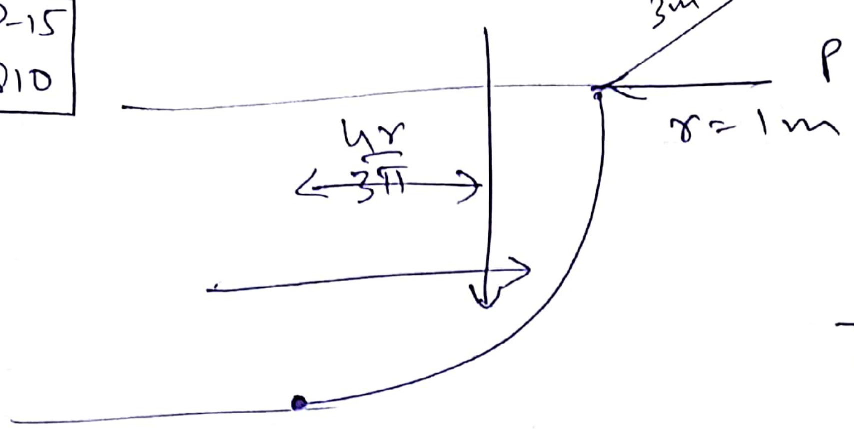
$= 7 \times 25 + 400$

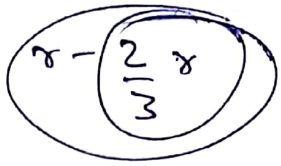
$= 175 + 400$

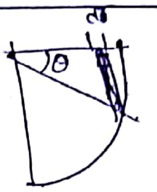
$= 575\ kg/m^3$

Ans
798.3 kg/m3

P-15
Q10







$dA = y\,dx$

$r\sin\theta$

$$P \times 1 = F_H \times \frac{1}{3} + F_V \times \frac{4r}{3\pi}$$

$F_V = \left(\frac{\pi}{4} r^2 \times 3\right)\rho g$

$\rho g \times \frac{\pi}{4} r^2 \times 3$

$wA\bar{x}$

$9810 \times 3 \times \frac{1}{2} \times \frac{1}{3}$

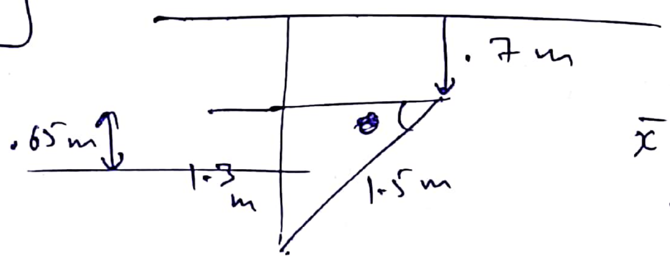
$\frac{\cancel{3\pi} \times 9810 \times \cancel{4}}{\cancel{4}} \cancel{3\pi}$

$\Rightarrow$ ~~P = 44.145 KN~~

$P = 9810 \times (.5 + 1)$

$= 14.715$ KN

P 15
Q1



$\bar{x} = .7 + .65$

$= 1.35$

$F = wA\bar{x} = \rho g \times \pi r^2 \times \bar{x}$

$= 23.403$ KN

$F\uparrow = .5 \times 1 \times \cancel{2} \times 700 \times g + \frac{\pi}{4} \times (1)^2 \times \frac{1}{2} \times \cancel{2} \times \cancel{800}\,200 \times g$

$= 13030.804$ N

$F\downarrow = (.5 \times .5 \times 2 - \frac{1}{4} \times (.5)^2 \times \pi \times 2) \times 700 \times g$

$= 736.83$ N

$F_{net} = 12293.969 \text{ N} = \frac{\pi}{4} \times (1)^2 \times \cancel{2}\,2 \times \rho \times g$

$\Rightarrow \rho = 797.817 \text{ kg/m}^3$

c) Buoyancy & Floatation

- whenever a body is either Partially or fully submerged in a fluid it experiences a net vertical force and this force is known as bouyancy force.

- According to Archemedies buoyancy force is equal to weight of the fluid displaced and this is known as Archemedies Principle.

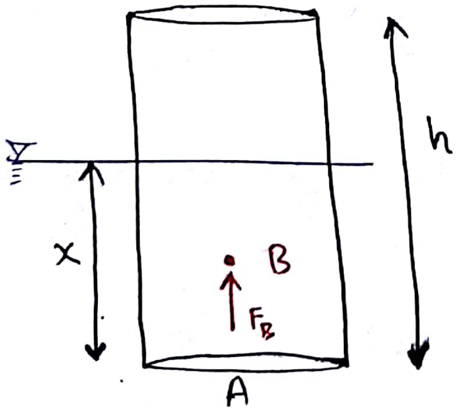
Center of buoyancy (B)

Center of buoyancy is defined as the Point of application of buoyancy force and this force will act at the centroid of volume of fluid displaced.

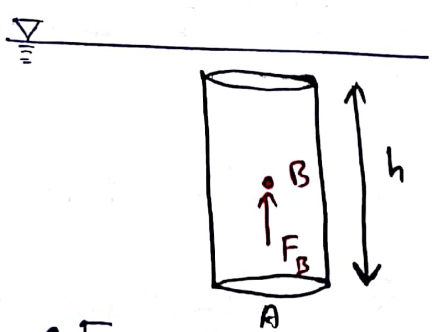
$$F_B = \text{wt. of fluid displaced}$$

$$F_B = \rho_f \cdot g \cdot V_{\text{fluid displaced}}$$

$$\boxed{F_B = \rho_f \cdot g \cdot V_{\text{body Submerged}}}$$



$\uparrow F_B = \rho_f \cdot g \cdot A \cdot x$

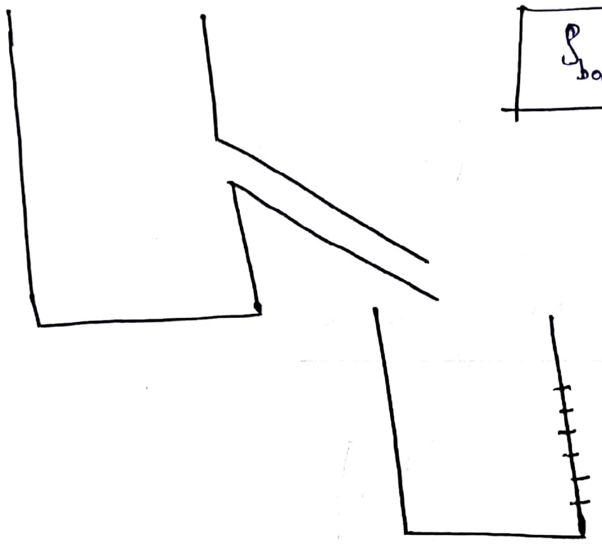


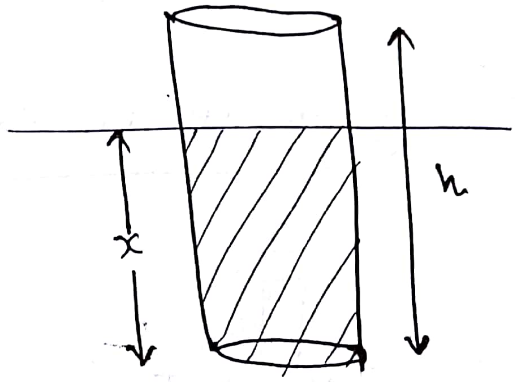
$\uparrow F_B = \rho_f \cdot g \cdot A \cdot h$

Principal of floatation

weight of Body = F_B (↑)

$$\boxed{\rho_{body} \cdot g \cdot A \cdot h = \rho_f \cdot g \cdot A \cdot x}$$





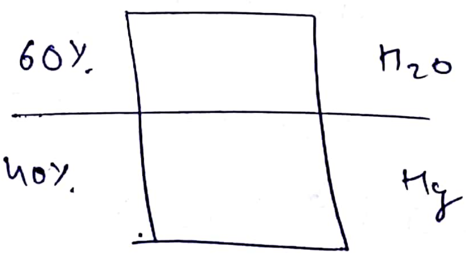
P-17 Q1

$w_b = 8976 \ N/m^3$ $\qquad$ $w_f = 10104 \ N/m^3$

$$\frac{V_{sub}}{V_{total}} = \frac{8976}{10104}$$

$$\% \ V_{visible} = \left(1 - \frac{8976}{10104}\right) \times 100 = 11.16 \%$$

P-17 Q2

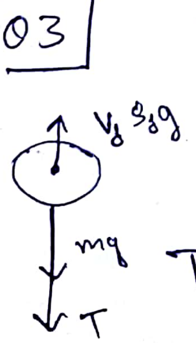


$$\rho = .6 \times 1000 + .4 \times 13600$$

$$= 6040 \ kg/m^3$$

P-17 Q3

$$T + \cancel{1} \times 100\cancel{0} \times g = \cancel{1} \times 200\cancel{0} g$$



$$\Rightarrow T = 100 g = \underline{981} \ N$$

$$T + 100 \times g \times 1 = \frac{1000 \times g \times 1 \times x}{100}$$

$$T + 100 g = 10 x g$$

$\Rightarrow x = .20$

$\Rightarrow 20\%$

P-17
Q4

$$V_b \rho_b \times g = 100\,N$$

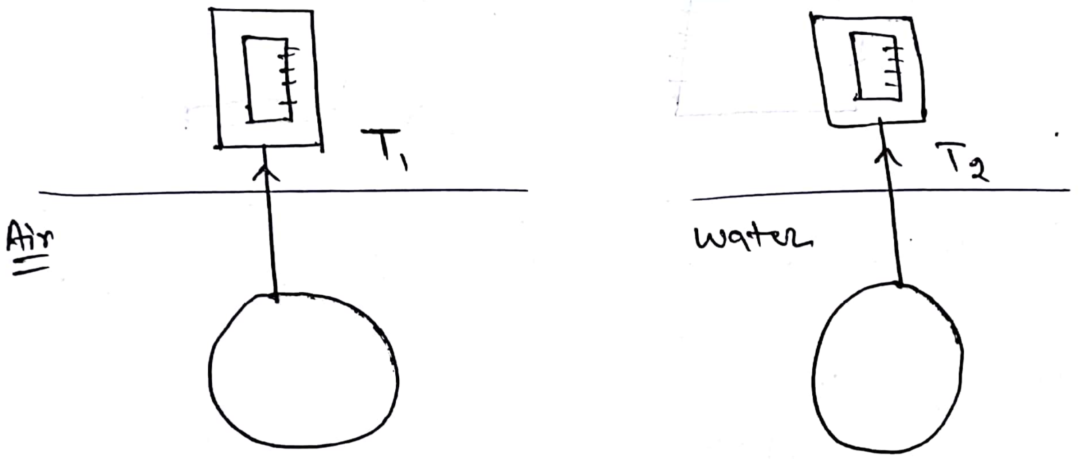
$$F_B = 20 = V_b \times 1000 \times 9.81$$

$$\Rightarrow V_b = \frac{20}{1000 \times 9.81}$$

$$\rho_b = \frac{100}{V_b \times g}$$

$$= \frac{\cancel{100}^{5} \times 1000 \times \cancel{9.81}}{\cancel{20} \times \cancel{9.81}} = 5000\ kg/m^3$$

Loss of weight due to buoyancy force



$$T_1 = W - F_{B_1}$$

F_{B_1} ⇓ negligible

$$T_2 = W - F_{B_2}$$

loss of weight $= T_1 - T_2 = W - W + F_{B_2} = F_{B_2}$

P-17
Q5

$$T_1 = \cancel{\rho g (V_b)}\ \cancel{V_b} \times \cancel{g}(\rho_b - 800) = 30$$

$$T_2 = \cancel{V_b} \times \cancel{g}(\rho_b - 1200) = 15$$

$$V_b = \frac{30}{g \times 800}$$

$$= 3.82 \times 10^{-3}\ m^3$$

$$\frac{x - 800}{x - 1200} = 2$$

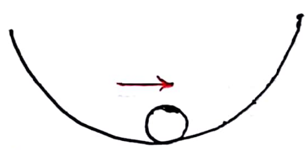
$$x - 800 = 2x - 2400$$

$$\boxed{1600 = x}$$

Note:
- True weight of the body is measured in Air.
- while measuring weight the body should be completely submerged.
- The loss of weight in a fluid is equal to buoyancy force.

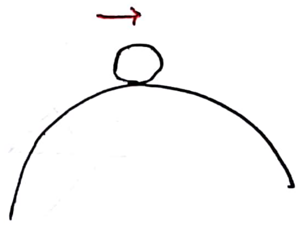
Types of Equilibrium

1. Stable Equi.



Body will stop at its original Position

2. unstable Equi.



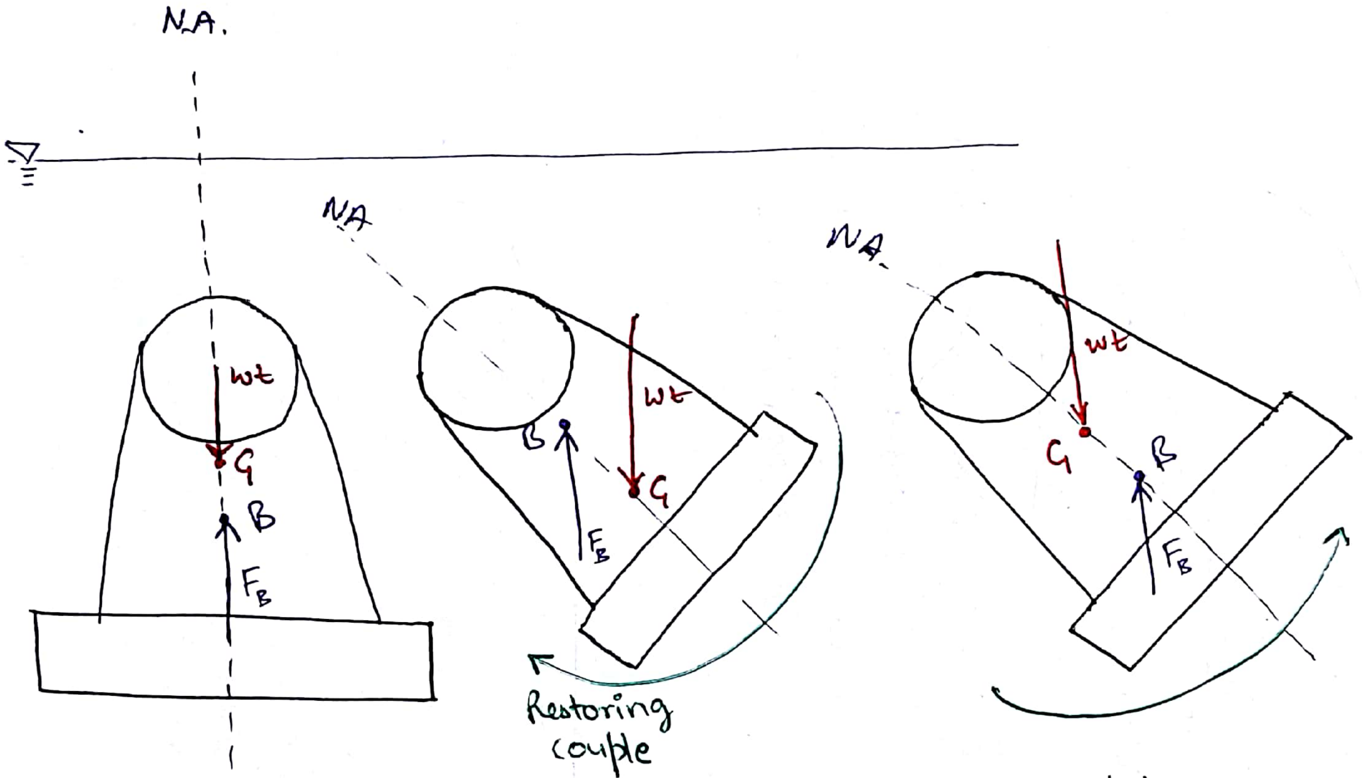
Body will neither stop nor return to its original Position

3. Neutral Equi

Body will stop but at some other position than original.

Note:
- A floating body under vertical displacement is an example of stable equilibrium.
- A floating body under horizontal displacement is an example of neutral equilibrium.

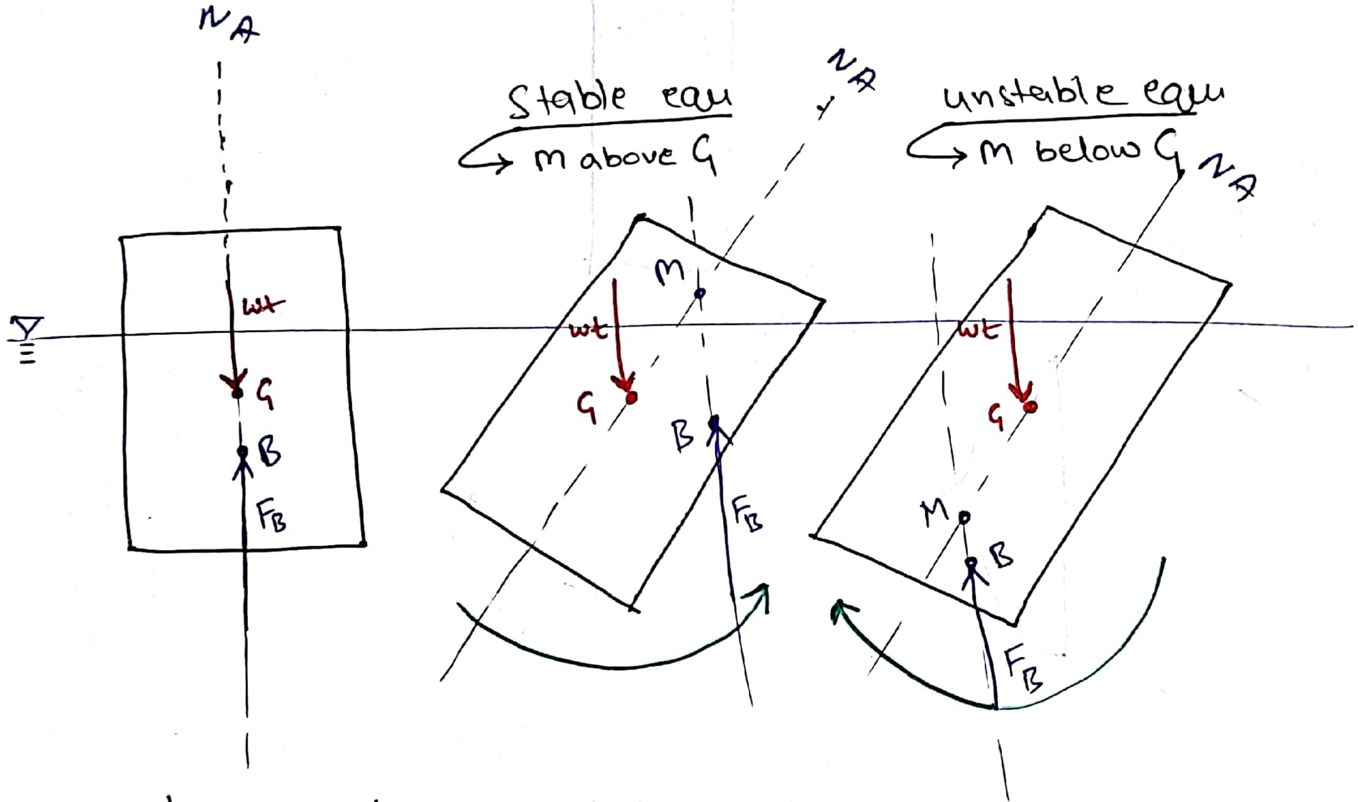
Stability conditions of a completely Submerged bodies under angular deflection

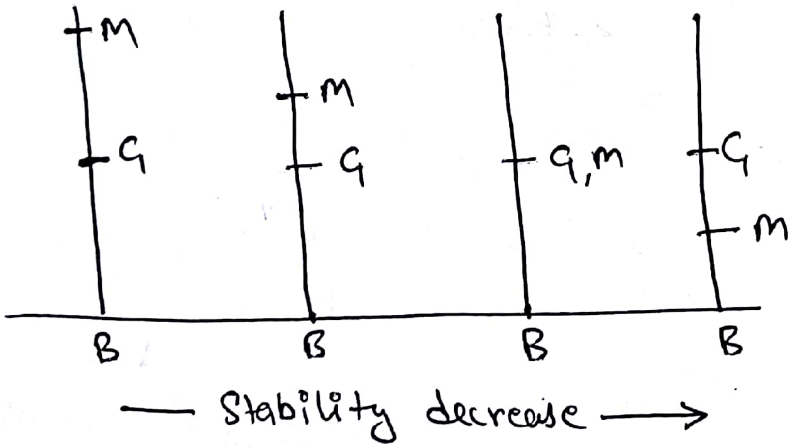


Stable Equ.
⇓
B above G

unstable equ
⇓
B below G

Stability conditions of partially submerged bodies under angular deflection





Equilibrium

Stable

- M above G
- $BM > BG$
- $GM = +ve$

Unstable

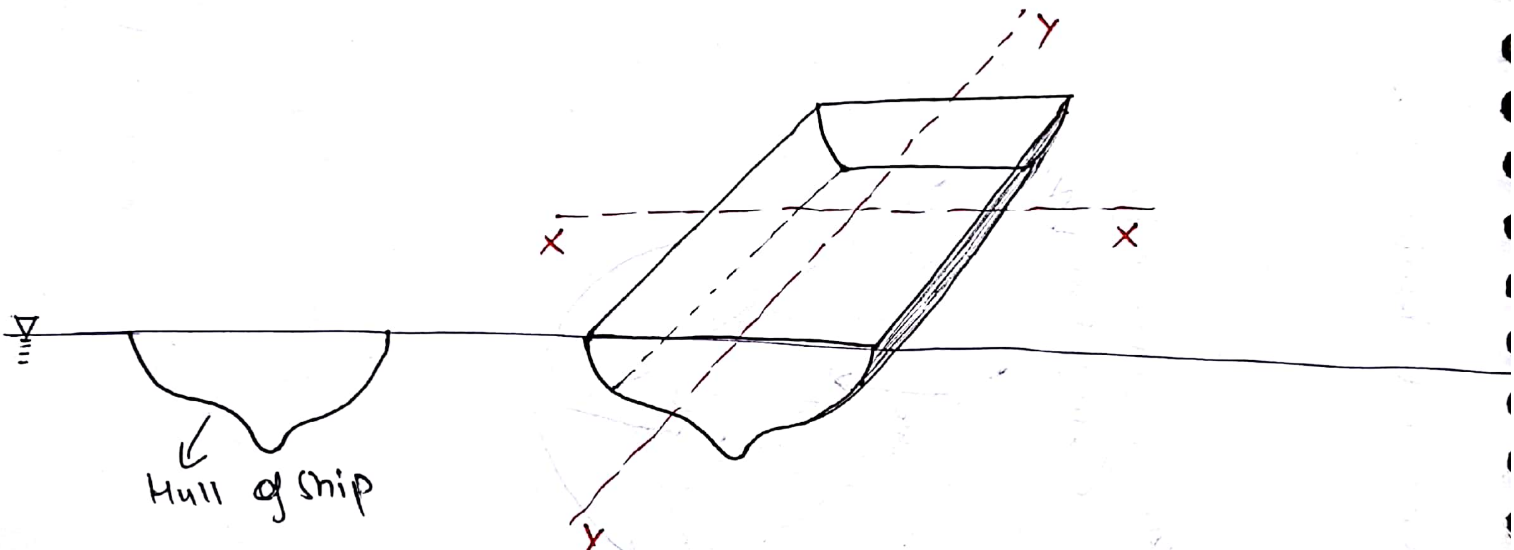
- M below G
- $BM \lneq BG$
- $GM = -ve$

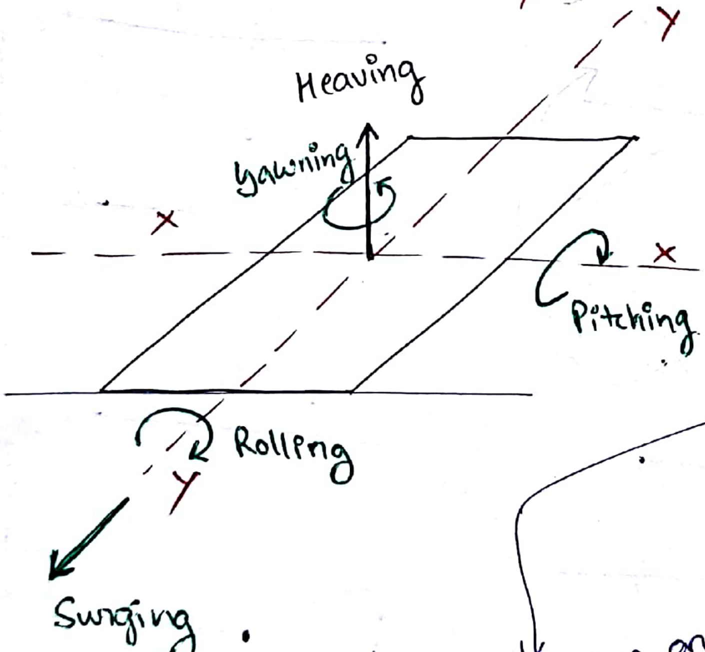
Calculation of metacentric height (GM)

Metacenter (M) it is defined as the point of intersection of normal Axis and new line of Action of buoyancy force.

Metacentric Height (GM) it is defined as the vertical distance b/w metacenter "M" and center of gravity "G"

- For stable equilibrium GM should be positive.





$$\boxed{BM = \frac{I}{V}}$$

I = least M.O.I. of the surface formed at the free surface

taken for area which is in plane cut by free liquid surface

V = Volume of fluid displaced.

$$I_{xx} = \frac{bl^3}{12} \quad ; \quad I_{yy} = \frac{lb^3}{12}$$

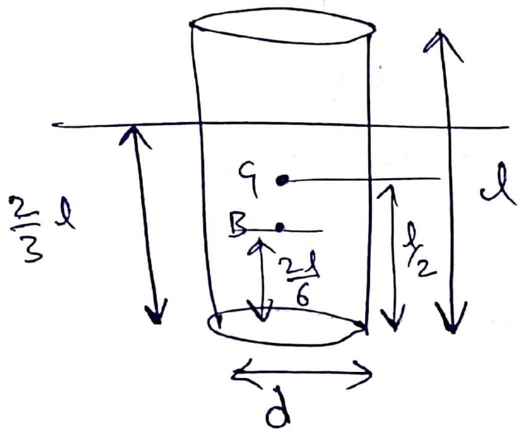
$$(BM)_{xx} = \frac{I_{xx}}{V} \quad ; \quad (BM)_{y-y} = \frac{I_{yy}}{V}$$

$$(BM)_{xx} > (BM)_{yy}$$

$$(BM)_{pitching} > (BM)_{rolling}$$

Note: • while designing boats or ships, $(BM)_{rolling}$ must always be taken bcz if by designing by rolling condition, the metacentric height comes true then the boat will be stable in all the cases.

P-17
Q7



$$\frac{I}{V} \ge \frac{l}{2} - \frac{l}{3} = \frac{l}{6}$$

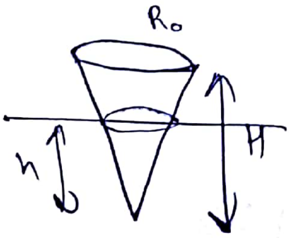
$$\frac{\cancel{\pi} d^{\cancel{4}} \times \cancel{4} \times 3}{\cancel{64}_{16} \times \cancel{\pi} \cancel{d^2} \times 2l} \ge \frac{l}{6}$$

$$d^2 \ge \frac{32 l^2}{18}$$

$$\boxed{d \ge \frac{4}{3} l} \Leftarrow d^2 \ge \frac{16}{9} l^2$$

P-18
Q8

Note: Center of gravity of cone is $\frac{3H}{4}$ from the apex



$$h = SH$$

$$r = \frac{hR}{H} = SR_o$$

$$I = \frac{\pi}{64} \times \frac{r^4}{16} \quad ; \quad V = \frac{1}{3}\pi r^2 h$$

$$\frac{I}{V} = \frac{\cancel{\pi}}{64} \times \frac{r^{\cancel{4}\,2}}{16} \times \frac{3}{\cancel{\pi} \cancel{r^2} h}$$

$$BG = \frac{3H}{4} - \frac{3SH}{4} = \frac{3H}{4}(1-S) < \frac{3}{16 \times 64} \times \frac{S^{\cancel{2}} R^2}{\cancel{S} H}$$

$$\frac{\cancel{3}H^2}{\cancel{4}}(1-S) < \frac{\cancel{3}}{\cancel{64}\times 16}\, S R^2$$

16

$$R^2 > 16\times 16\; H^2 \left(\frac{1-S}{S}\right)$$

$$\boxed{R > 16H\sqrt{\frac{1-S}{S}}}$$

P-1
Q4

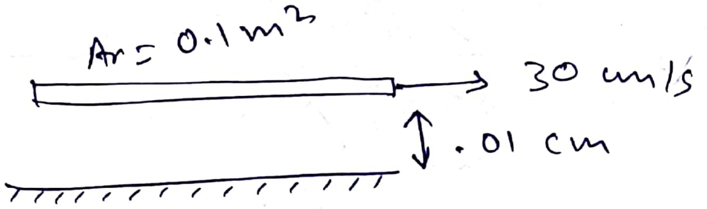
$\Delta P = .5\ \text{mPa} = 5\times 10^5\ \text{Pa}$

$\Delta \rho = 1\ \text{kg/m}^3$

$K = \rho \frac{dP}{d\rho} = 500 \times \frac{5\times 10^5}{1}$

$= 250\ \text{mPa}$

P-2
Q6

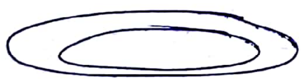


$\mu = 10^{-3}\ \frac{Ns}{m^2}$ $\quad F = \mu \frac{v}{y} \times A$

$P = F\times v = \frac{\cancel{10^{-3}} \times .30}{\cancel{.01\times 10^{-2}}} \times \cancel{0.1} \times 0.3 = 0.09\ W$

P-2
Q7

d = 60 mm



$y = 0.05$ mm

D = 60.1 mm $\quad F = K\mu$

$F = \mu \frac{v}{y} A$ $\quad \% \downarrow = \frac{\cancel{.00}206 - \cancel{.}0182}{\cancel{0.}0182}$

$\Rightarrow 88.68$

P-2 Q8

$F = \mu N = .02 \times 800 = \mu \frac{v}{y} \times Ar$

$16 = 10^{-3} \times \frac{v \rightarrow 15}{y} \times 10 \times \cancel{10^{-4}} \times 10^{-4}$

$\cancel{16} \times y = \frac{15 \times 10^{-6}}{16}$

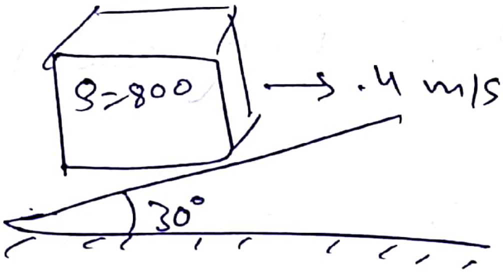
$= 9.375 \times 10^{-7}$ m

P-2 Q9

$F = \overset{10}{\cancel{130}} \times \frac{5}{\cancel{13}} = \mu \times \frac{\overset{100}{\cancel{0.500}} \times 1}{\cancel{0.005}}$

$\mu = \frac{50}{100} = 0.5$

P-3 Q10



$y = 0.4 \times 10^{-3}$ m

$mg = 64$ N

$\overset{32}{\cancel{64}} \times \frac{1}{\cancel{2}} = \mu \times \frac{\cancel{0.4}}{\cancel{0.4 \times 10^{-3}}} \times \overset{40}{\cancel{0.04}}$

$\mu = \frac{32}{40} = 0.8$

$\nu = \frac{\overset{4}{\cancel{32}}}{\cancel{40} \times \overset{10}{\cancel{800}}} \; 100 = 10^{-3}$

P-3 Q11

$F = 150 = .1 \times \frac{v}{.005} \times 5 + .3 \times \frac{v}{.005} \times 5$

$150 = \frac{100}{\cancel{5}} \times \cancel{5} v + \frac{300}{\cancel{5}} \times \cancel{5} v$

$15\cancel{0} = 40\cancel{0} v$

$v = \frac{15}{40} = \frac{3}{8}$

CHAPTER – 3

FLUID KINEMATICS

- Now in fluid mechanics the fluid will start to flow and we will study only the motion of the fluid without considering the reason behind that motion (forces).
- According to ~~Eulerian~~ Lagrangean technique he considered a fluid partical and by tracing the path of fluid partical and studying the properties, the fluid flow analysis was done.
- Lagrangean technique was TDS and complex and hence it didn't gain popularity.
- According to Eulerian technique a X-S/c was considered in a fluid flow and the fluid properties studied at this given X-S/c & fluid flow was analysed. This technique became more famous due to its simplicity and hence it is used in fluid mechanics.

Types of fluid flow

1. Steady / unsteady flow

A steady flow is the one in which fluid properties are not changing w.r.t. but at a given X-S/c.

$$\left(\frac{\partial v}{\partial t}\right)_{\text{at a } X\text{-}S/c} = 0 \quad ; \quad \left(\frac{\partial \rho}{\partial t}\right)_{\text{at } X\text{-}S/c} = 0$$

2. Uniform / non-uniform flow

A uniform flow is one in which velocity of fluid is not changing w.r.t. X-S/c but at a given interval of time.

$$\left[\frac{\partial v}{\partial (X\text{-}S/c)}\right]_{\text{at given time}} = 0$$

Examples

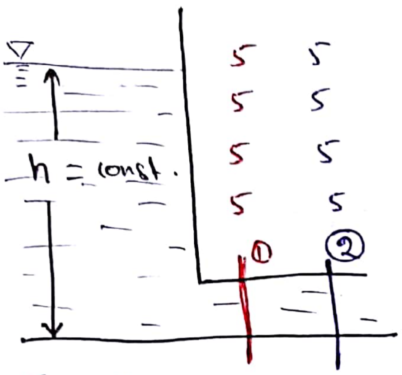


Fig:- Steady uniform flow

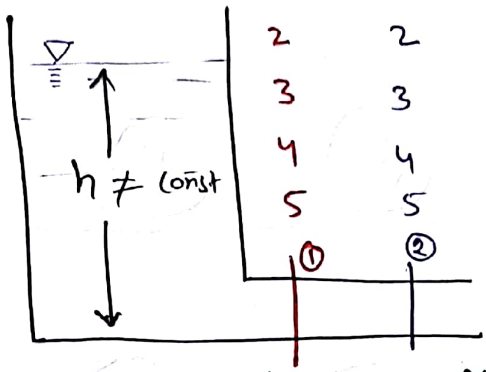


Fig: unsteady uniform flow

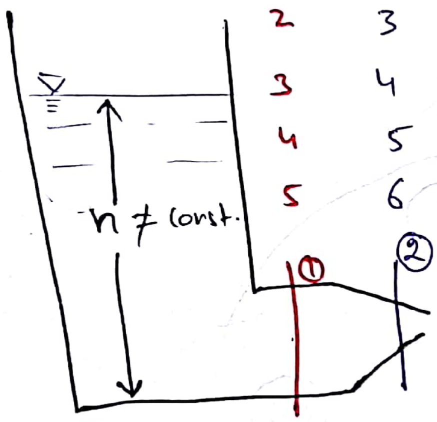


Fig: unsteady & non-uniform flow

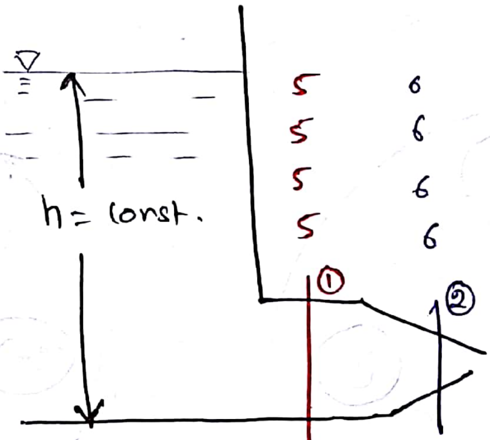
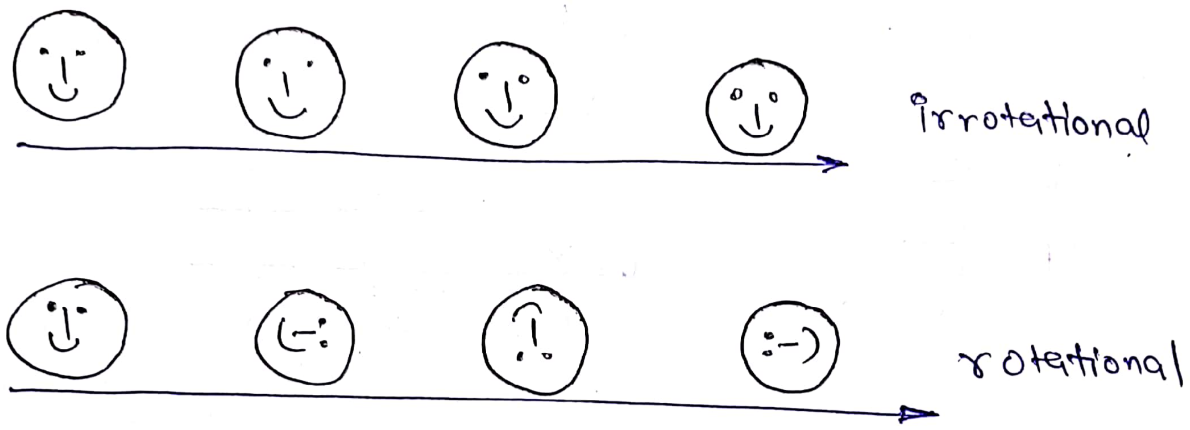


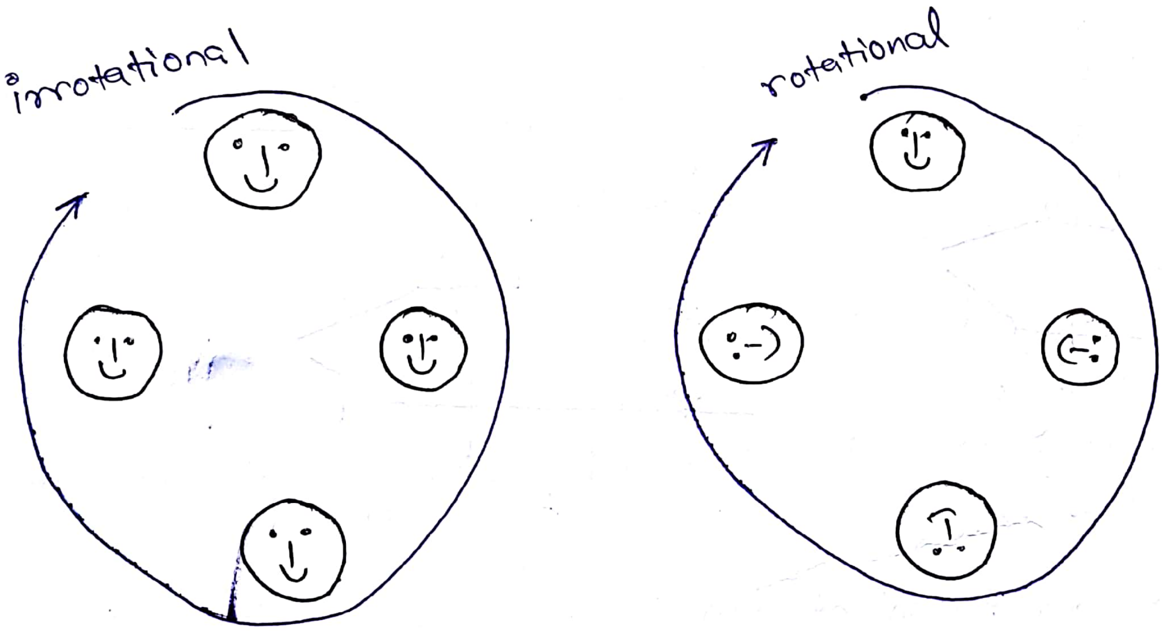
Fig: Steady & non-uniform flow

3. Laminar and turbulent flow

- A Laminar flow is the one in which fluid moves in the form of layers and there is no intermixing of fluid particals or molecular momentum transfer.
  ex:- flow of blood in viens.
- Laminar flow generally occurs at low velocity & high viscosity.
- A turbulent flow is the one in which there is huge order intermixing of fluid particals or molecular momentum transfer.
- Turbulent flow generally occurs at high velocity & low viscosity [ex:- flow of water in rivers]
  ex:- flow of exhaust gases through chimney.

4. Rotational & irrotational flow

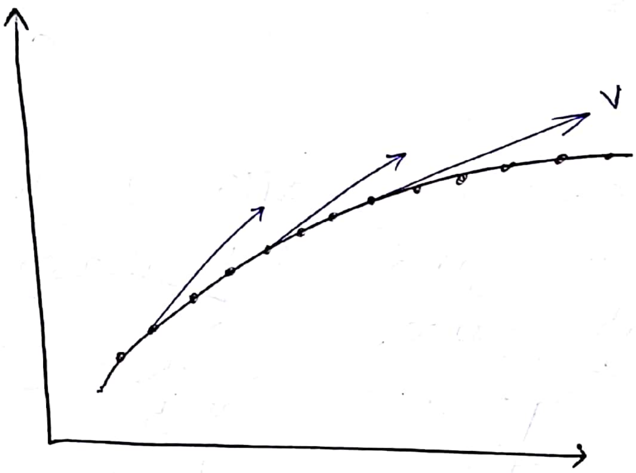




- A rotational flow is the one in which the fluid particals rotate about their mass centers otherwise the flow is irrotational.
- For a fluid to be rotational a certain tangential shear is required to produce torque on the fluid partical, which is provided by viscosity and hence it can be concluded that non viscous fluids can never be rotational

Tools for studying fluid flow

1. **Streamline** it is an imaginary curve drawn in space such that tangent drawn to it at any point will give velocity of fluid flow. Streamline gives an instentenous picture of the fluid flow. As there is no component of velocity across the stream line, the flow always takes place along the stream line. Two streamlines can never intersect with itself, neither a streamline can never intersect with itself.



at a given time

Velocity of a fluid Partical

$$\vec{V} = u\hat{i} + v\hat{j} + w\vec{k}$$

where u, v, w are the components of velocity in x, y, z dir^n.

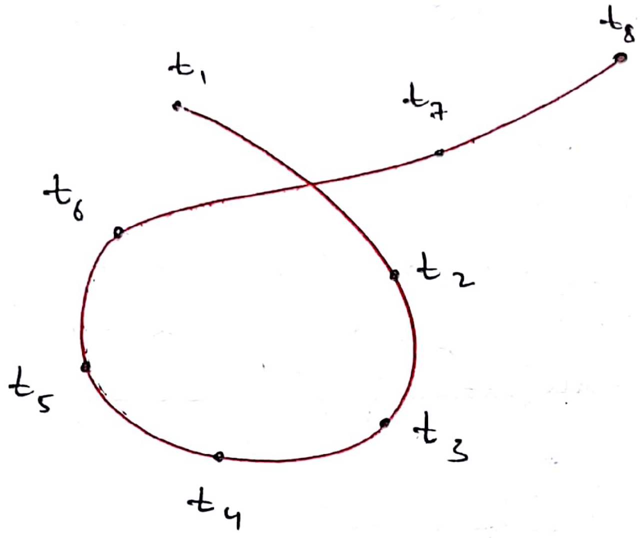
$$V = \sqrt{u^2 + v^2 + w^2}$$

Equation of streamline

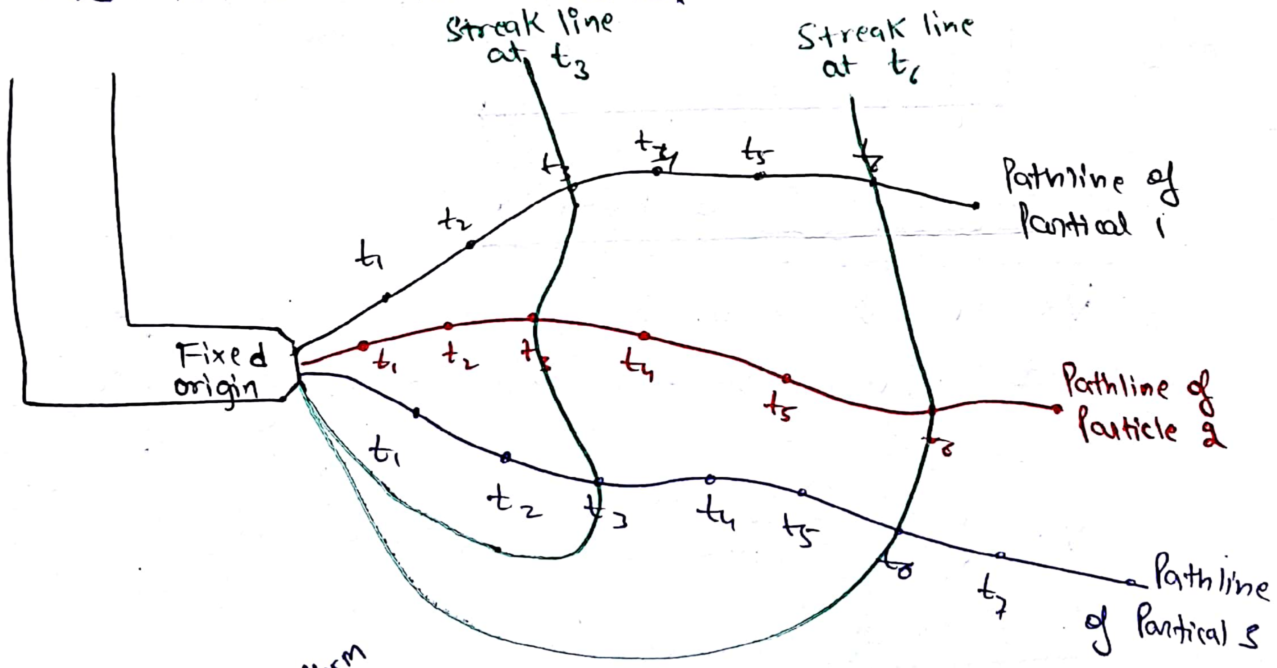
$$\boxed{\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}} \rightarrow 3D$$

$$\boxed{\frac{dx}{u} = \frac{dy}{v}} \longrightarrow 2D$$

2. **Path line** A path line is the line formed by joining the locations of single fluid partical at different interval of time. A path line can intersect with itself.



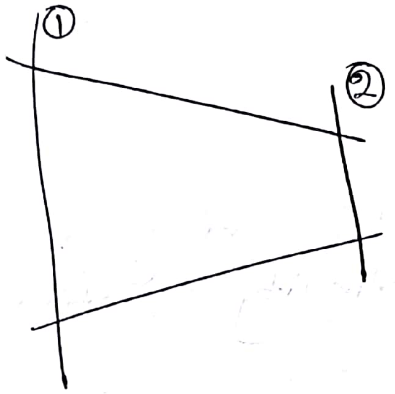
3. **Streak line** it is the locus of different fluid particals originating from a single fixed point at a given interval of time is known as streak line.



Note: • For a steady, uniform flow stream line, streak line & Path line will concide whereas for an unsteady flow they will never coincide

Continuity equation [conservation of mass]

- Steady, 1D → $\dot{m}_{entering} = \dot{m}_{leaving}$
  → $E_{entering} = E_{leaving}$



$\dot{m}_1 = \dot{m}_2$

$$\boxed{\rho_1 A_1 V_1 = \rho_2 A_2 V_2}$$

T&C
1) Steady
2) 1D

- for an incompressible fluid

$$\boxed{A_1 V_1 = A_2 V_2}$$

T&C
1) Steady
2) 1D
3) incompressible

- Discharge (Q)

$Q = AV \ m^3/s$

$$\boxed{Q = Q_1 = Q_2 = A_1 V_1 = A_2 V_2 = \text{constant}}$$

T&C
1) Steady
2) 1D
3) incompressible

- $\rho AV = \text{const.}$

differentiating both side

$\rho A\,dV + AV\,d\rho + V\rho\,dA = 0$

dividing by ρAV

$$\boxed{\frac{dV}{V} + \frac{dA}{A} + \frac{d\rho}{\rho} = 0}$$

T&C
1) Steady
2) 1D

Generalised continuity equation

$$\boxed{\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0}$$

valid for all cases

$$\Rightarrow \quad \frac{d\rho}{dt} + \rho(\text{div.}\,\vec{v}) = 0$$

$$\Rightarrow \quad \boxed{\frac{d\rho}{dt} + \rho(\nabla.\vec{v}) = 0}$$

vector form of generalised continuity equation.

here, $\nabla.\vec{v} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$

$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + u.\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z}$$

Case-1 Steady flow

$$\boxed{\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0}$$

Case-2 incompressible flow ($\rho =$ const.)

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0} \longrightarrow 3D$$

$$\Rightarrow \quad \nabla.\vec{v} = 0$$

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0} \longrightarrow 2D$$

Note: • Every fluid flow must satisfy its continuity eqn otherwise the flow is not possible.

P-20 Q5

$u = 1.5x \qquad v = ?$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$-1.5 = \frac{\partial v}{\partial y}$$

$$-1.5\,\partial y = \partial v \Rightarrow -1.5y = v + c$$

at $(1, 0)$

$$0 = 0 + c \Rightarrow c = 0$$

$$\boxed{v = -1.5y}$$

$2A_1 = 3A_2 + 5A_3$

$3A_1 = 4A_2 + xA_3$

$A_1 = A_2 + (x-5)A_3$

$2A_1 = 8A_2$

$3A_1 = (4+x)A_2$

$\frac{2}{3} = \frac{8}{4+x}$

$\Rightarrow x = 8$

P-20 Q6

$$\frac{d\rho}{\rho} + \frac{dv}{v} + \frac{dA}{A} = 0$$

$$-.15 + \frac{dv}{v} - 0.40 = 0$$

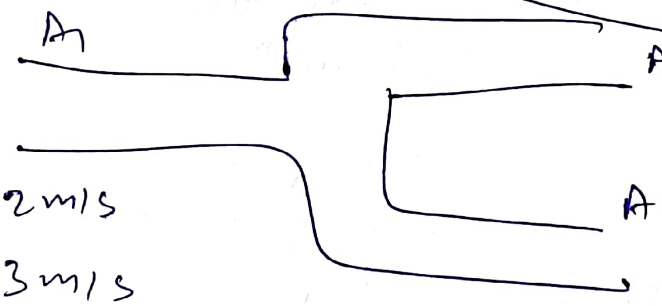
$$\frac{dv}{v} = 0.55$$

$$\frac{v_2 - v_1}{v_1} = .55 \Rightarrow \frac{v_2}{v_1} - 1 = 0.55$$

$$\frac{v_2}{v_1} = 1.55$$

$$.18 = \frac{\pi}{4} \times (.3)^2 \times v$$

$$\boxed{v = 2.546 \text{ m/s}}$$



$\frac{3}{4} = \frac{5}{x}$

$\cancel{2}A_1 = \frac{3}{2}A_2 + \frac{5}{2}A_3$

$\cancel{3}A_1 = \frac{4}{3}A_2 + \frac{x}{3}A_3$

$174 = \rho \times .18$

$$\boxed{\rho = \frac{174}{.18} = 966.6}$$

P-20 Q7

$2A_1 = 3A_2 + 5A_3$

$3A_1 = 4A_2 + xA_3$

$\frac{3/2}{5/2} = \frac{4/3}{x/3}$

$x = \frac{4 \times 5}{3} = \frac{20}{3} = 6.66$

$150 + \cancel{800} \times \cancel{0.03} = \dot m$

$174 = \dot m = \rho v$

$150 + 30 = 180$

Acceleration of a fluid Partical

$$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

where, a_x, a_y, a_z are the components of acceleration of fluid partical in x, y, z dirⁿ

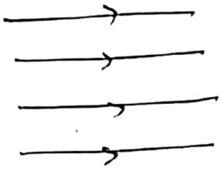
$$a_x = \underbrace{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}} + \boxed{\frac{\partial u}{\partial t}}$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t} \Rightarrow \text{local component}$$

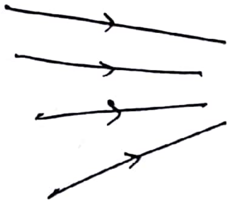
$$a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

↳ convective component

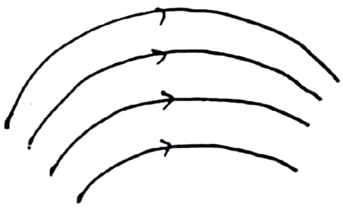
	local accⁿ	convective accⁿ
1. Steady & uniform	0	0
2. unsteady & uniform	exist	0
3. Steady & nonuniform	0	exist
4. unsteady & nonuniform	exist	exist

1) 

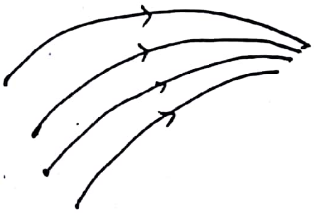
straight & parallel — no. acceleration

2) 

straight & converging — tangential acc.

3) 

curved & parallel — Normal acc.

4) 

Curved & converging — both acc.

$\boxed{\begin{matrix} P\,21 \\ Q\,9 \end{matrix}}$

Stead
1 D

$V = u$

$$a = a_x = u\frac{\partial u}{\partial x} = V\frac{\partial V}{\partial x}$$

$$u = \frac{5\sqrt{3}}{(\sqrt{3}+x)}$$

$x = 0$

$$\left.\frac{\partial u}{\partial x}\right|_{x=0} = \frac{-5\sqrt{3}}{(\sqrt{3})^2}$$

$$a = \frac{5\sqrt{3}}{(\sqrt{3})}\left(-\frac{5\sqrt{3}}{(\sqrt{3})^2}\right)$$

$a = -14.45$

P-21
Q10

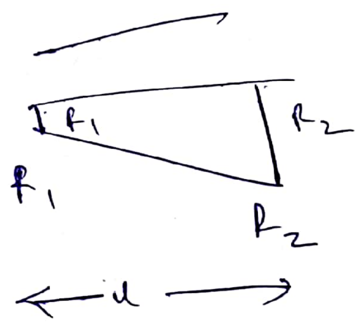
Steady flow

$$\vec{V} = u_0\left(1 + \frac{3x}{l}\right)\hat{i}$$

$$\frac{dx}{dt} = V \qquad \int \frac{dx}{u_0\left(\frac{l+3x}{l}\right)} = \int dt \qquad \ln\frac{4l}{l} = \ln 4$$

$$\frac{l}{3u_0} \times \Big[\ln (l+3x)\Big]_0^l = \frac{l}{3u_0} \times \ln 4$$

P-21
Q11



$$Q_1 = Q_2$$

$$A_1V_1 = A_2V_2$$

$$R_1^2 V_1 = R_2^2 V_2$$

$$u = \frac{Q}{A} = \frac{Ql}{lR_1 + (R_2-R_1)x}$$

$$a_x = u\frac{\partial u}{\partial x}$$

$$K = \frac{(R_2-R_1)}{l} \qquad r = \frac{(R_2-R_1)x}{l} + R_1$$

$$\cancel{a_x = \frac{Ql}{lR_1 + (R_2-R_1)x} \times \left(-\boxed{Ql}\right)}$$

$$\cancel{K^2x^2 + R_1^2 + 2KR_1x}$$

$$\cancel{2(Kx+R_1)\times K}$$

$$\cancel{2R_1 \times (R_2-R_1)}$$

$$r = Kx + R_1$$

$$u = \frac{Q}{\pi \times (Kx+R_1)^2}$$

$$x^{-2}$$

$$-\frac{2}{x^3}$$

$$a_x = \frac{Q}{\pi (Kx+R_1)^2} \times \frac{Q}{\pi} \times \frac{-2}{(Kx+R_1)^3} \times K$$

$$= \frac{-2Q^2}{\pi^2} \times \frac{1}{(R_2-\cancel{R_1}+\cancel{R_1})^2} \times \frac{(R_2-R_1)}{(R_2)^3} = \frac{2Q^2(R_1-R_2)}{\pi^2 l R_2^5}$$

Rotation of a fluid Partical

$$\vec{\omega} = \omega_x \hat{i} + \omega_y \hat{j} + \omega_z \hat{k}$$

where, $\omega_x, \omega_y, \omega_z$ are the components of rotation vector $\vec{\omega}$ in x, y, z dir^n

$$\boxed{\vec{\omega} = \frac{1}{2} \text{ curl } \vec{V}} = \frac{1}{2} \nabla \times \vec{V} = \frac{1}{2} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

$$\vec{\omega} = \frac{1}{2}\left[\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right]\hat{i} - \frac{1}{2}\left[\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}\right]\hat{j} + \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]\hat{k}$$

$$\Rightarrow \boxed{\begin{aligned} \omega_x &= \frac{1}{2}\left[\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right] \\ \omega_y &= \frac{1}{2}\left[\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right] \\ \omega_z &= \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right] \end{aligned}}$$

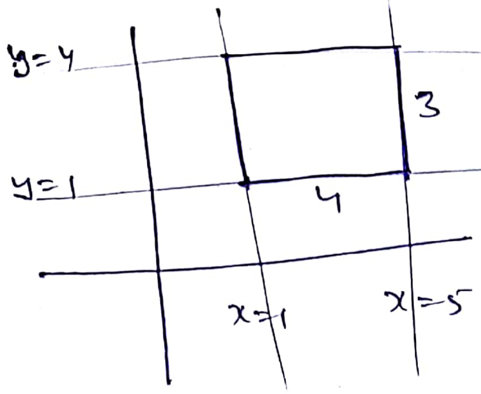
- if $\omega_x = \omega_y = \omega_z = 0 \Rightarrow$ flow is irrotational
- Vorticity(ζ) $2\vec{\omega} = \text{curl } \vec{V} \Rightarrow \boxed{\text{Vorticity} = \nabla \times \vec{V}} = \zeta$
- $\boxed{\text{Circulation} = \text{Vorticity} \times \text{Area}} = \Gamma$
- For 2D case

  if $\omega_z = 0 \Rightarrow$ $\omega = 0$

  Vorticity $= 0$

  Circulation $= 0$

P-21
Q 12



$Ar = 12$

Γ = circulation = $\zeta \times Ar$

ζ = vorticity = $2\omega_z$

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$= \frac{1}{2}(0 - 3) = -\frac{3}{2}$$

$$\Rightarrow \zeta = 2\omega_z = 2 \times -\frac{3}{2} = -3$$

$$\Gamma = \zeta \times Ar = -3 \times 12 = -36$$

P-22
Q 13

$$\zeta = -3$$

$$\Gamma = \zeta \times Ar = -3 \times \pi \times 4 = -12\pi$$

P-23
Q 22

$h = 5$ mm

$\rho = 860$

$\mu = 2 \times 10^{4}$

$V = .05$ m/s

$Ar = .25$ m2

$$u = \frac{V}{h} \times y$$

$$\frac{\partial u}{\partial y} = \frac{V}{h}$$

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$= -\frac{1}{2} \times \frac{V}{h}$$

$$= -\frac{1}{\cancel{2}} \times \frac{\cancel{.05}}{\cancel{.005}}\ \cancel{10}\ 5$$

$$P = FV$$

$$= \mu \frac{V}{y} \times Ar \times V$$

$$= 2 \times 10^{4} \times \frac{.25}{.005} \times (0.05)^2$$

$$= 2500\ \underline{W} \times \cancel{10^{-8}}$$

$$\cancel{= 2.5 \times 10^{5}}$$

P-24
Q24

$$Vol = \pi \times R^2 \times h$$

$$\frac{dVol}{dt} = \pi \left[h 2R\, dR + R^2 dh\right]$$

$$-2h\cancel{R}\frac{dR}{dt} = R^{\cancel{2}}\frac{dh}{dt}$$

$$-2h\frac{dR}{dt} = VR$$

$$v_r = \boxed{\frac{dR}{dt} = -\frac{VR}{2h}}$$

P-24
Q25

$$\cancel{a_r = V\frac{d^2R}{dt^2} = \frac{V^2R^2}{4h}}$$

$$a_r = V_r\frac{\partial V_r}{\partial r} + \frac{\partial V_r}{\partial t}$$

$$= \cancel{-\frac{VR}{2h}}\left(-\frac{V}{2h}\right) + -\frac{VR}{2} \times -\frac{1}{h^2} \times V$$

$$= \cancel{\frac{V^2R^2}{8h^2}} \quad \frac{V^2R}{4h^2} + \frac{2V^2R}{2\times 2h^2}$$

$$= \frac{3V^2R}{4h^2}$$

$$a_r = \frac{VR}{2h}\cdot\frac{V}{2h} + \frac{\partial}{\partial t}\left(\frac{VR}{2h}\right]$$

$$= \frac{V^2R}{4h^2} + \frac{VR}{2}\frac{\partial}{\partial t}(h^{-1})$$

$$= \frac{3}{4}\frac{V^2R}{h^2}$$

$\boxed{\begin{array}{l} P-25 \\ Q\,27 \end{array}}$

$$\zeta_z = 2\omega_z$$

$$2\omega_z = \left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$$

$$= -2xz - 2x$$

$$= -4\hat{k}$$

$$2\omega_x = \left[\cancel{\frac{\partial w}{\partial y}}^{\,0} - \frac{\partial v}{\partial z}\right]$$

$$= +x^2 = 1\hat{i}$$

$$\zeta = 2\omega = \hat{i} - 4\hat{k}$$

$\boxed{\begin{array}{l} P-25 \\ Q\,28 \end{array}}$

$$\zeta_z = 2\omega_z = \left[\cancel{\frac{\partial v}{\partial x}}^{\,0} - \frac{\partial u}{\partial y}\right]$$

$$= k(-1)$$

$\boxed{\begin{array}{l} P-25 \\ Q\,29 \end{array}}$

$$a_1 + b_2 + c_3 = 0$$

$$2 + b_2 - 4 = 0$$

$$b_2 = 2$$

$\boxed{\begin{array}{l} P-25 \\ Q\,30 \end{array}}$

$$\frac{dx}{u} = \frac{dy}{v} \Rightarrow \frac{dx}{x^2\cancel{t}} = \frac{dy}{-2\cancel{x}y\cancel{t}}$$

$$2\ln x = \ln\frac{c}{y}$$

$$\Rightarrow x^2 y = c$$

Velocity Potential function (ϕ)

it is a function of space such that the −ve derivative of this function w.r.t. a given direction will give component of velocity in that direction.

$$\boxed{-\frac{\partial \phi}{\partial x} = u} \quad \boxed{-\frac{\partial \phi}{\partial y} = v} \quad \boxed{-\frac{\partial \phi}{\partial z} = w}$$

- For 2D → incompressible flow

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$LHS = \frac{\partial}{\partial x}\left(\frac{-\partial \phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{-\partial \phi}{\partial y}\right)$$

$$= -\frac{\partial^2 \phi}{\partial x^2} - \frac{\partial^2 \phi}{\partial y^2} = -\underbrace{\left[\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}\right]}_{\text{Laplace eqn}}$$

⇒ Case-1

if ϕ satisfies Laplace ⇒ flow is possible

⇒ Case-2

if ϕ doesnot satisfies Laplace ⇒ flow is not possible

- $$w_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$$

$$= \frac{1}{2}\left[\frac{\partial}{\partial x}\left(\frac{-\partial \phi}{\partial y}\right) - \frac{\partial}{\partial y}\left(\frac{-\partial \phi}{\partial x}\right)\right]$$

$$= \frac{1}{2}\left(\cancel{-\frac{\partial^2 \phi}{\partial x \partial y}} + \cancel{\frac{\partial^2 \phi}{\partial x \partial y}}\right)$$

$$= 0$$ ⇒ $\boxed{\text{if } \phi \text{ exists} \Rightarrow \text{flow is irrotational}}$

Stream function (ψ) [only defined for 2D]

- it is a function of space such that it satisfies continuity equation.

$$\boxed{u = -\frac{\partial \psi}{\partial y}} \qquad \boxed{v = \frac{\partial \psi}{\partial x}}$$

- 2D incompressible flow

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$LHS = \frac{\partial}{\partial x}\left(-\frac{\partial \psi}{\partial y}\right) + \frac{\partial}{\partial y}\left(\frac{\partial \psi}{\partial x}\right)$$

$$= \cancel{-\frac{\partial^2 \psi}{\partial x \partial y}} + \cancel{\frac{\partial^2 \psi}{\partial x \partial y}} = 0$$

$$\Rightarrow w_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

Laplace

$$= \frac{1}{2}\left[\frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial x}\right) - \frac{\partial}{\partial y}\left(-\frac{\partial \psi}{\partial y}\right)\right] = \frac{1}{2}\underbrace{\left[\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}\right]}_{\text{Laplace}}$$

** Case-1: if ψ satisfy laplace $\rightarrow$ flow is irrotational

** Case-2: if ψ doesnot satisfy Laplace $\rightarrow$ flow is rotational

Note:
- A line along which stream func^n ψ is constant is known as stream line
- A line along which velocity Potential fun^n (ϕ) is const. is known as equi Potential line.

- The grid formed by joining the series of equi-potential lines and stream lines is known as flow net.
- Equi-Potential lines & stream lines are orthogonal to each other.
- The difference b/w any two stream lines will give discharge per unit length. (discharge = $|\psi_1 - \psi_2|$)

Slope of ~~eq~~ stream ~~line~~ function

$$\frac{dx}{u} = \frac{dy}{v}$$

$v\,dx - u\,dy = 0 \longrightarrow$ eqn of stream line

$$u = -\frac{\partial \psi}{\partial y} \qquad v = \frac{\partial \psi}{\partial x}$$

$$\frac{\partial \psi}{\partial x} \cdot dx + \frac{\partial \psi}{\partial y} \cdot dy = 0$$

- $\psi = f(x, y) \Rightarrow d\psi = \frac{\partial \psi}{\partial x} \cdot dx + \frac{\partial \psi}{\partial y} \cdot dy$

$\Rightarrow d\psi = 0 \Rightarrow \psi =$ constant

$$\Rightarrow 0 = \frac{\partial \psi}{\partial x} \cdot dx + \frac{\partial \psi}{\partial y} \cdot dy$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\partial \psi / \partial x}{\partial \psi / \partial y} = \frac{-v}{-u} = \frac{v}{u}$$

$$\Rightarrow \boxed{\left(\frac{dy}{dx}\right)_{\text{stream line}} = \frac{V}{u}} \quad \text{—— (1)}$$

Slope of Equipotential lines

- $\phi = f(x, y)$

$$d\phi = \frac{\partial \phi}{\partial x} . dx + \frac{\partial \phi}{\partial y} . dy$$

Equipotential $\Rightarrow d\phi = 0 \Rightarrow \phi =$ constant

$$0 = \frac{\partial \phi}{\partial x} . dx + \frac{\partial \phi}{\partial y} . dy$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\partial \phi / \partial x}{\frac{\partial \phi}{\partial y}} = -\frac{u}{v}$$

$$\Rightarrow \boxed{\left(\frac{dy}{dx}\right)_{\text{Equipotential}} = \frac{-u}{v}} \quad \text{--- (2)}$$

- from (1) & (2)

we can say equipotential lines & streamlines are $\perp$ar.

Cauchy - Reiman Eqn

$$\boxed{\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}}$$

P-22
Q16

$u = \frac{u_0 x}{l}$ $v = \frac{u_0 y}{l}$

$l = 0.2\,m$

$a = 10\,m/s^2 = \sqrt{a_x + a_y}$

$a_x = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = u_0 \frac{x}{l} \times \frac{u_0}{l} + \frac{u_0 y}{l} \times 0$

$= \left(\frac{u_0}{l}\right)^2 x = \frac{u_0^2}{l} = \frac{u_0^2}{.2}$

$= 5u_0^2$

$a_y = u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = \frac{u_0 \cancel{x}}{\cancel{l}} \times 0 + \frac{u_0 \cancel{y}}{\cancel{l}} \times \frac{u_0}{l}$

$= \frac{u_0^2}{.2} = 5u_0^2$

$10 = \sqrt{(5u_0^2)^2 + (5(u_0)^2)^2}$

$10 = u_0^2 \times \sqrt{25+25}$

$u_0 = 1.19$

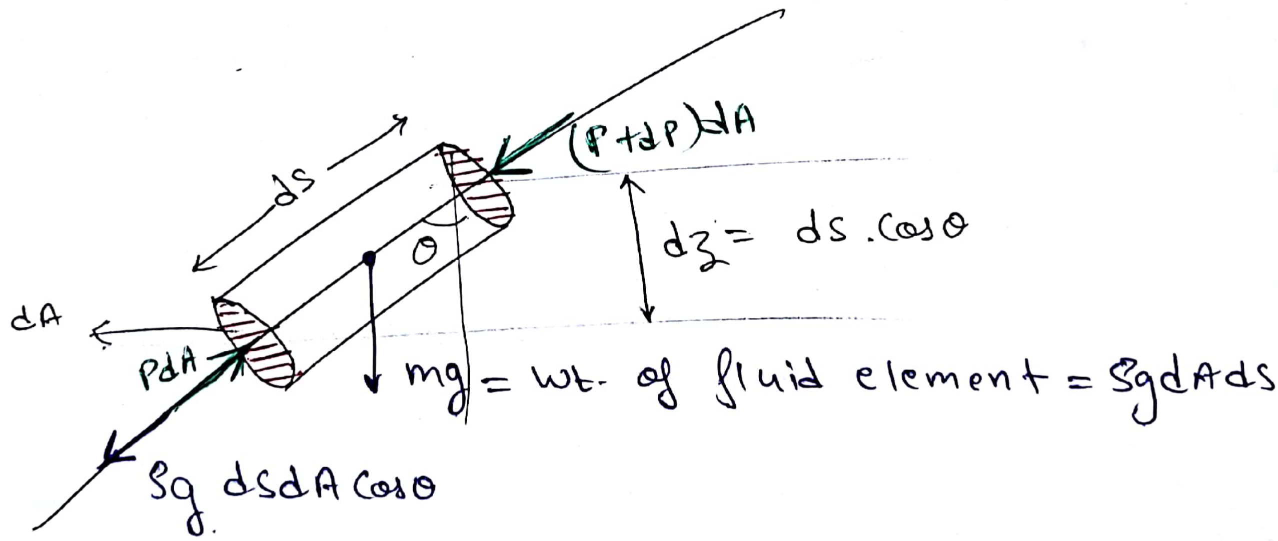
CHAPTER - 4

FLUID DYNAMICS

- Now in fluid dynamics we will consider the various forces behind the fluid flow.
- According Navier - stoke he considered Pressure force F_p, gravity force F_g, viscous force F_v, and derived the Navier's - Stoke eqn.
- Euler further assumed the fluid to be non-viscous and by considering the Pressure and gravity force, he derived the Euler's eqn.
- Then by further assuming the fluid to be steady & incompressible Bernauly derived his famous eqn Bernaulli's eqn, which became popular as conservation of energy eqn.

Euler's Equation $[F_P + F_g]$

Assumption : 1) flow is non-viscous



$$m = \frac{wt}{g} = \rho dA ds$$

$$a = a_x = u\frac{\partial u}{\partial x} + \frac{\partial u}{\partial t}$$

$$a = v\frac{\partial v}{\partial s} + \frac{\partial v}{\partial t}$$

$$\Sigma F = m \times a$$

$$\cancel{PdA} - \cancel{PdA} - dPdA - \rho g\, dA\, ds \cos\theta = \rho\, dA\, ds\left[v\frac{\partial v}{\partial s} + \frac{\partial v}{\partial t}\right]$$

$$-dP\cancel{dA} + \rho g \cancel{dA}\, ds \cos\theta = \rho \cancel{dA}\, ds\left(v\frac{\partial v}{\partial s} + \frac{\partial v}{\partial t}\right)$$

$$\Rightarrow \boxed{dP + \rho g\, dz + \rho\, ds\left(v\frac{\partial v}{\partial s} + \frac{\partial v}{\partial t}\right) = 0}$$ Euler's eqn

Asumption : 2) flow is steady $\left[\frac{\partial v}{\partial t} = 0\right]$

$$dP + \rho g\, dz + \rho \cancel{ds} \cdot v\frac{dv}{\cancel{ds}} = 0$$

$$dP + \rho g\, dz + \rho v\, dv$$

Dividing by ρ

$$\frac{dP}{\rho} + g\, dz + v\, dv = 0$$

Asumption : 3) Considering incompressible fluid & integrating

$$\int\frac{dP}{\rho} + \int g\, dz + \int v\, dv = 0$$

$$\boxed{\frac{P}{\rho} + gz + \frac{v^2}{2} = \text{Const.}}$$ $\frac{\text{Joules}}{\text{Kg}}$

Dividing by "g"

$$\boxed{\frac{P}{\rho g} + \frac{V^2}{2g} + z = \text{constant}}$$

meters

→ Classical Bernaulli's equation

Bernaulli's Theorem

- Bernaulli's theorem state that for a non-viscous steady & incompressible flow along a streamline the sumation of Kinetic energy, P.E., pressure energy is a constant and such a equation is known as bernaulli's equation
- Classical Bernaulli's eqn is also known as mechanical energy conservation eqn whereas S.F.E.E is known as total energy conservation equation.

$$\underbrace{\frac{P}{w}}_{\text{Pressure head}} + \underbrace{\frac{V^2}{2g}}_{\text{Kinetic head}} + \underbrace{z}_{\text{Potential head}} = \text{const.}$$

⇒ Classical Bernaulli's eqn [mechanical energy conservation eqn].

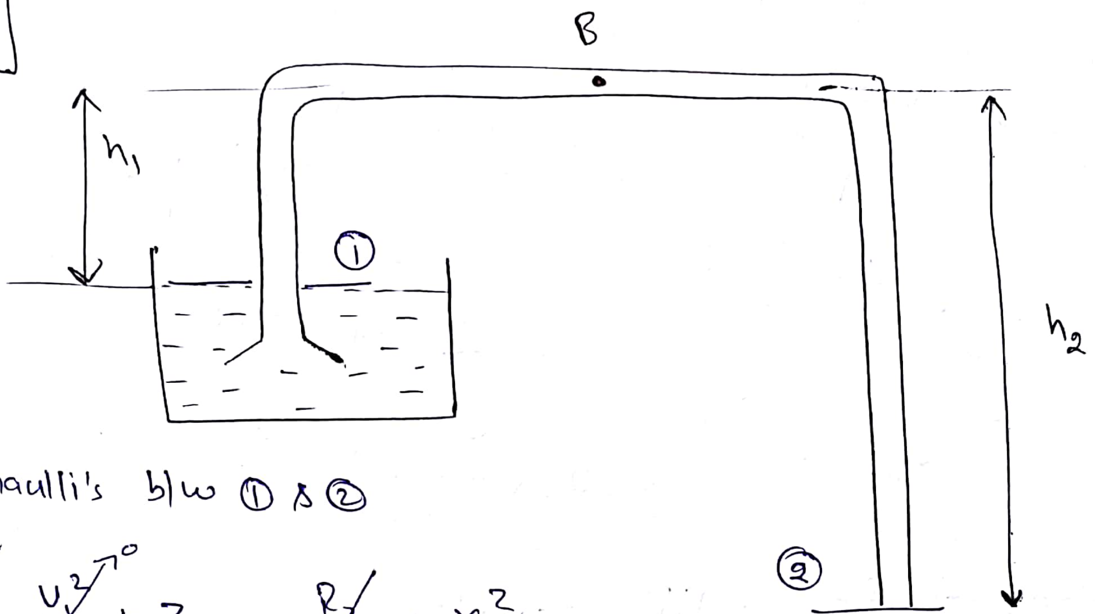
Assumptions : 1) Non viscous
2) Steady flow
3) incompressible
4) ideal fluid
5) irrotational

$$\frac{P}{w} + z = \text{Piezometric head}$$

Ideal $\Rightarrow$ $$\frac{P_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{w} + \frac{V_2^2}{2g} + z_2$$

Real $\Rightarrow$ $$\frac{P_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{w} + \frac{V_2^2}{2g} + z_2 + \text{head losses}$$

$\boxed{\text{P-27 Q2}}$



Bernaulli's b/w 1 & 2

$$\cancel{\frac{P_1}{w}} + \cancel{\frac{V_1^2}{2g}}^{\to 0} + z_1 = \cancel{\frac{P_2}{\rho g}} + \frac{V_2^2}{2g} + z_2$$

$$h_2 - h_1 = \frac{V_2^2}{2g}$$

$$\Rightarrow V_2 = \sqrt{2g(h_2-h_1)}$$

$A_1V_1 = A_2V_2$

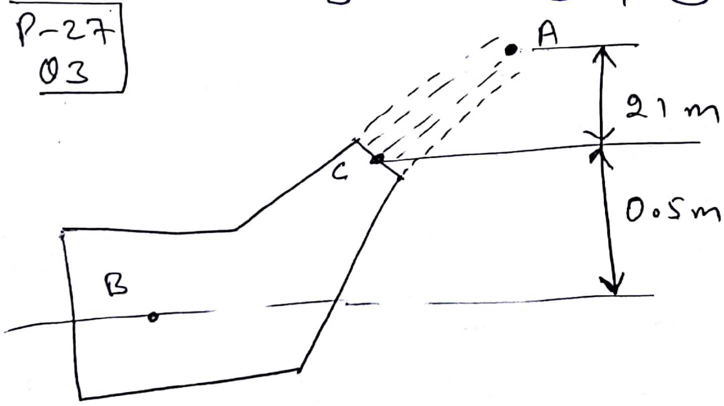
$V_1 = \frac{A_2}{A_1}V_2$

very small

continuity b/w B & 2

$\cancel{A_B}V_B = \cancel{A_2}V_2 \Rightarrow V_B = V_2$

$\boxed{\text{P-27 Q3}}$



B. eqn b/w A & C

$$\cancel{\frac{P_c}{w}} + \frac{V_c^2}{2g} + \cancel{z_c} = \cancel{\frac{P_A}{w}} + \frac{V_A^2}{2g} + z_A$$

$$\frac{V_c^2}{2g} = \frac{(18)^2}{2g} + 21$$

$$\Rightarrow V_c = 27.1 \text{ m/s}$$

Continuity b/w B & C

$$A_B V_B = A_C V_C$$

$$\frac{\pi}{4} \times (.225)^2 \times V_B = \frac{\pi}{4} \times (.075)^2 \times 27.1$$

$$V_B = 3.01 \text{ m/s.}$$

B. eqn b/w B & C

$$\frac{P_B}{w} + \frac{V_B^2}{2g} + \cancel{Z_B} = \cancel{\frac{P_C}{w}} + \frac{V_C^2}{2g} + Z_C$$

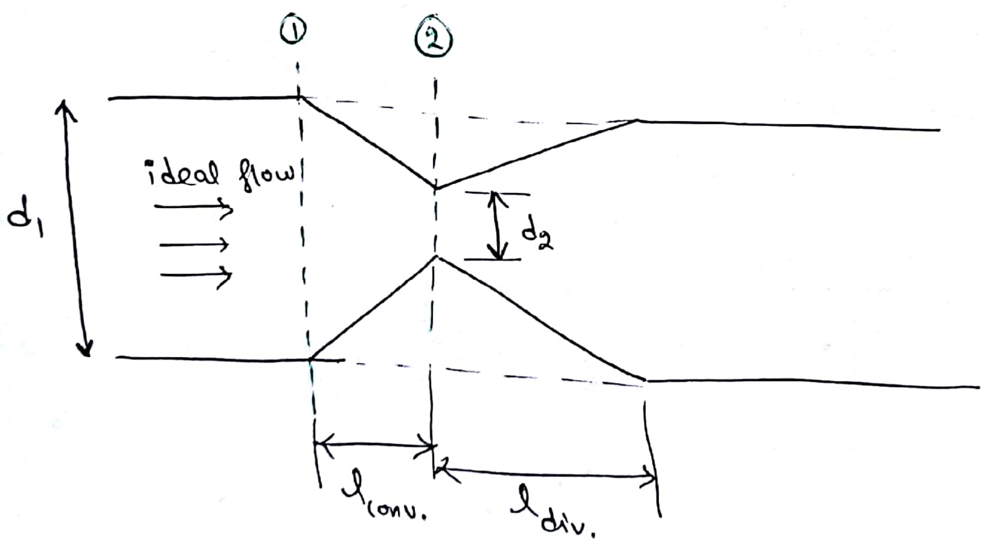
$$\frac{P_B}{9810} + \frac{(3.01)^2}{2g} = \frac{(27.1)^2}{2g} + 0.5$$

$$\Rightarrow P_B = 367.579 \text{ KPa}$$

Applications of Bernaulli's equation

1) Venturimeter

it is a converging diverging device that is used to measure discharge



B- equⁿ b/w ① & ②

$$\frac{P_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{w} + \frac{V_2^2}{2g} + z_2$$

for horizontal pipe $(z_1 = z_2)$

$$\frac{P_1}{w} + \frac{V_1^2}{2g} = \frac{P_2}{w} + \frac{V_2^2}{2g}$$

h

$$\frac{P_1 - P_2}{w} = \frac{V_2^2 - V_1^2}{2g} \Rightarrow V_2^2 - V_1^2 = 2gh$$

$\Rightarrow h = \frac{P_1 - P_2}{w} \longrightarrow$ for horizontal Pipe [i.e. Pressure head difference]

$h = \left(\frac{P_1}{w} + z_1\right) - \left(\frac{P_2}{w} + z_2\right) \longrightarrow$ for inclined pipe [i.e. Piezometric head difference]

$\Rightarrow$ for incompressible fluid

$$A_1V_1 = A_2V_2 = Q_1 = Q_2 = Q$$

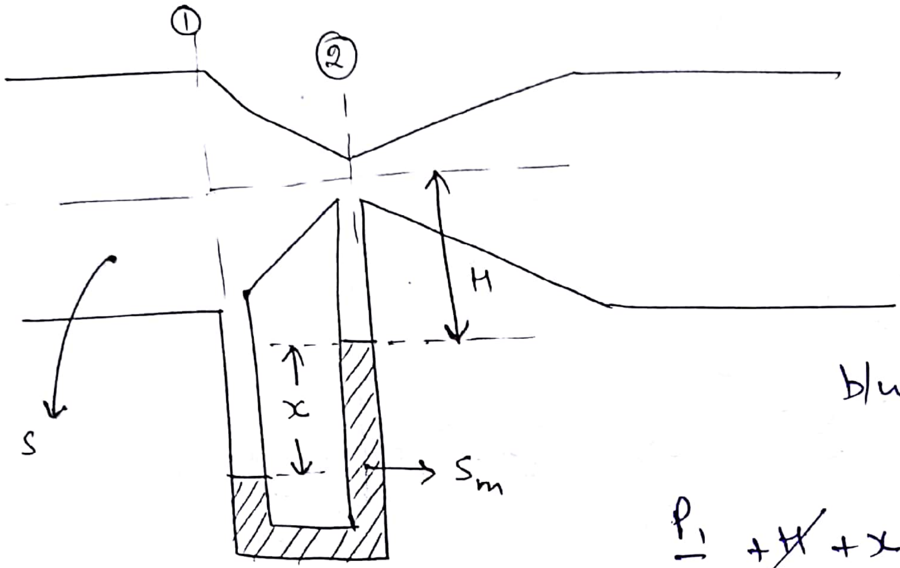
$$\Rightarrow V_2 = \frac{Q}{A_2}, \quad V_1 = \frac{Q}{A_1}$$

$$\frac{Q^2}{A_2^2} - \frac{Q^2}{A_1^2} = 2gh$$

$$\Rightarrow Q^2\left[\frac{A_1^2 - A_2^2}{A_1^2 A_2^2}\right] = 2gh$$

$$\Rightarrow \boxed{Q_{theo.} = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh}}$$

Calculation of "h"



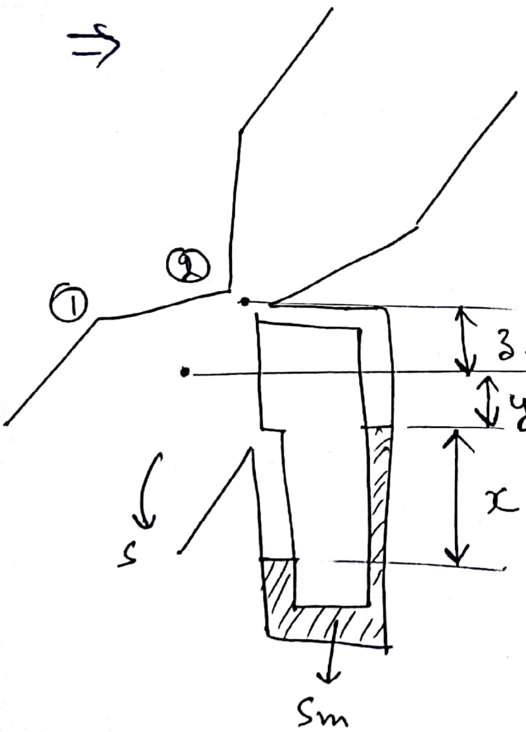
b/w ① & ②

$$\frac{P_1}{w} + \cancel{H} + x - \frac{S_m}{S}x - \cancel{H} = \frac{P_2}{w}$$

$$\Rightarrow \frac{P_1}{w} - \frac{P_2}{w} = x\left[\frac{S_m}{S} - 1\right]$$

$$\Rightarrow \boxed{h = x\left[\frac{S_m}{S} - 1\right]} \Rightarrow \text{for horizontal pipe}$$

⇒



$$\frac{P_1}{w} + \cancel{y} + x - \frac{S_m}{S}x - \cancel{y} - z_2 + z_1 = \frac{P_2}{w}$$

$$\left(\frac{P_1}{w} + z_1\right) - \left(\frac{P_2}{w} + z_2\right) = h = x\left\{\frac{S_m}{S} - 1\right\}$$

$$\Rightarrow \boxed{h = x\left[\frac{S_m}{S} - 1\right]}$$

Note:- For inverted type U-tube manometer,

$$\boxed{h = x\left(1 - \frac{S_m}{S}\right)}$$

Coefficient discharge (C_d) → 0.94 to .98
→ losses are less
→ Accuracy is high
→ expansive

it is defined as ratio of actual discharge to theoretical discharge.

$$\boxed{C_d = \frac{Q_{actual}}{Q_{theoretical}}}$$

$$\Rightarrow \boxed{Q_{actual} = C_d \times \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gh}} \quad \text{—— (1)}$$

$$\Rightarrow \frac{P_1}{w} + \frac{V_1^2}{2g} = \frac{P_2}{w} + \frac{V_2^2}{2g} + h_e$$

$$\frac{P_1}{w} - \frac{P_2}{w} - h_e = \frac{V_2^2 - V_1^2}{2g}$$

$$(h - h_e) = \frac{V_2^2 - V_1^2}{2g} \Rightarrow Q_{act.} = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2g(h - h_e)} \quad \text{(2)}$$

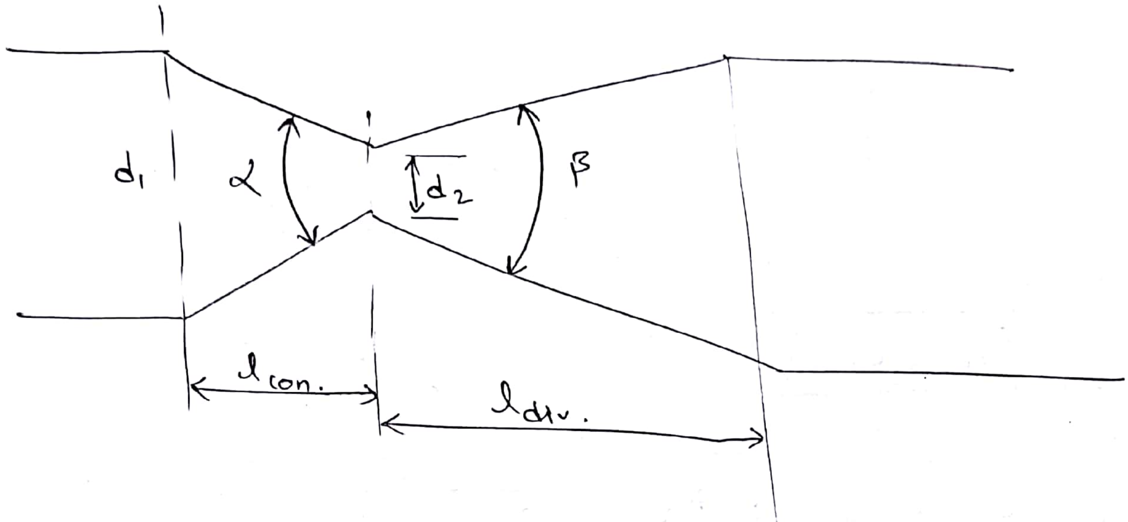
comparing (1) & (2)

$$\boxed{C_d = \sqrt{\frac{(h - h_e)}{h}}}$$

here, h_e = head loss

$$h = x\left(\frac{S_m}{S} - 1\right)$$

General dimensions of a venturimeter



- $d_2 = \left(\frac{1}{3} \text{ to } \frac{1}{2}\right) d_1$ → To avoid cavitation
- α = angle of convergence $= 20\text{-}22^\circ$ → To avoid cavitation

  β = angle of divergence = must be below 7° [To avoid Boundary layer seperation]

P-27
Q4

$$\frac{P_1}{w} + \frac{v_1^2}{2g} + \cancel{z_1} = \frac{P_2}{w} + \frac{v_2^2}{2g} + \cancel{z_2}$$

$$\frac{P_1 - P_2}{\cancel{w}\ \rho \cancel{g}} = \frac{v_2^2 - v_1^2}{2\cancel{g}}$$

$$\left(30 \times \cancel{10^3} \times \frac{2}{\cancel{1000}}\right) = v_2^2 - v_1^2 = 60$$

$$A_1 v_1 = A_2 v_2$$

$$v_1 = \frac{A_2}{A_1} v_2$$

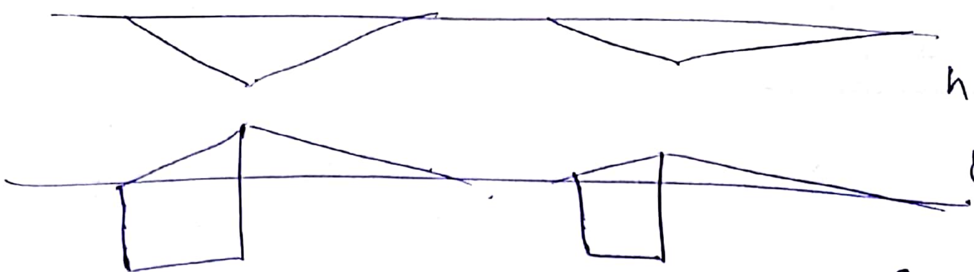
$$v_2 = \frac{A_1}{A_2} \times v_1$$

$$= 4 v_1$$

$$\Rightarrow 16 v_1^2 - v_1^2 = 60$$

$$\Rightarrow \boxed{v_1 = 2 \text{ m/s}}$$

P-28
05



$h = x\left(\frac{S_m}{S} - 1\right)$

$Q \propto \sqrt{h}$

(h) $\frac{1}{2}\frac{A_1}{A_a} = 2A_a$ $\quad x\frac{A_1}{A_b} = A_b$ if $(5h)$

$\frac{A_1}{A_a} = 2$

$\frac{A_1}{A_b} = x$

$\frac{A_1 A_a}{\sqrt{A_1^2 - A_a^2}}\sqrt{h} = \frac{A_1 A_b}{\sqrt{A_1^2 - A_b^2}} \times \sqrt{5}\sqrt{h}$

$\frac{A_1^2}{\cancel{8}} - \frac{A_1^2}{4}$

$\frac{3}{4}$

$\frac{A_1^2/2}{\sqrt{\frac{3}{4}}\,A_1} = \frac{x A_1^2 \sqrt{5}}{(1-x^2)A_1}$

$\left(\frac{1-x^2}{x}\right) = \sqrt{\frac{15}{4\times4}}$

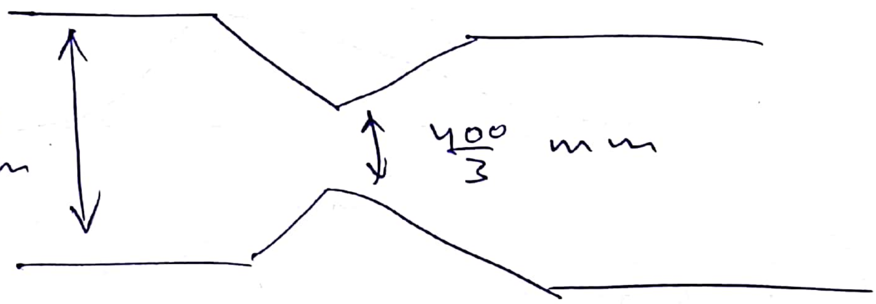
$\frac{A_1}{\sqrt{4-1}} \times 1 = \frac{A_1\sqrt{5}}{\sqrt{x^2-1}}$

$\frac{1}{3} = \frac{5}{x^2-1}$

$x^2 - 1 = 15$

$x^2 = 16 \Rightarrow x = 4$

P-28
06



400 mm

$\frac{400}{3}$ mm

$h - h_L = .964$

$P_1 = 1.405 \times 9.81 \times 10^4$ Pa

$P_2 = (\cancel{760} - 375)$ mm Hg $\quad -50031$ Pa

$= -.375 \times 13600 \times 9.81 = \cancel{51365.16\ Pa}$

$Q = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2g(h - h_L)} \longrightarrow .964$

ms
.96

$h = \frac{P_1 - P_2}{w} = \cancel{8.814\ m}$ 19.15 m

$.339 \times \frac{\pi}{4}$

$= .266\ m^3/s$

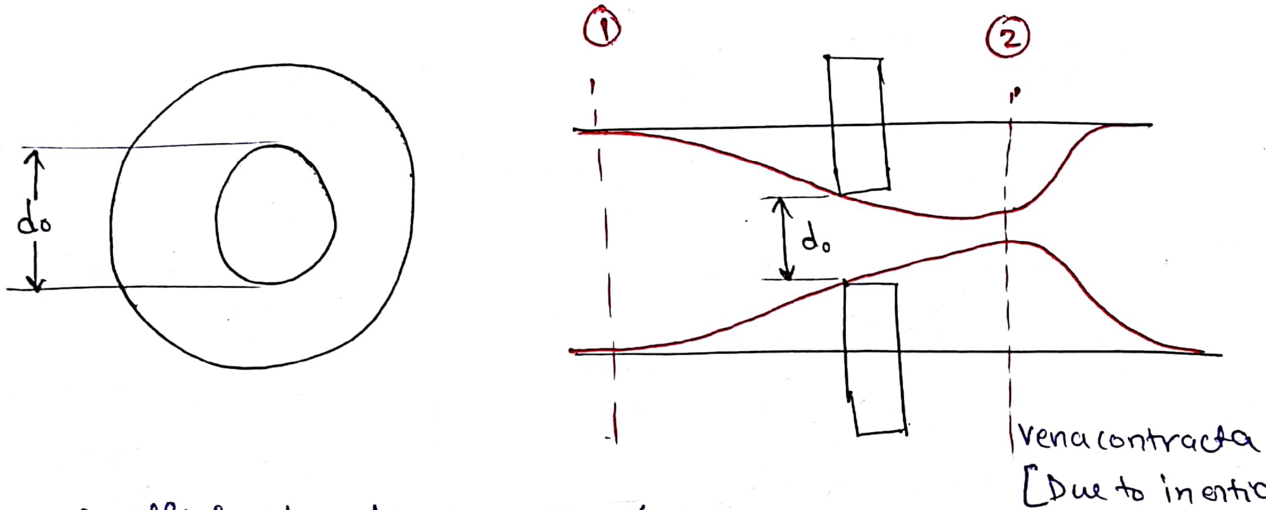
$Q = \frac{\frac{\pi}{4}(.4)^2 \times \frac{\pi}{4}(\frac{.4}{3})^2}{\sqrt{\frac{\pi}{4}\left(\frac{\pi}{4}.4^4 - \frac{\pi}{4}(\frac{.4}{3})^4\right)}} \times \sqrt{2g \times .96 \times \cancel{8.814}\ 19.15}$

$= \cancel{0.23048\ m^3}$

$= 230.48$ liters

2) Orifice meter

- Orifice meter is used to measure the discharge of fluid flow and it is the cheapest device to measure discharge.
- The losses are very high in case of orificemeter and that is why the value of C_d is low.
- C_d of an orifice meter lies b/w 0.68 to 0.74



Coefficient of contraction (C_c)

$$C_c = \frac{\text{Actual minimum Area}}{\text{Theoretical mini. Area}} = \frac{A_2}{A_0}$$

$$\Rightarrow \quad A_2 = C_c A_0$$

Continuity eqn b/w ① & ②

$$A_1 V_1 = A_2 V_2 \implies V_1 = \frac{A_2 V_2}{A_1} = \frac{C_c A_0 V_2}{A_1}$$

B-eqn b/w ① & ②

$$\frac{P_1}{w} + \frac{V_1^2}{2g} = \frac{P_2}{w} + \frac{V_2^2}{2g}$$

$$h = \frac{P_1 - P_2}{w} = \frac{V_2^2 - V_1^2}{2g}$$

$$\boxed{V_2^2 - V_1^2 = 2gh}$$

$$V_2^2 - \frac{C_c^2 A_0^2 V_2^2}{A_1^2} = 2gh$$

$$V_2^2 = \frac{\sqrt{2gh}}{\sqrt{1 - \frac{C_c^2 A_0^2}{A_1^2}}}$$

$$Q = A_2 V_2$$

$$= \frac{C_c A_0 \sqrt{2gh}}{\sqrt{1 - \frac{C_c^2 A_0^2}{A_1^2}}} \times \frac{\sqrt{1 - \frac{A_0^2}{A_1^2}}}{\sqrt{1 - \frac{A_0^2}{A_1^2}}} \Rightarrow C_d$$

$$\boxed{Q = C_d \cdot \frac{A_1 A_0}{\sqrt{A_1^2 - A_0^2}} \times \sqrt{2gh}}$$

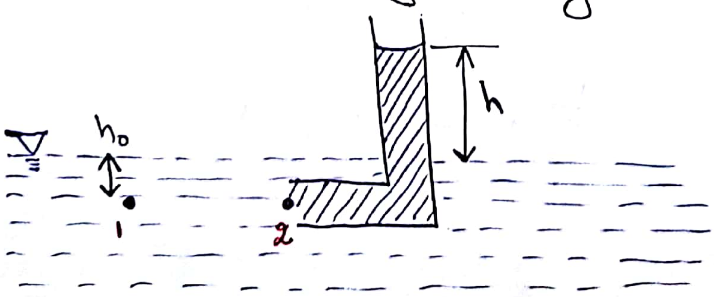
A_0 = Area of orifice
A_1 = Area of Pipe.

- The losses in case of orifice meter are high bcz the convergence of area is sudden and hence leading to losses
- The losses in case of ventury meter are less bcz the convergence of area is gradual and slow and hence losses are less.

3) Pitot tube

- A Pitot tube is a 90° bend glass tube that is used to measure ~~of~~ velocity of fluid flow.

Case-1 measurement of velocity in open channels



$P_1 = \rho g h_0$

$h_0 = \frac{P_1}{w}$

$V_2 = 0$ [fluid isentropically Bought to rest]

$$P_2 = \rho g (h + h_0)$$

$$h + h_0 = \frac{P_2}{w}$$

Applying B-eqn b/w ① & ②

$$\frac{P_1}{w} + \frac{V_1^2}{2g} = \frac{P_2}{w} + \cancel{\frac{V_2^2}{2g}}$$

$$\boxed{\frac{P_1}{w}} + \boxed{\frac{V_1^2}{2g}} = \boxed{\frac{P_2}{w}}$$

$\Downarrow$ Static head

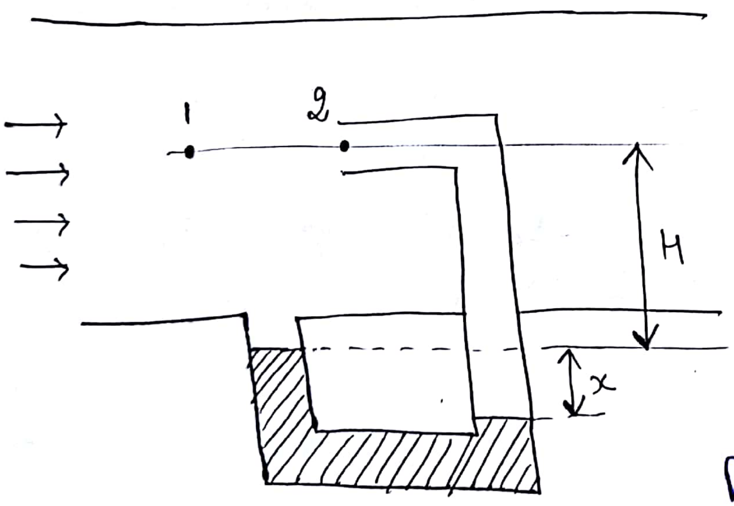
$\Downarrow$ Dynamic head

$\Downarrow$ Stagnation Head

$$\frac{V_1^2}{2g} = \frac{P_2}{w} - \frac{P_1}{w} = h + \cancel{h_0} - \cancel{h_0}$$

$$\Rightarrow \boxed{V_1 = \sqrt{2gh}} ** = \sqrt{2g\left(\text{Stagnation Head} - \text{Static Head}\right)}$$

Case-2 Velocity in Pipes



Applying manometry

$$\frac{P_1}{w} + H + \frac{S_m}{S}x - x - H = \frac{P_2}{w}$$

$$\frac{P_2 - P_1}{w} = x\left[\frac{S_m}{S} - 1\right]$$

Applying B-eqn b/w ① & ②

$$\frac{P_1}{w} + \frac{V_1^2}{2g} = \frac{P_2}{w} + \cancel{\frac{V_2^2}{2g}}^{\,0}$$

$$\frac{V_1^2}{2g} = \frac{P_2 - P_1}{w} = x\left[\frac{S_m}{S} - 1\right]$$

$$\Rightarrow \boxed{V_1 = \sqrt{2gh}} \quad \& \quad \boxed{h = x\left[\frac{S_m}{S} - 1\right]}$$

P-29
Q5

$$Q_v = Q_o$$

$$C_{d_v} \times K\sqrt{h_v} = C_{d_o} \times K\sqrt{h_o}$$

$$\frac{.98}{.61} = \sqrt{\frac{h_o}{h_v}} \Rightarrow \frac{h_o}{h_v} = 2.58$$

$$\Rightarrow \frac{h_v}{h_o} = 0.387$$

P-30
Q6

$$v = \sqrt{2g\,x\left(\frac{S_m}{S} - 1\right)}$$

$$(1.2)^2 = 2g \times x\,(0.4)$$

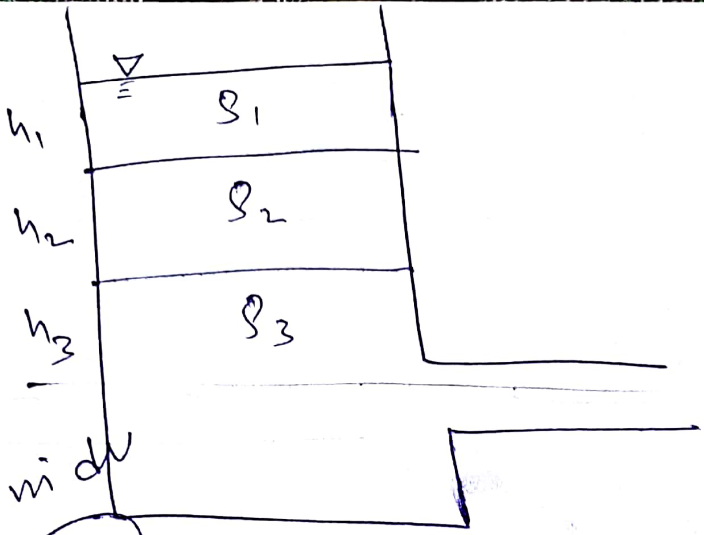
$$\Rightarrow x = 0.183 \text{ m}$$

P-31
Q7

$$Q \propto \sqrt{h}$$

$$\% = \left(\frac{\sqrt{h} - \sqrt{1.05\,h}}{\sqrt{h}}\right) \times 100 = -2.469\ \%$$

P-29
Q2



$\rho_3 h_3 = \rho_2 h_2$

$$\boxed{\cancel{h_2 = \frac{\rho_3 h_3}{\rho_2}} \quad \cancel{h_1 = \frac{\rho_3 h_3}{\rho_1}}}$$

$\rho g h$

$F = m\,dv$

$= \rho Q V$

$\frac{p}{\rho g}$

$\frac{1}{2}mv^2$

$\frac{\rho Q v^2}{2}$

$$\cancel{v = \sqrt{2g\left(h_3 + \frac{\rho_3 h_3}{\rho_2} + \frac{\rho_3 h_3}{\rho_1}\right)}}$$

$$\cancel{v = \sqrt{2gh_3\left[1 + \frac{\rho_3}{\rho_2} + \frac{\rho_3}{\rho_1}\right]}}$$

$$\frac{h_2 \rho_2}{\rho_3} = y_2 \qquad y_1 = \frac{h_1 \rho_1}{\rho_3}$$

$$v = \sqrt{2gh_3\left[\frac{h_3}{h_3} + \frac{h_2 \rho_2}{\rho_3 h_3} + \frac{h_1 \rho_1}{\rho_3 h_3}\right]}$$

$$v = \sqrt{2gh_3\left[1 + \frac{\rho_2 h_2}{\rho_3 h_3} + \frac{\rho_1 h_1}{\rho_3 h_3}\right)}$$

P-30
Q8

$$v = \sqrt{2g \times .01\left(\frac{1000}{1.2} - 1\right)}$$

$= 12.779$ m/s

CHAPTER – 5

LAMINAR FLOW [Viscous incompressible flow]

- Reynold's no. $= \frac{\rho V D}{\mu} = \frac{VD}{\nu}$

D = characteristic length
= diameter ⟶ for pipe [$D = \frac{4A}{P}$ for non circular x-s/c]
= length ⟶ for plate

Pipe

$Re < 2000$ laminar
$2000 < Re < 4000$ Transient
$Re > 4000$ turbulent

Plate

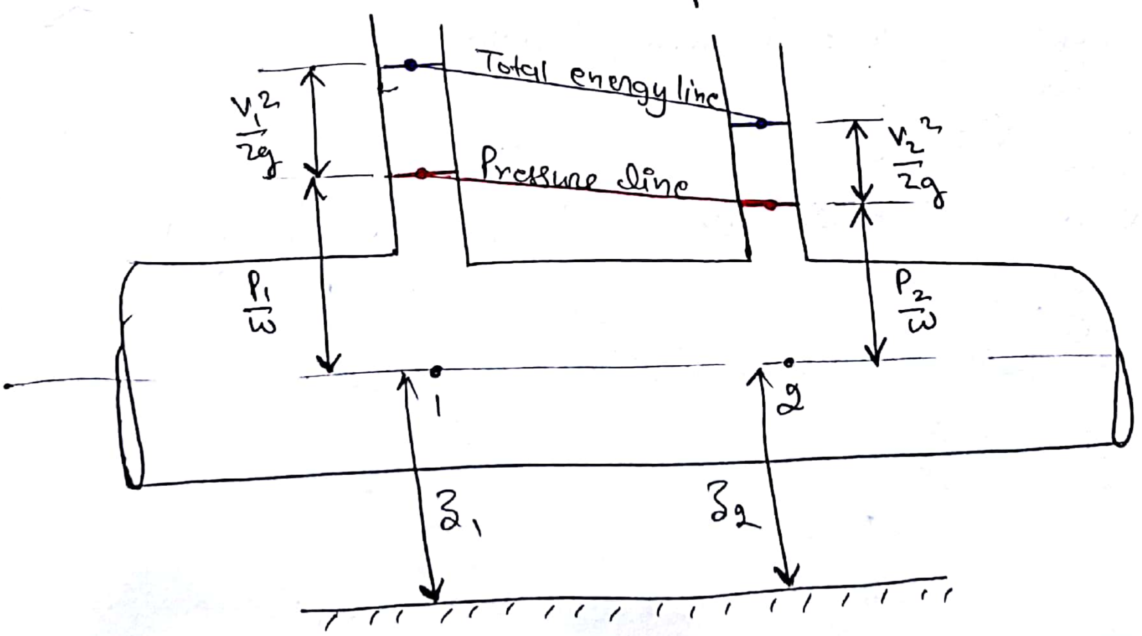
$Re < 5\times10^5$ Laminar
$Re > 5\times10^5$ turbulent
[transient is small so neglected]

open channels

$Re < 500$ Laminar
$500 < Re < 1000$ transient
$Re > 1000$ turbulent

Soil

$Re < 1$ Laminar
$1 < Re < 2$ Transient
$Re > 2$ turbulent



Continuity eqn
$A_1V_1 = A_2V_2$
$\boxed{V_1 = V_2}$

⇒ B-eqn for horizontal pipe

$$\frac{P_1}{w} + \cancel{\frac{V_1^2}{2g}} = \frac{P_2}{w} + \cancel{\frac{V_2^2}{2g}} + h_e$$

$$\frac{P_1 - P_2}{w} = h_e$$

⇒ B-eqn for inclined pipe

$$\frac{P_1}{w} + \cancel{\frac{V_1^2}{2g}} + z_1 = \frac{P_2}{w} + \cancel{\frac{V_2^2}{2g}} + z_2 + h_e$$

$$\left(\frac{P_1}{w} + z_1\right) - \left(\frac{P_2}{w} + z_2\right) = h_e$$

⇒ <u>Darcy weisbach eqn</u>

$$\boxed{h_e = \frac{f l v^2}{2gD}}$$

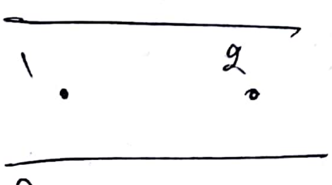
valid for all
- laminar
- turbulent
- horizontal
- inclined

here, f = friction factor

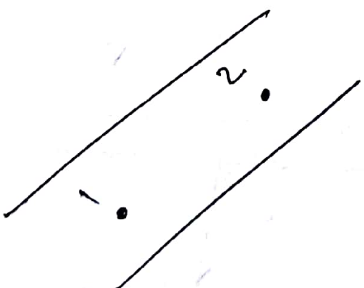
$f = 4f'$; f' = fanning factor

⇒ <u>Direction of fluid flow</u>

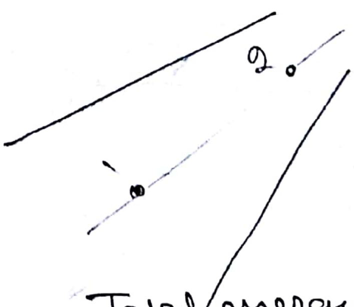
The fluid always flows from higher energy to lower energy.



Pressure energy will decide the flow.



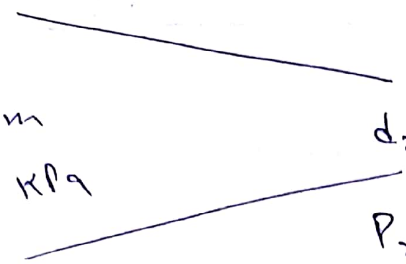
Piezometric head will decide the flow



Total energy will decide the flow

P-31
Q10

$\rho g = w = 5000 \ N/m^3$



$d_1 = .2 \, m$ $\quad$ $d_2 = .1 \, m$

$P_1 = 150 \ KPa$ $\quad$ $P_2 = 50 \ KPa$

$$\frac{P_1}{w} + \frac{v_1^2}{2g} = \frac{P_2}{w} + \frac{v_2^2}{2g}$$

$A_1 V_1 = A_2 V_2$

$V_2 = 4 V_1$

$$\frac{P_1 - P_2}{w} = \frac{v_2^2 - v_1^2}{2g}$$

$$\frac{(150 - 50) \times \cancel{1000}}{5 \times \cancel{1000}} = \frac{16 v_1^2 - v_1^2}{2g}$$

$$20 = \frac{15}{2 \times 9.81} v_1^2 \Rightarrow v_1 = 5.1146 \ m/s$$

$$Q = \frac{\pi}{4} \times (.2)^2 \times V = 0.16 \ m^3/s$$

P-31
Q 11

$\boxed{A_1 V_1 = A_2 V_2}$

$A_1 = \frac{\pi}{4} \times (.02)^2$

$V_1 = 2 \, m/s$

$$\frac{v_1^2}{2g} + z_1 = \frac{v_2^2}{2g} + z_2$$

$d_1^2 v_1 = d_2^2 V_2$

$$\frac{v_2^2 - v_1^2}{\cancel{2g}} = .5 \times 2g$$

$A_2 = \frac{A_1 V_1}{V_2}$

$v_2^2 = 4 + \frac{1}{\cancel{2}} \times \cancel{2} \times 9.81$

$V_2 = 3.716 \ m/s$

$$d_2^2 = \frac{d_1^2 \times V_1}{V_2}$$

$$d_2^2 = \frac{d_1^2 \times v_1}{v_2}$$

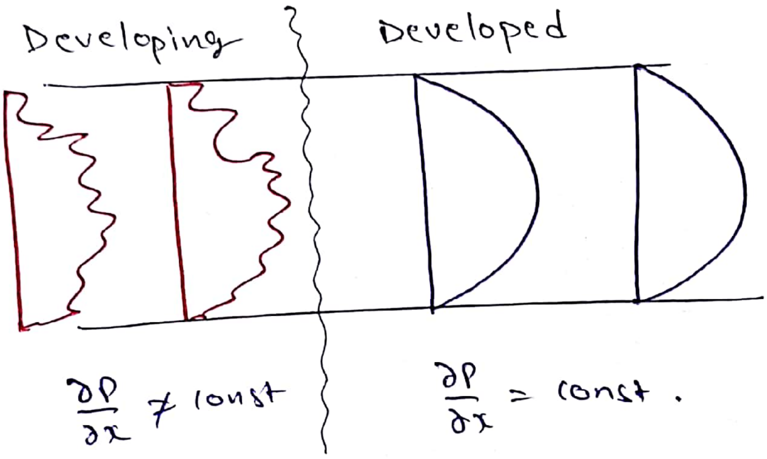
$\Rightarrow d_2 = 14.67$

Haigen - Poiseuelle flow [Laminar flow in pipe]

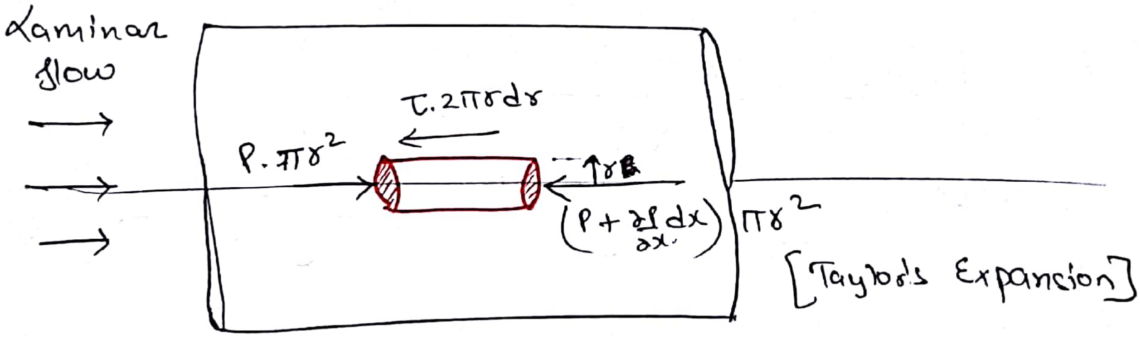
Assumptions : 1) steady flow

2) flow is fully developed → velocity Profile is const. w.r.t. length

→ $\frac{\partial P}{\partial x} = \text{const.}$



1) shear stress distribution



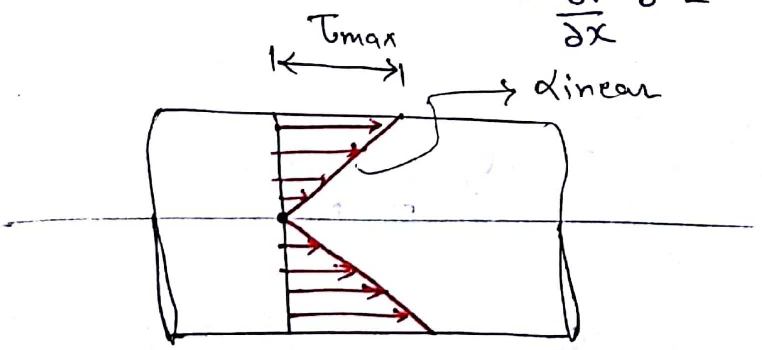
$$\Sigma F = ma \Rightarrow \Sigma F = 0$$

$$\cancel{P\pi r^2} - \cancel{P\pi r^2} - \frac{\partial P}{\partial x} dx\, \pi r^2 = \tau\, 2\pi r\, dr$$

1

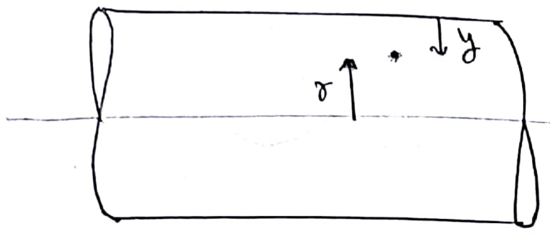
$$-\frac{\partial P}{\partial x} r = \tau \times 2 \Rightarrow \boxed{\tau = -\frac{\partial P}{\partial x} \times \frac{r}{2}}$$

$$\boxed{\tau_{max} = -\frac{\partial P}{\partial x} \times \frac{R}{2}}$$



Shear stress distribution in Pipe [Haigen-Poiseuelle flow]

2) Velocity distribution in Laminar flow through Pipe



$$r + y = R$$

$$dr + dy = 0$$

$$dr = -dy$$

$$\tau = \mu \frac{du}{dy} = -\mu \frac{dy}{dr} = \text{equation } (1)$$

$$\cancel{-}\mu \frac{du}{dr} = \cancel{-}\frac{\partial p}{\partial x} \cdot \frac{r}{2}$$

$$du = \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right) \cdot r\,dr \Rightarrow u = \frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right) r^2 + c$$

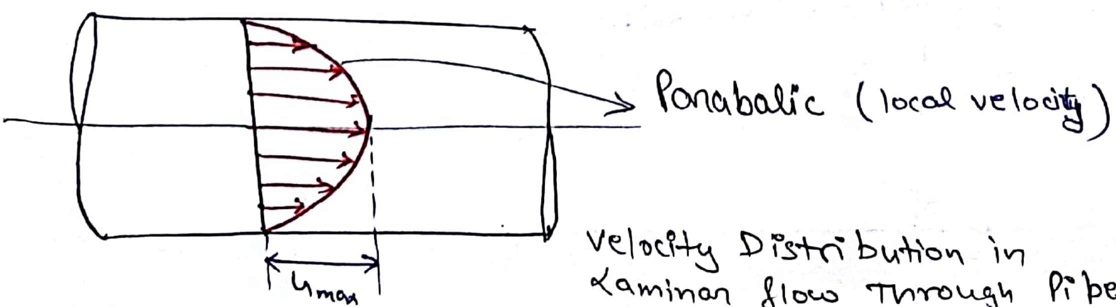
at $r = R$
$u = 0$

$$0 = \frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right) R^2 + c \Rightarrow c = \frac{-1}{4\mu}\left(\frac{\partial p}{\partial x}\right) R^2$$

$$u = \frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right) r^2 - \frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right) R^2$$

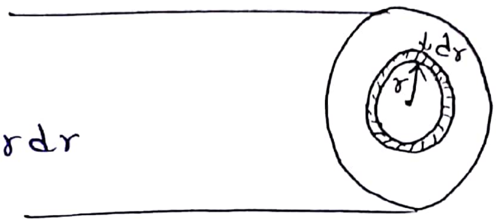
$$\boxed{u = -\frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right) R^2 \left(1 - \frac{r^2}{R^2}\right)} \quad \text{—— (2)}$$

$$\boxed{u_{max} = -\frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right) R^2} \Rightarrow \boxed{\frac{u}{u_{max}} = 1 - \frac{r^2}{R^2}}$$



Velocity Distribution in Laminar flow Through Pipes

3) Discharge (Q)



$$dQ = u\,dA$$

$$= \frac{-1}{4\mu}\left(\frac{\partial P}{\partial x}\right) R^2\left(1-\frac{r^2}{R^2}\right) 2\pi r\,dr$$

$$= u_{max} \times 2\pi \int_0^R \left(r - \frac{r^3}{R^2}\right) dr$$

$$= u_{max} \times 2\pi \left[\frac{r^2}{2} - \frac{r^4}{4R^2}\right]_0^R$$

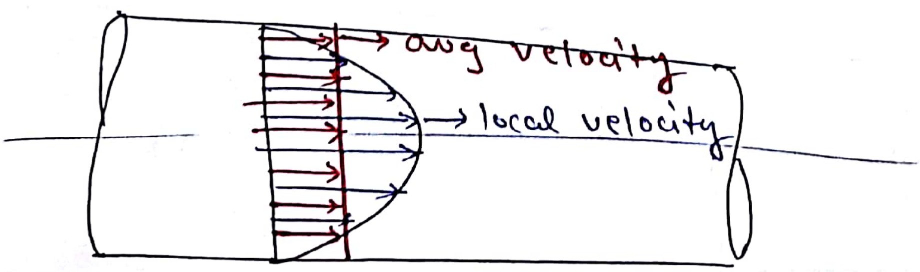
$$= u_{max} \cdot 2\pi \cdot \left(\frac{R^2}{2} - \frac{R^2}{4}\right) = u_{max} \cdot \frac{\cancel{2}\pi R^2}{\cancel{2}\times 2}$$

$$\boxed{Q = u_{max} \times \frac{A}{2}} = V_{avg.} \times A$$

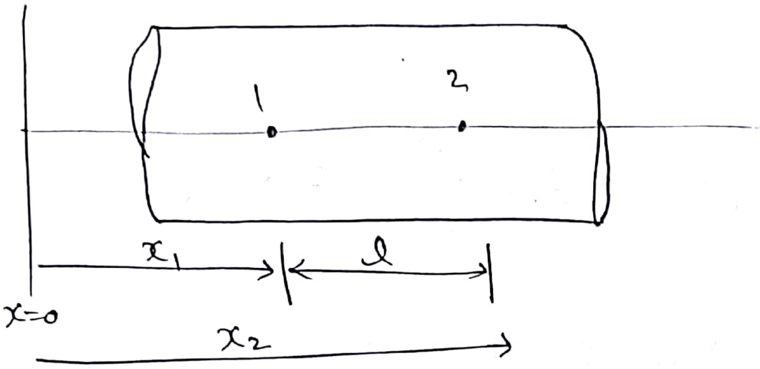
$$\Rightarrow \quad V_{avg} = \boxed{V = \frac{u_{max}}{2}}$$ ⇒ for laminar flow through pipe

⇒ $Q = V.A$

Note: in a pipe flow the velocity varries in y-dirn whereas the pressure varries in x-dirn. And hence a pipe flow is a two Dimensional flow but a pipe flow is converted from 2D to 1D flow by the concept of average velocity. The variable velocity "u" are replaced by a constant average velocity that gives the same result and hence 2D is converted to 1D.



4) Pressure drop in Laminar flow through Pipe



$$V = \frac{u_{max}}{2}$$

$$u_{max} = 2V$$

$$-\frac{1}{4\mu}\left(\frac{\partial P}{\partial x}\right) R^2 = 2V$$

$$-\partial P = \frac{8\mu V(\partial x)}{R^2}$$

$$\Rightarrow \boxed{P_1 - P_2 = \frac{8\mu V l}{R^2}} \Rightarrow \boxed{P_1 - P_2 = \frac{32\mu V l}{D^2}}$$

Valid for only horizontal pipe [Laminar]

$$P_1 - P_2 = w h_l$$

$$\rho g \times \frac{f l V^2}{2gD} = \frac{32\mu V l}{D^2} \Rightarrow \boxed{f = \frac{64}{Re}}$$

Valid for both horizontal & inclined pipe [Laminar]

5) Shear velocity (V_*)

$$\tau = -\frac{\partial P}{\partial x} \cdot \frac{r}{2}$$

at $r = r_1$, $\tau = \tau_0$

$$\tau_0 = -\frac{\partial P}{\partial x} \cdot \frac{R}{2}$$

$$\tau_0 = -\frac{(P_2 - P_1)}{(x_2 - x_1)} \cdot \frac{R}{2}$$

$$\tau_0 = \frac{(P_1 - P_2)}{l} \times \frac{D}{4}$$

$$\tau_0 = \rho g \times \frac{f l V^2}{2gD} \times \frac{D}{4} \times \frac{1}{l}$$

$$\boxed{\tau_0 = \frac{\rho f V^2}{8}}$$

$$\boxed{V_* = V \cdot \sqrt{\frac{f}{8}} = \sqrt{\frac{\tau_0}{\rho}}}$$

→ valid for both → laminar
→ turbulent

P-32
Q1

$r = .1\ m$

$V = 5\ m/s$

$u = u_{max} \times \left(1 - \frac{r^2}{R^2}\right)$

$= 10 \times \left(1 - \frac{1}{4}\right) = \frac{3}{\cancel{4}} \times 1\cancel{0}^{2.5} = 7.5\ m/s$

P-32
Q2

$\cancel{V/V_0}\ \cancel{\frac{u_{max}}{2}} = \cancel{u_{max}}\left(1 - \frac{r^2}{R^2}\right)$

$R^2 = 2R^2 - 2r^2$

$2r^2 = R^2 \qquad r = \frac{R}{\sqrt{2}} \Rightarrow r = .707R$

P-32
Q3

$l = 100\ m$

$D = .15\ m$

$P_1 = 1.08 \times 10^6\ Pa$

$P_2 = .95 \times 10^6\ Pa$

$\nu = \frac{\mu}{\rho} = 412.5 \times 10^{-6}\ \frac{m^2}{s}$

$\rho = 918\ kg/m^3$

$V = \frac{1}{8\mu}\left(\frac{\partial P}{\partial x}\right) \times R^2$

$= 241.38\ m/s$

$Re = \frac{VD}{\nu}$

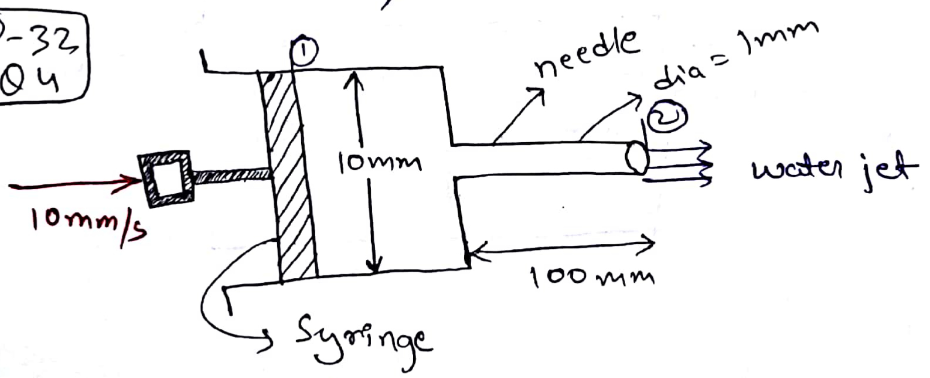
$= \frac{2.4138 \times .15}{412.5 \times 10^{-6}}$

$= 877.76$

$Q = VA$

$= 0.0426\ m^3/s$

P-32
Q4



A) B-eqn ① & ②

$$\frac{P_1}{w} + \frac{V_1^2}{2g} + \cancel{z_1} = \cancel{\frac{P_2}{w}} + \frac{V_2^2}{2g} + \cancel{z_2}$$

$$\frac{P_1}{w} + \frac{V_1^2}{2g} = \frac{V_2^2}{2g}$$

$$\frac{P_1}{9810} = \frac{(1)^2 - (.01)^2}{2 \times 9.81} \Rightarrow P_1 = 499.95 \text{ Pa}$$

$$\Rightarrow F = P_1 A = .039 \text{ N}$$

B) B-eqn b/w ① & ②

$$\frac{P_1}{w} + \frac{V_1^2}{2g} + \cancel{z_1} = \cancel{\frac{P_2}{w}} + \frac{V_2^2}{2g} + \cancel{z_2} + h_{l\,needle}$$

$$h_l = \frac{f l V^2}{2gD} \qquad f = \frac{64}{Re} = \frac{64\mu}{\rho V D} \qquad Re = \frac{VD}{\nu}$$

$$= 0.326 \text{ m}$$

$$\cancel{P_1 - P_2 = \frac{32\mu V l}{D^2}}$$

$$\frac{64\mu V l}{\rho V D \, 2g D}$$

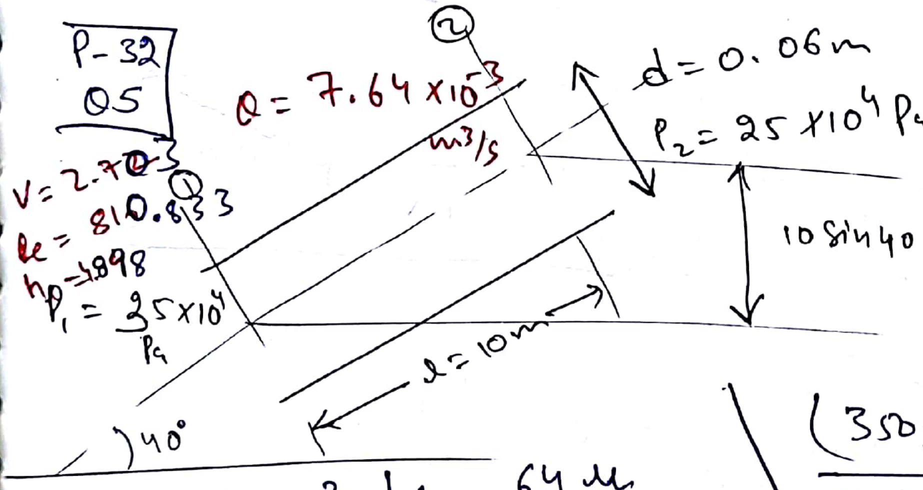
$$\frac{P_1}{w} = \frac{(1)^2 - (.01)^2}{2g} + 0.326$$

$$P_1 = 3699.95 \text{ Pa} \Rightarrow F = P_1 A = 0.29 \text{ N}$$

$$h = \frac{64 \mu V l}{\rho D \times 2g}$$

$$\mu = \rho\nu$$

P-32
Q5



B-eqn b/w ① & ②

$$\frac{P_1}{w} + \cancel{\frac{V_1^2}{2g}} + z_1 = \frac{P_2}{w} + \cancel{\frac{V_2^2}{2g}} + z_2 + h_l$$

$$\frac{(350 - 250) \times 10^3}{900 \times 9.81} - 10 \sin 40 = h_l$$

$$11.326 - 6.4278 = h_f$$

$$\Rightarrow h_l = 4.898 \text{ m}$$

$$h_l = \frac{f l V^2}{2gD} \quad \Bigg| \quad f = \frac{64\mu}{\rho V D} = \frac{2.37 \times 10^{-4}}{V}$$

$$4.898 = \frac{64 \nu}{\cancel{V} D} \times \frac{l V^2}{2gD} \Rightarrow V =$$

$h_l =$

Correction factors

There are 2 correction factors
1) momentum correction factor (β)
2) Kinetic energy correction factor (α)

1. momentum correction factor (β)

it is defined as the momentum ~~beget~~ per second based on actual velocity to the momentum per second ~~to~~ based on average velocity.

2. Kinetic energy correction factor (α)

it is defined as the ratio of KE./s based on actual velocity to the KE/s based on average velocity.

Let P = momentum

$\dot{P}$ = momentum / sec.

$$\beta = \frac{\dot{P}_{actual}}{\dot{P}_{avg}} = \frac{\int_0^R \rho\, dA \times u . u}{\rho A V \times V} \Rightarrow \boxed{\beta = \frac{1}{AV^2}\int_0^R u^2 dA}$$

$$\alpha = \frac{\dot{KE}_{actual}}{\dot{KE}_{avg}} = \frac{\int_0^R \frac{u^3 dA}{2}\rho}{\frac{\dot{m} V^2}{2}} \Rightarrow \boxed{\alpha = \frac{1}{AV^3}\int_0^R u^3 dA}$$

Ques: find β for Laminar flow through Pipe

Soln

$$\beta = \frac{1}{AV^2}\int_0^R u^2 dA = \frac{1}{AV^2} \times \int_0^R u_{max}^2 \left(1 - \frac{r^2}{R^2}\right)^2 . 2\pi r\, dr$$

$$= \frac{1}{AV^2} \times \left[u_{max}^2 \times \left(r + \frac{r^5}{R^4} - 2\frac{r^3}{R^2}\right) 2\pi\, dr \right.$$

$$\beta = \frac{2\pi u_{max}^2}{AV^2}\left[\cancel{\frac{R^2}{2}} + \frac{R^2}{6} - \cancel{\frac{R^2}{2}}\right] = \frac{2\pi u_{max}^2 \times \frac{R^2}{6}}{\pi R^2 \times \frac{u_{max}^2}{4}}$$

$$\Rightarrow \boxed{\beta = 1.33}$$

Que: find KE correction factor for flow through pipe

$$\alpha = \frac{KE_{actual}}{KE_{avg}} \; ; \; KE_{avg} = \frac{1}{2}\dot{m}V^2 = \frac{1}{2} \times \frac{\rho A V^3}{1}$$

$$\frac{KE_{actual}}{KE_{avg}} = \frac{1}{AV^3}\int_0^R u_{max}^3 \times \left(1 - \left(\frac{r^2}{R^2}\right)^3 - 3 \times 1^2 \times \frac{r^2}{R^2} + 3\frac{r^4}{R^4}\right) 2\pi r\, dr$$

$$= \frac{8\cancel{u_{max}^3} \times 2\cancel{\pi}}{\cancel{\pi} R^2 \times \cancel{u_{max}^3}}\int_0^R \left(r - \frac{r^7}{R^6} - \frac{3r^3}{R^2} + 3\frac{r^5}{R^4}\right) dr$$

$$= \frac{16}{R^2} \times \left[\frac{R^2}{2} - \frac{R^2}{8} - \frac{3R^2}{4} + \frac{R^2}{2}\right]$$

$$= 16 \times \left[\frac{4 - 1 - 6 + 4}{8}\right] = 16 \times \frac{1}{8} = 2$$

for flow through pipe

	Laminar	turbulent
α	2	1.33
β	1.33	1.2

- from the above we can see that the value of correction factor for laminar is more than turbulent this is bcz in laminar flow the difference b/w actual and average velocities is large as there is no intermixing of fluid particals.
- whereas in case of turbulent flow there is huge order intermixing of fluid particals which leads to less difference b/w actual and average velocities and hence correction factors are less.

CHAPTER - 6

DIMENSIONAL ANALYSIS

Buckingham π - theorem

↳ covered in Heat transfer

Various forces in fluid mechanics

1. Inertia force (F_i)

$$m = \rho V \Rightarrow m = \rho l^3$$

$$F_i = m \times a = \rho l^3 \times \frac{V}{t} = \rho l^2 \frac{l}{t} . V \Rightarrow \boxed{F_i = \rho l^2 V^2}$$

2. Surface tension force (F_s)

$$\boxed{F_s = \sigma \times l}$$

3. Gravity force (F_g)

$$F_g = m \times g = \rho l^3 \times g \Rightarrow \boxed{F_g = \rho l^3 g}$$

4. Pressure force (F_p)

$$F_p = P \times A \Rightarrow \boxed{F_p = P l^2}$$

5. Viscous force (F_v)

$$F_v = \frac{\mu A V}{y} = \frac{\mu l^2 V}{l} \Rightarrow \boxed{F_v = \mu l V}$$

6. Elastic force (F_e)

$$\boxed{F_e = K l^2}$$

Dimensionless numbers

1. **Reynold's no. (Re)** [Pipe flow, fully submerged]

$$Re = \frac{F_i}{F_v} = \frac{\rho l^2 v^2}{\mu l v} \Rightarrow \boxed{Re = \frac{\rho l v}{\mu}}$$

2. **Froude no. (F_r)** [Spillway, Notches, weirs]

$$F_r = \sqrt{\frac{F_i}{F_g}} = \sqrt{\frac{\rho l^2 v^2}{\rho l^3 g}} = \sqrt{\frac{v^2}{l g}} \Rightarrow \boxed{F_r = \frac{V}{\sqrt{l g}}}$$

3. **Euler's no. (E_u)** [cavitation]

$$E_u = \sqrt{\frac{F_i}{F_p}} = \sqrt{\frac{\rho l^2 v^2}{P l^2}} = \sqrt{\frac{\rho v^2}{P}} \Rightarrow \boxed{E_u = \frac{V}{\sqrt{P/\rho}}}$$

4. **weber's no. (W_e)** [capillarity]

$$W_e = \sqrt{\frac{F_i}{F_s}} = \sqrt{\frac{\rho l^2 v^2}{\sigma l}} = \sqrt{\frac{\rho l v^2}{\sigma}} \Rightarrow \boxed{W_e = \frac{V}{\sqrt{\frac{\sigma}{\rho l}}}}$$

5. **Mach no. (M)** [Hammer blow, Sonic, Super Sonic, Subsonic]

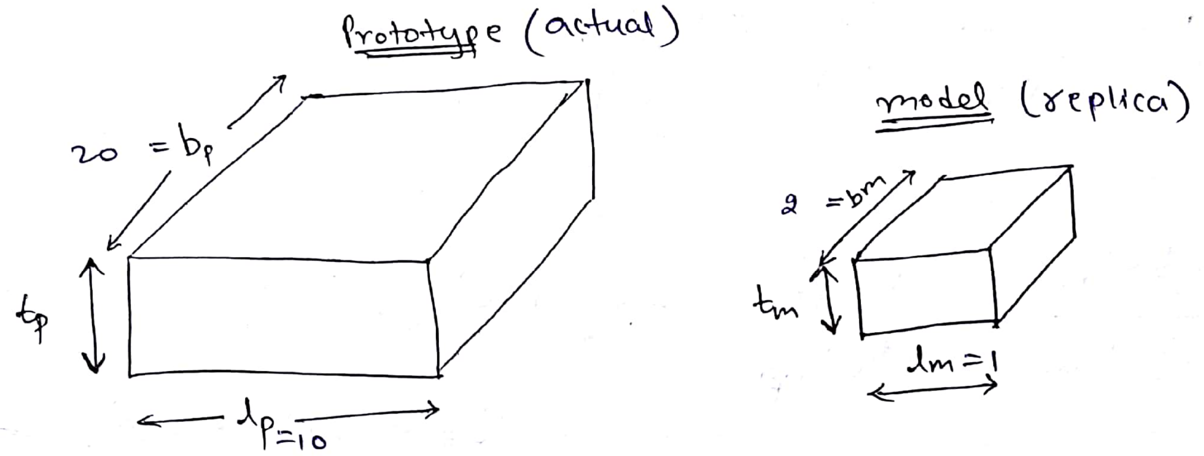
$$M = \sqrt{\frac{F_i}{F_e}} = \sqrt{\frac{\rho l^2 v^2}{K l^2}} = \sqrt{\frac{\rho v^2}{K}} \Rightarrow \boxed{M = \frac{V}{\sqrt{\frac{K}{\rho}}}} = \frac{V}{C}$$

$$\boxed{\frac{K}{\rho} = C^2}$$

Similitude & modeling

1. Geometric Similarity

A model & Prototype are said to be in Geometric similarity if the ratio of corresponding dimensions of model & Prototype are same



$$\boxed{\frac{l_m}{l_p} = \frac{b_m}{b_p} = \frac{t_m}{t_p} = L_r}$$

$$\boxed{\frac{V_m}{V_p} = L_r^3}$$

$$\boxed{\frac{A_m}{A_p} = \frac{l_m b_m}{l_p b_p} = L_r^2}$$

2. Kinematic Similarity

A model and Prototype are said to be in Kinematic similarity if the ratio of velocities at corresponding points in model and prototype are same.

For Kinematic similarity Geometric similarity is mendatory.

$$\boxed{\frac{V_{1m}}{V_{1p}} = \frac{V_{2m}}{V_{2p}} = \frac{V_{3m}}{V_{3p}}}$$

3) <u>Dynamic Similarity</u>

A model and prototype are said to be in dynamic similarity if the ratio of forces at corresponding points in model and prototype are same.

- For dynamic similarity geometric and kinematic similarity are mendatory.

$$\frac{(F_v)_{1m}}{(F_v)_{1p}} = \frac{(F_i)_{1m}}{(F_i)_{1p}} = \frac{(F_g)_{1m}}{(F_g)_{1p}} = \frac{(F_p)_{1m}}{(F_p)_{1p}}$$

$$\frac{(F_i)_{1m}}{(F_v)_{1m}} = \frac{(F_i)_{1p}}{(F_v)_{1p}} \Rightarrow (Re)_{1m} = (Re)_{1p}$$

↳ Reynolds model law

Ch-8

Q4

$$(Re)_m = (Re)_p$$

$$\left(\frac{\rho V D}{\mu}\right)_m = \left(\frac{\rho V D}{\mu}\right)_p$$

$$\left(\frac{V D}{\nu}\right)_m = \left(\frac{V D}{\nu}\right)_p$$

$$\frac{V_m}{V_p} = \frac{\nu_m D_p}{\nu_p D_m} = \frac{\nu_r}{l_r}$$

$$\frac{Q_m}{Q_p} = \frac{V_m}{V_p} \times \frac{l_m^2}{l_p^2} = \frac{\nu_r \cdot l_r^2}{l_r} = \nu_r \cdot l_r$$

05

$$(F_r)_m = (F_r)_p$$

$$\left(\frac{V}{\sqrt{lg}}\right)_m = \left(\frac{V}{\sqrt{lg}}\right)_p$$

$$\frac{V_m}{V_p} = \left(\frac{l_m}{l_p}\right)^{1/2} = l_r^{1/2}$$

$$\frac{Q_m}{Q_p} = \frac{V_m \times l_m^2}{V_p \times l_p^2} = l_r^{1/2} \times l_r^2 = l_r^{5/2}$$

06

$$l_r = \frac{1}{9}$$

$$Q_p = 10^3 \ m^3/s$$

$$\frac{Q_m}{Q_p} = (l_r)^{5/2} = Q_m = 10^3 \times \left(\frac{1}{9}\right)^{5/2}$$

$$= 10^3 \times \frac{1}{3^5}$$

$$= 0.0411 \ m^3/s \times 100$$

$$= 4.11 \ m^3/s$$

07

Prototype

$\rho = 917 \ kg/m^3$

$\mu = .29 \ N \cdot s/m^2$

$D = 15 \ cm$

$V = 2 \ m/s$

model

$\rho = 998 \ kg/m^3$

$\mu = 1.31 \times 10^{-3} \ N \ s/m^2$

$D = 1 \ cm$

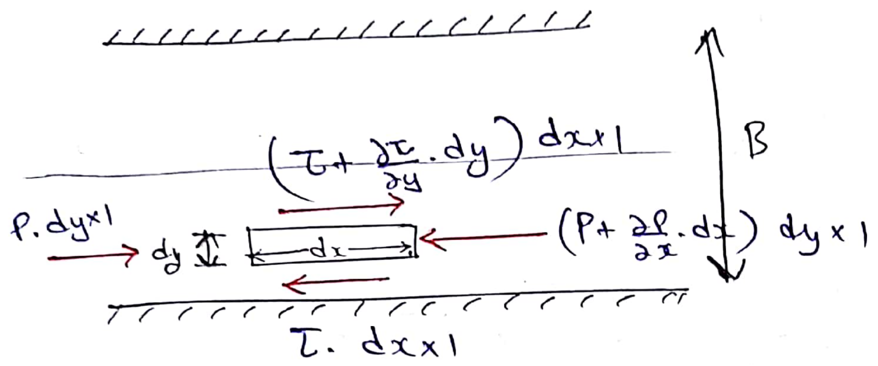
$V = ?$

$$\frac{\rho V D}{\mu} = \frac{\rho V D}{\mu}$$

$$\frac{917 \times 2 \times 15}{.29} = \frac{998 \times V \times 1}{.00131}$$

$$\Rightarrow V = 0.124 \ m/s$$

Laminar flow b/w two parallel plate [width = 1 unit]



$$\cancel{P dy} + \cancel{\tau dx} + \frac{\partial \tau}{\partial y} dy\, dx - \cancel{P dy} - \frac{\partial P}{\partial x} dx dy - \cancel{\tau dx} = 0$$

$$\frac{\partial \tau}{\partial y} \cancel{dx}\,\cancel{dy} = \frac{\partial P}{\partial x} \cancel{dx}\,\cancel{dy}$$

$$\boxed{\frac{\partial \tau}{\partial y} = \frac{\partial P}{\partial x}}$$

1. Velocity Distribution (u)

$$\tau = \mu \frac{du}{dy} \quad ; \quad \frac{\partial P}{\partial x} = \frac{\partial \tau}{\partial y}$$

$$\Rightarrow \frac{\partial P}{\partial x} = \mu \frac{\partial^2 u}{\partial y^2}$$

$$\Rightarrow \frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\left(\frac{\partial P}{\partial x}\right)$$

$$\Rightarrow \frac{\partial u}{\partial y} = \frac{1}{\mu}\left(\frac{\partial P}{\partial x}\right) y + C_1$$

$$\Rightarrow u = \frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right) y^2 + C_1 y + C_2$$

at $y = 0$; $u = 0$

$$0 = \frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right) 0 + C_1 0 + C_2 \Rightarrow C_2 = 0$$

at $y = B$; $u = 0$

$$0 = \frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right) B^{\cancel{2}} + C_1 \cancel{B}$$

$$\Rightarrow C_1 = -\frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right) B$$

$$\Rightarrow u = \frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right) y^2 - \frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right) By$$

$$\boxed{u = \frac{1}{2\mu}\left(-\frac{\partial P}{\partial x}\right)\left(By - y^2\right)}$$

at $y = \frac{B}{2}$; $u = u_{max}$

$$u_{max} = \frac{1}{2\mu}\left(-\frac{\partial P}{\partial x}\right)\left(B \times \frac{B}{2} - \frac{B^2}{4}\right)$$

$$u_{max} = \frac{1}{2\mu}\left(-\frac{\partial P}{\partial x}\right)\frac{B^2}{4} \Rightarrow \boxed{u_{max} = \frac{1}{8\mu}\left(-\frac{\partial P}{\partial x}\right) B^2}$$

2. <u>Discharge (Q)</u>

$$dQ = u\, dy \times 1$$

$$dQ = -\frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right)\left(By - y^2\right) dy$$

$$Q = -\frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right) \times \int_0^B \left(By - y^2\right) dy$$

$$Q = -\frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right) \times \left(\frac{By^2}{2} - \frac{y^3}{3}\right)_0^B$$

$$Q = -\frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right) \times \frac{B^3}{6} \Rightarrow \boxed{Q = -\frac{1}{12\mu}\left(\frac{\partial P}{\partial x}\right) B^3}$$

Average velocity

$Q = AV$

$$-\frac{1}{12\mu}\left(\frac{\partial P}{\partial x}\right) B^{\cancel{3}\,2} = \cancel{B} \times 1 \times V$$

$$V = -\frac{1}{12\mu}\left(\frac{\partial P}{\partial x}\right) B^2 \quad ; \quad u_{max} = -\frac{1}{8\mu}\left(\frac{\partial P}{\partial x}\right) B^2$$

$$\Rightarrow \boxed{V_{avg} = \frac{2}{3} u_{max}}$$

3. Pressure difference

$$V = -\frac{1}{12\mu}\left(\frac{\partial P}{\partial x}\right) B^2$$

$$-\partial P = \frac{12\mu V}{-B^2} \partial x$$

$$\boxed{P_1 - P_2 = \frac{12\mu V l}{B^2}}$$

4. Shear stress distribution

$$\tau = \mu \frac{\partial u}{\partial y} = \mu \frac{\partial}{\partial y}\left[-\frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right)(By - y^2)\right]$$

$$\tau = -\frac{1}{2}\left(\frac{\partial P}{\partial x}\right)(B - 2y)$$

$$\Rightarrow \boxed{\tau = \left(-\frac{\partial P}{\partial x}\right)\left(\frac{B}{2} - y\right)} \qquad \boxed{\tau_{max} = \left(-\frac{\partial P}{\partial x}\right) \cdot \frac{B}{2}}$$

Note:- • Laminar flow b/w parallel plates when the upper plate is moving is known as "Couette" flow.

CHAPTER – 6

FLOW THROUGH PIPES

- There are two types of losses in pipe i.e. major loss and minor loss.
- Major loss is due to friction whereas minor losses are divided into five types of losses that includes sudden contraction loss, sudden expansion loss, Exit loss, entry loss, bend loss.

Major losses

(friction ⇒ 95%)

1. Darcy weisbach eqnation

$$\boxed{h_l = \frac{f l v^2}{2gD}}$$

$$Q = AV \Rightarrow V = \frac{4Q}{\pi D^2}$$

$$h_l = \frac{4 \times 4\, f l Q^2}{\pi^2 \times 2g D^5} = \frac{f l Q^2}{\left(\frac{\pi^2 2g}{16}\right) D^5} \Rightarrow \boxed{h_l = \frac{f l Q^2}{12 D^5}}$$

2. Chez's formula

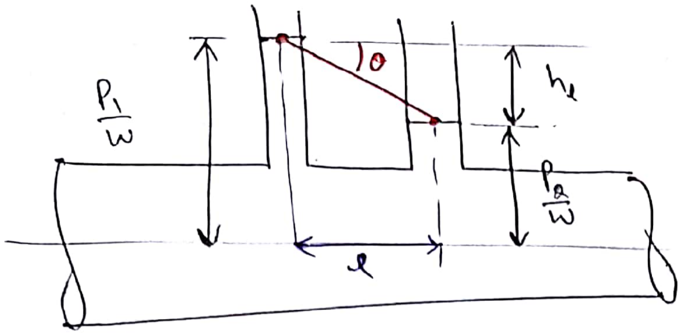
$$\boxed{V = C\sqrt{mi}}$$

C = Chez's constant

V = Average velocity

m = hydraulic mean depth $= \frac{\text{Area}}{\text{wetted perimeter}} = \frac{\frac{\pi}{4}D^2}{\pi D} = \frac{D}{4}$

i = hydraulic slope



$$i = \tan\theta = \frac{h_L}{l}$$

$$V = C\sqrt{m\,i}$$

$$V = C\sqrt{\frac{D}{4} \times \frac{h_L}{l}}$$

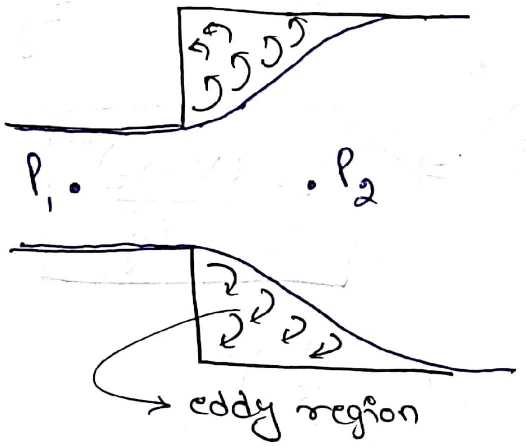
$$\boxed{h_L = \frac{4lV^2}{C^2 D}} \Leftarrow \frac{V^2}{C^2} = \frac{D h_L}{4l}$$

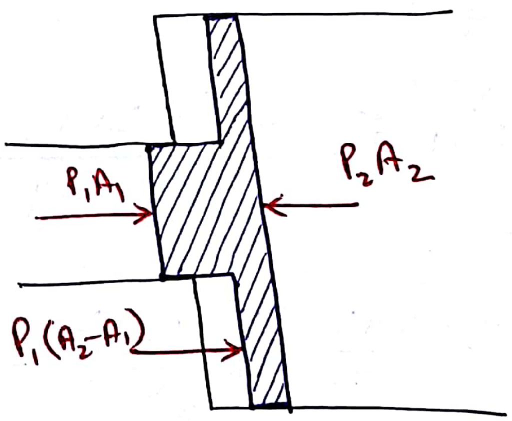
Comparision b/w C & f

$$\frac{4lV^2}{C^2 D} = \frac{flV^2}{2gD} \Rightarrow \boxed{C = \sqrt{\frac{8g}{f}}} \Rightarrow \frac{\sqrt{m}}{s} \Rightarrow L^{1/2}T^{-1}$$

Minor losses [it carries 5% → generally neglected]

1. Sudden expansion loss





Assumptions : 1) Steady
2) The pressure in eddy region is taken as upstream pressure (i.e. P_1)

Continuity eqn

$$A_1V_1 = A_2V_2 = Q$$

$$V_2 = \frac{Q}{A_2}$$

momentum integral equation

$$\Sigma F = \rho Q (v - u)$$

$$P_1 A_1 + P_1 A_2 - P_1 A_1 - P_2 A_2 = \rho Q (v - u)$$

$$(P_1 - P_2) A_2 = \rho Q (V_2 - V_1)$$

$$(P_1 - P_2) = \rho V_2 (V_2 - V_1)$$

$$\frac{P_1 - P_2}{\rho g} = \frac{V_2^2 - V_1 V_2}{g} \quad \text{——} \ (1)$$

B-eqn b/w (1) & (2)

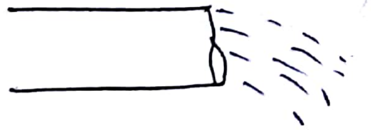
$$\frac{P_1}{w} + \frac{V_1^2}{2g} = \frac{P_2}{w} + \frac{V_2^2}{2g} + h_e$$

$$(h_e)_{S.E.} = \frac{P_1 - P_2}{w} + \frac{V_1^2 - V_2^2}{2g} = \frac{V_2^2 - V_1 V_2}{g} + \frac{V_1^2 - V_2^2}{2g}$$

$$(h_e)_{S.E.} = \frac{2V_2^2 - 2V_1V_2 + V_1^2 - V_2^2}{2g} \Rightarrow \boxed{h_e = \frac{(V_1 - V_2)^2}{2g}}$$

$$h_e = \frac{V_1^2}{2g}\left(1 - \frac{V_2}{V_1}\right)^2 = \frac{(V_1)^2}{2g} \times \left(1 - \frac{A_1}{A_2}\right)^2$$

2. Exit losses

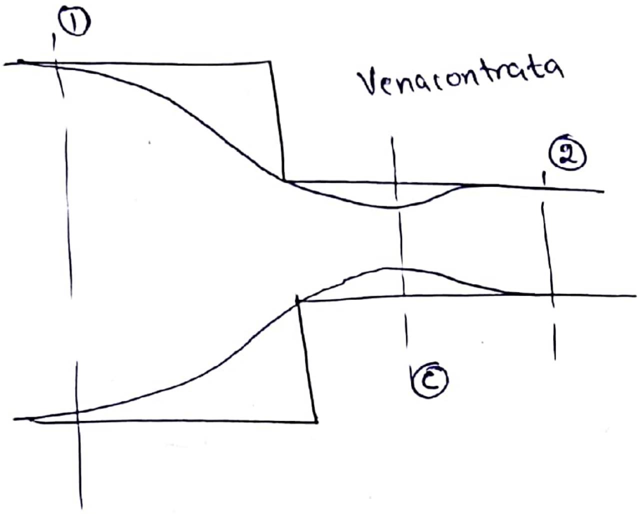


In the above eqn $A_2 = \infty$

$$h_e = \frac{V_1^2}{2g} \times \left(1 - \frac{A_1}{\infty}\right)^2$$

$$\Rightarrow \boxed{h_e = \frac{V^2}{2g}}$$

3. Sudden Contraction loss



1 to c

losses are neglected as area decreases Boundary layer region is small so losses are negligible

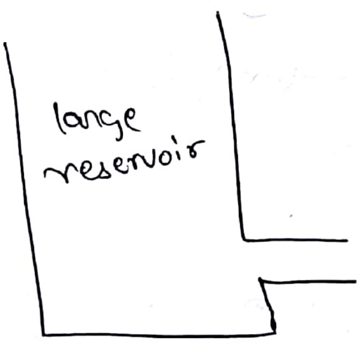
$$(h_l)_{s.c.} = \frac{(V_c - V_2)^2}{2g} = \frac{V_2^2}{2g}\left[\frac{V_c}{V_2} - 1\right]^2$$

$$A_c V_c = A_2 V_2$$

$$\frac{V_c}{V_2} = \frac{A_2}{A_c} = \frac{1}{C_c} \Rightarrow \boxed{(h_l)_{s.c.} = \left(\frac{1}{C_c} - 1\right)^2 \frac{V_2^2}{2g}}$$

$$\boxed{(h_l)_{s.c.} = 0.5\,\frac{V_2^2}{2g}}$$

4. Entry losses



$$\boxed{(h_l)_{entry} = 0.5\,\frac{V^2}{2g}}$$

5. Bend losses

$$\boxed{(h_l)_{Bend} = K\,\frac{V^2}{2g}}$$

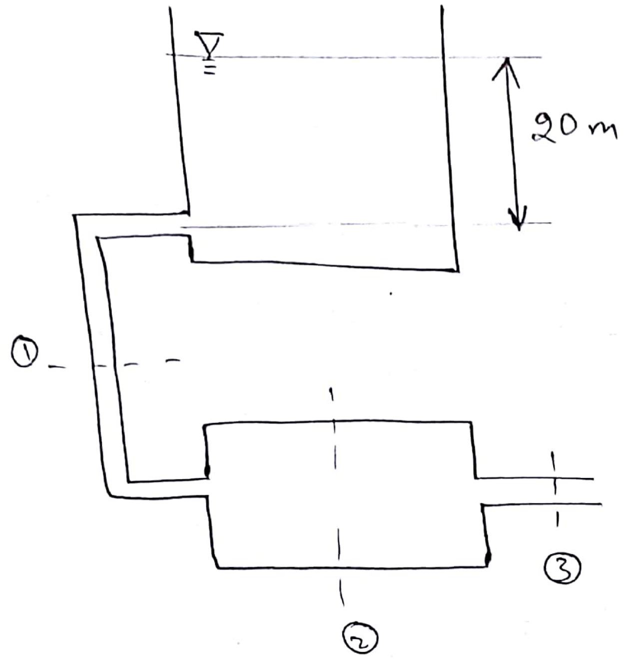
K = Bending coefficient

Angle of Bend $\theta \uparrow \Rightarrow K \uparrow$

Radius of curvature $\uparrow \Rightarrow K \downarrow$

Radius of Pipe $\uparrow \Rightarrow K \downarrow$

Ch-10
Q1



$K = 1$
$l_1 = 100\ m$
$d_1 = .1\ m$

$l_2 = 200\ m$
$d_2 = .2\ m$

$l_3 = 100\ m$
$d_3 = .1\ m$

$f = 0.02$

$V_1 = 4V_2 = V_3$

considering all losses (major + minor)

$$2g \times 20 = \frac{0.5 V_1^2}{2g} + \frac{KV_1^2}{2g} + \frac{f l_1 V_1^2}{2g D_1} + \frac{KV_1^2}{2g} + \frac{(V_1 - V_2)^2}{2g} + \frac{f l_2 V_2^2}{2g D_2}$$
$$+ \frac{0.5 V_3^2}{2g} + \frac{f l_3 V_3^2}{2g D_3} + \frac{V_3^2}{2g}$$

$$2g \times 20 = \left[.5 + 1 + \frac{.02 \times 100}{0.1} + 1 + \frac{9}{16} + \frac{0.02 \times 200}{16 \times 0.2} + 0.5 + \frac{0.02 \times 100}{0.1} + 1 \right] V_1^2$$

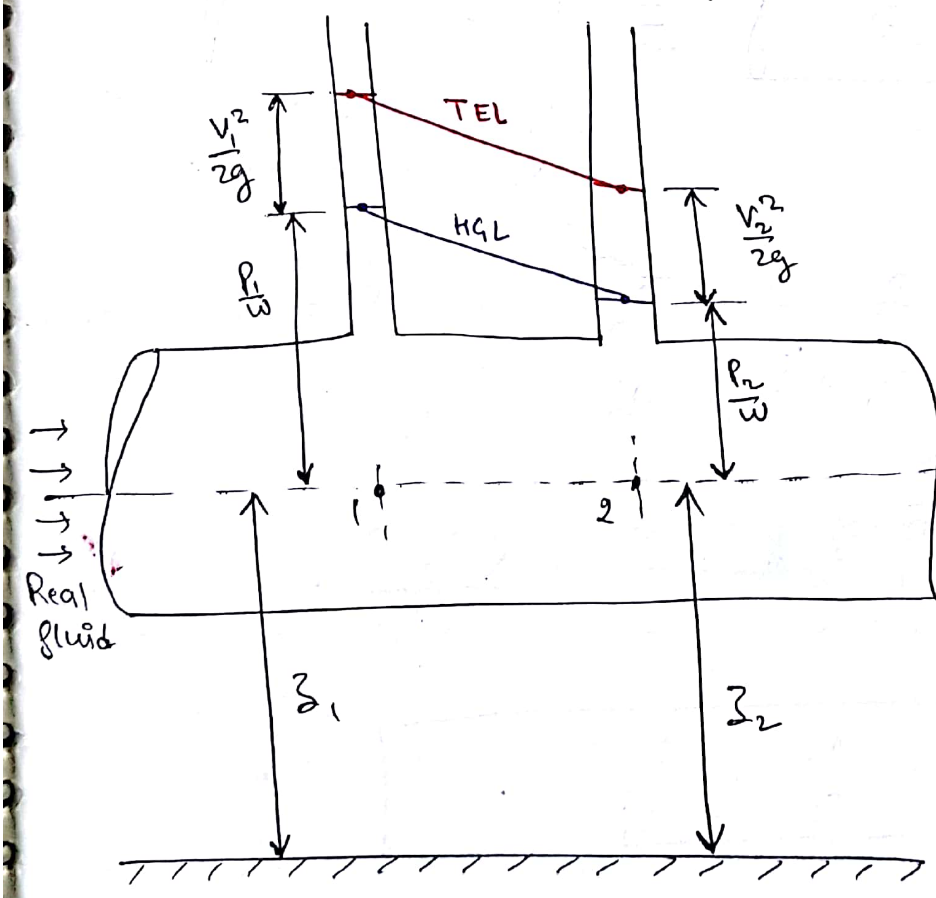
$\Rightarrow V_1^2 = 6.077 \Rightarrow V_1 = 2.465 \Rightarrow Q_{\text{all losses}} = V_1 \times A_1 = 0.0199\ \frac{m^3}{s}$

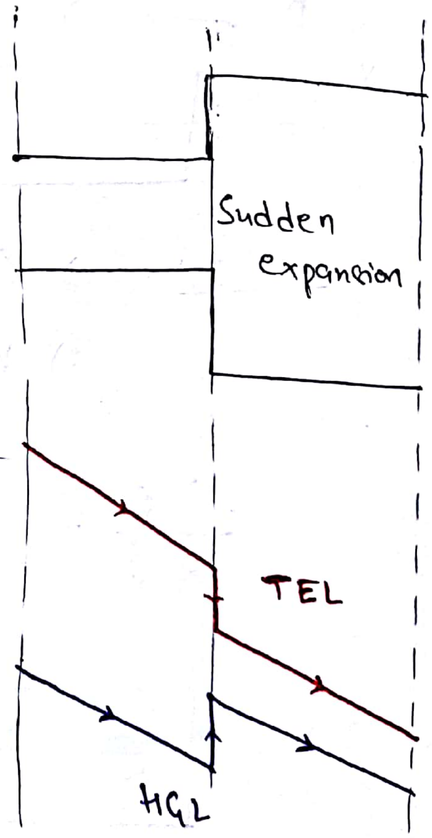
considering only major losses

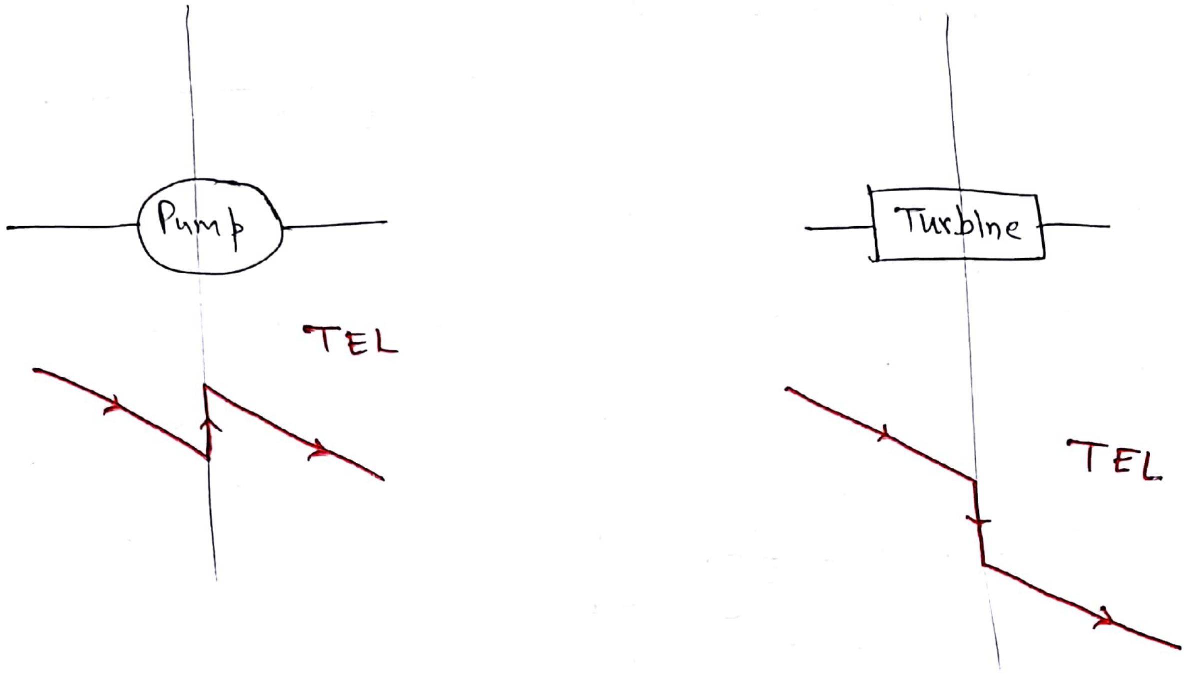
$$20 = \frac{f l_1 Q^2}{12 D_1^5} + \frac{f l_2 Q^2}{12 D_2^5} + \frac{f l_3 Q^2}{12 D_3^5} \Rightarrow Q = 0.0241\ m^3/s$$

Hydraulic gradient line & Total Energy line
(HGL) (TEL)

- Hydraulic Gradient line is Defined as the line formed by joining the piezometric heads at various points in a fluid flow. $\left[\frac{p}{w} + z\right]$
- TEL is defined as the line formed by joining the total energies $\left[\frac{p}{w} + \frac{V^2}{2g} + z\right]$ at various points in a fluid flow.
- HGL may rise or fall ~~up~~ whereas total energy line always falls.
- TEL can only rise when energy is provided to the fluid from outside
- For an ideal fluid HGL and TEL are straight, parallel and horizontal.



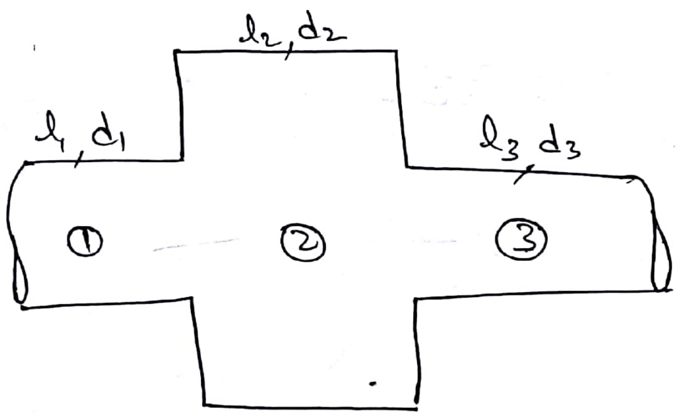




Pipes in Series & Pipes in parallel

1) Pipes in Series

$$h_{l_e} = \frac{f\, l_e Q^2}{12 D_e^5}$$



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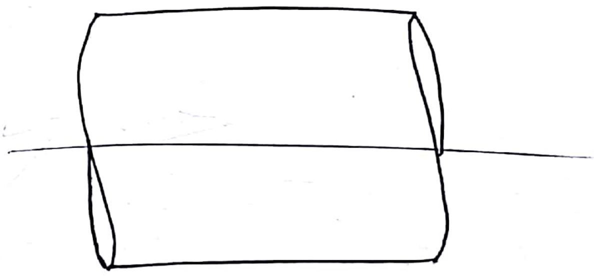


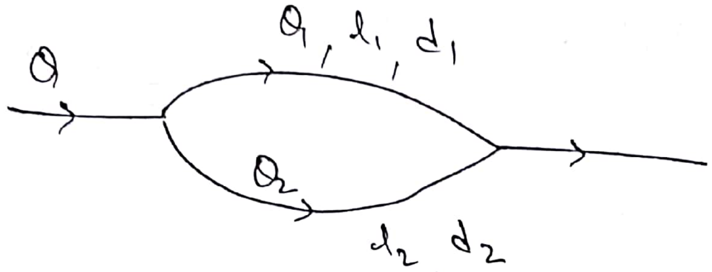
Conditions:
1) $Q_1 = Q_2 = Q_3 = Q_4 \text{ ---- } = Q_n = Q = \text{Const.}$
2) $h_l = h_{l_1} + h_{l_2} + h_{l_3} \text{ --- } h_{l_n}$

$$\frac{f\, l_e Q^2}{12 D_e^5} = \frac{f\, l_1 Q^2}{12 D_1^5} + \frac{f\, l_2 Q^2}{12 D_2^5} + \frac{f\, l_3 Q^2}{12 D_3^5} + \text{---}$$

$$\boxed{\frac{l_e}{D_e^5} = \frac{l_1}{D_1^5} + \frac{l_2}{D_2^5} + \frac{l_3}{D_3^5} + \text{---}}$$

Dupuit's formula

2. Pipes in Parallel



Conditions: 1) $Q = Q_1 + Q_2 + Q_3 + \cdots Q_n$

2) $h_{l_1} = h_{l_2} = h_{l_3} = \cdots h_{l_n}$

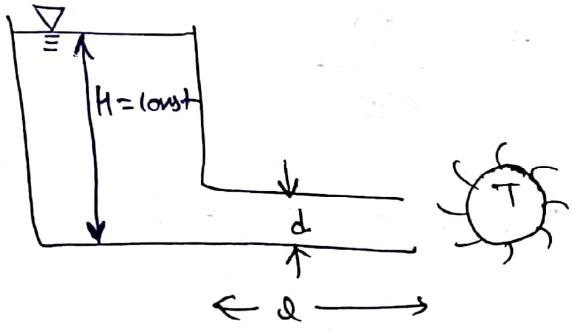
for "n" pipes → Discharge through 1 pipe $= \frac{Q}{n}$

$h_{le} = (h_l)$ through any pipe

$$\frac{f l_e Q^2}{12 D_e^5} = \frac{8 l Q^2}{n^2 \, 12 D^5} \Rightarrow \boxed{\frac{l_e}{D_e^5} = \frac{l}{n^2 D^5}}$$ when all pipes are similar

$$\boxed{\sqrt{\frac{D_e^5}{l_e}} = \sqrt{\frac{D_1^5}{l_1}} + \sqrt{\frac{D_2^5}{l_2}} + \sqrt{\frac{D_3^5}{l_3}} \cdots}$$ when pipes are diff. & h_l is same.

Power transmission through pipes



$(Power)_{th.} = \rho g Q H$

$(Power)_{act.} = \rho g Q (H - h_l)$

$$\boxed{\eta_{\text{Power trans.}} = \frac{P_{act}}{P_{th}} = \frac{H - h_l}{H}}$$

Condition for max. power transmission

$$P_{actual} = \rho g Q(H - h_L) = \rho g Q\left(H - \frac{f l Q^2}{12 D^5}\right)$$

$$\frac{dP}{dQ} = 0$$

$$\Rightarrow \quad H = 3h_L \quad \Rightarrow \quad \boxed{h_L = \frac{H}{3}}$$

$$\eta_{max} = \frac{H - h_L}{H} = 66.6\%$$

Ch-10
Q2

$$h_1 = h_2$$

$$\frac{f l V^2}{2gD} = \frac{f l V^2}{2gD}$$

$$\frac{\cancel{0.025} \times \cancel{200} \times V_1^2}{\cancel{.5}} = \frac{\cancel{0.02} \times \cancel{100} \times V_2^2}{\cancel{.1}}$$

$$\sqrt{5} = \frac{V_2}{V_1} = 2.236$$

Ch-10
Q3

$$l_e = l + 32 l + \frac{\cancel{4} l}{\cancel{32}_{8}}$$

$$= l\left(33 + \frac{1}{8}\right) = \frac{265}{8} l$$

Ch-10
Q4

$$\frac{12 \times 16}{.02} = \frac{f l Q^2}{\cancel{12} D^5} + \frac{f l Q^2}{\cancel{12} D^5} + \frac{f l Q^2}{\cancel{12} D^5} \quad \Rightarrow Q^2 = 0.01219$$

$$9600 = \left(\frac{400}{(.4)^5} + \frac{200}{(.2)^5} + \frac{300}{(.3)^5}\right) Q^2 \quad \Rightarrow Q = 0.11$$

$\boxed{05}$ 100 m 80 m

$$20 = \frac{.04 \times 3000 \times Q^2}{12 \times (0.3)^5} \Rightarrow Q_1 = 0.06971 \ m^3/s$$

$$\cancel{\frac{l}{n^2 D^5} = \frac{l_e}{D_e^5} \Rightarrow \frac{2000}{4 \times (.3)^5} \mp}$$

$$20 = \frac{f \ 1000 \times Q^2}{12 (.3)^5} + \frac{f \times 2000 \times Q^2}{4 \times 12 \times (.3)^5}$$

$$= \frac{f \times 1000 \times Q^2}{12 \times (.3)^5} \left[1 + \frac{1}{2} \right)$$

$$\Rightarrow Q_2 = 0.09859$$

$$\frac{Q_2}{Q_1} = 1.41428 \Rightarrow 41.4\% \uparrow$$

$\boxed{06}$

$$h_1 = h_2$$

$$\cancel{\frac{f \times l \times Q_1^2}{12 D_1^5} = \frac{f l \ Q_2^2}{12 (D_2)^5}}$$

$$20 = \frac{0.04 \times 1000 \times Q^2}{12 \times (0.3)^5}$$

$$\Rightarrow Q_2 = 0.12074$$

$$\frac{Q_2}{Q_1} = 1.732 \Rightarrow 73.2\% \uparrow$$

Q7

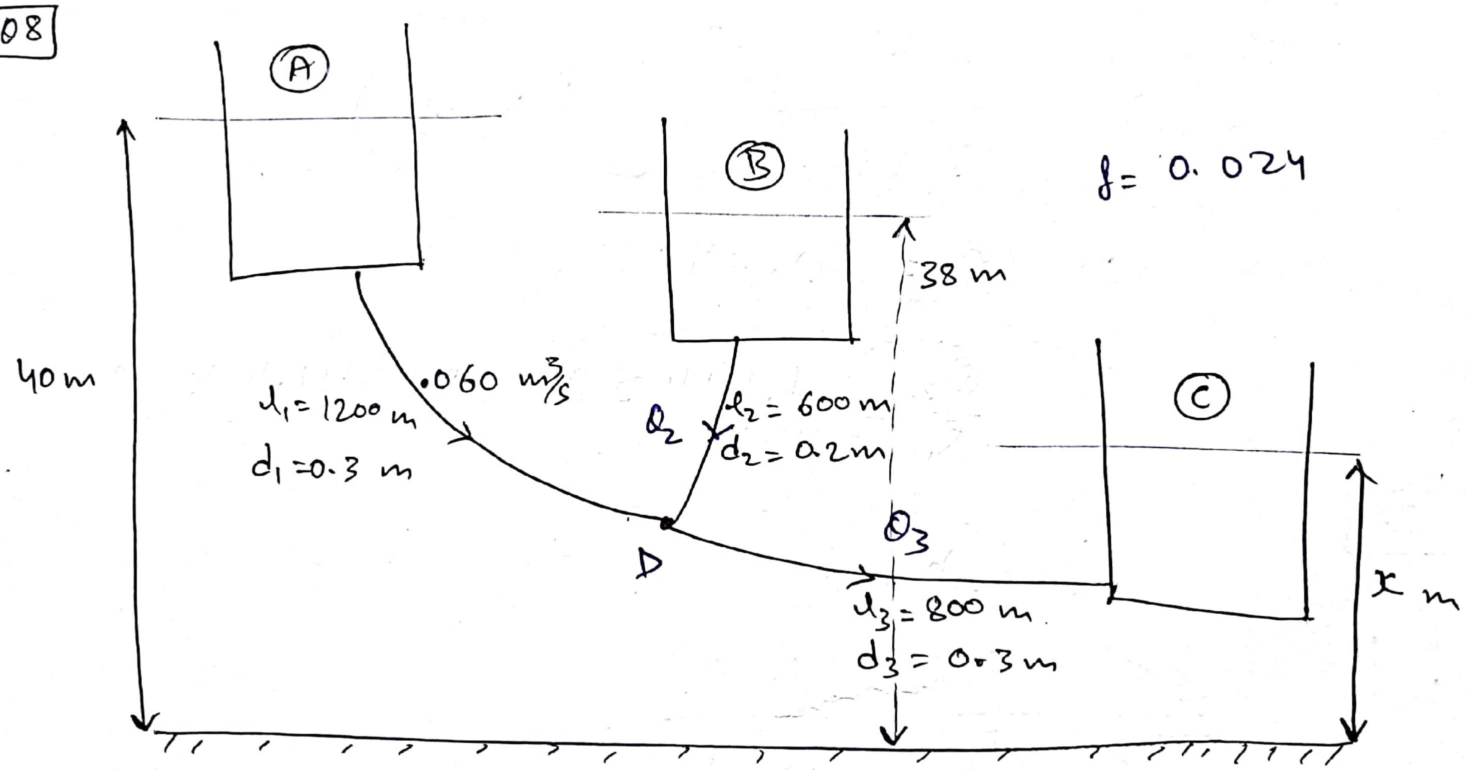
$$h_1 = h_2$$

$$\frac{\cancel{f} l_1 Q_1^2}{\cancel{12} D_1^5} = \frac{\cancel{f} l_2 Q_2^2}{\cancel{12} D_2^5}$$

$$\frac{500}{(.2)^5} \times Q_1^2 = \frac{500}{(0.3)^5} \times Q_2^2$$

$$\left(\frac{Q_1}{Q_2}\right)^2 = \left(\frac{.2}{.3}\right)^5 \Rightarrow \frac{Q_1}{Q_2} = 0.3628$$

Q8



$$h_1 = \frac{f l Q^2}{12 D^5} = 3.55 \text{ m}$$

Energy at D = 36.45 m

$$h_2 = 1.55 \text{ m} = \frac{0.024 \times 600 \times Q_2^2}{12 \times (.2)^5}$$

$$\Rightarrow Q_2 = 0.020366 \text{ m}^3/\text{s}$$

$$= 20.366 \text{ l/s}$$

$$Q_3 = Q_1 + Q_2 = 80.366 \ l/sec.$$

$$= 0.080366 \ m^3/s$$

$$h_3 = \frac{f l Q^2}{12 D^5} = \frac{0.024 \times 800 \times (.080366)^2}{12 \times (.3)^5}$$

$$= 4.2527 \ m$$

$$\Rightarrow x = 36.45 - 4.2527$$

$$= 32.19 \ m$$

VORTEX MOTION

[imp. for ESE]

- When a fluid moves in a curved path, it is known as Vortex motion.
- There are two types of vortex motion
  - → forced vortex motion
  - → free vortex motion

Forced vortex motion

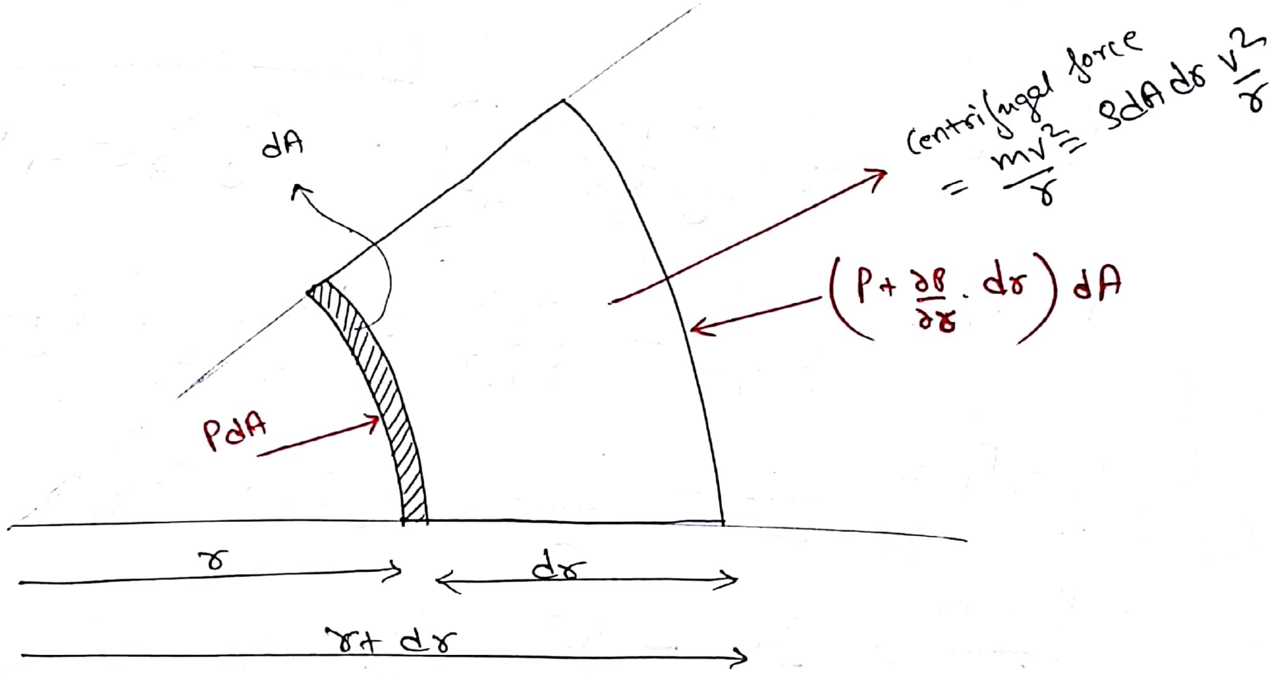
- It is the one in which fluid moves in a curved path by the action of some external agency (Torque)
- As there is continous dissipation of energy in forced vortex motion and hence Bernaulli's equation is not applicable.
- In forced vortex motion the flow is generally rotational.
- For forced vortex motion the equation $V = r\omega$ is aplicab
- Examples of forced vortex motion
  - a) water in bucket when rotated with a stick.
  - b) motion of a fluid in the impeller of centrifugal pump.

Free vortex motion

- It is one in which the fluid rotates in a curved path by some internal action and not by external torque.
- In free vortex motion ~~continuity~~ Bernaulli's eqn is applicable.
- free Vortex are generally irrotational.
- The equation $Vr = \text{Const.}$ is applicable to free vortex motion.

- examples of free vortex
  a) flow of fluid in wash basin.
  b) flow of fluid through pipe bend
  c) whirl pool
  d) motion of the fluid in the diffuser of centrifugal pump.

Generalised equation of vortex motion



$$\rho = \frac{m}{V} \Rightarrow m = \rho V$$

$$PdA - PdA - \frac{\partial P}{\partial r} dr dA = \frac{-\rho\, dA dr\, v^2}{r}$$

$$\cancel{-}\frac{\partial P}{\partial r} dr dA = \cancel{-}\frac{\rho dA dr\, v^2}{r}$$

$$P = f(r, z) \qquad \frac{\partial P}{\partial r} = \frac{\rho v^2}{r} \quad ; \quad \frac{\partial P}{\partial z} = \rho g$$

$$dP = \frac{\partial P}{\partial r} \cdot dr + \frac{\partial P}{\partial z} \cdot dz$$

$$\boxed{dP = \frac{\rho v^2}{r} \cdot dr - \rho g\, dz}$$ valid for both

Que Proofve that free vortex motion satisfies Bernaulli's equation.

Soluⁿ

$$dP = \frac{\rho v^2}{r} . dr - \rho g dz$$

for free vortex $vr = \text{const} = c \Rightarrow v = \frac{c}{r}$

$$\int_1^2 dP = \int_1^2 \frac{\rho c^2}{r^3} . dr - \int_1^2 \rho g dz$$

$$\boxed{z_2 - z_1 = \frac{c^2}{2g}\left[\frac{1}{r_1^2} - \frac{1}{r_2^2}\right]}$$

when both points on free surface

$$P_2 - P_1 = \frac{\rho c^2}{-2}\left[\frac{1}{r_2^2} - \frac{1}{r_1^2}\right] - \rho g (z_2 - z_1)$$

$$P_2 - P_1 = \frac{\rho c^2}{2r_1^2} - \frac{\rho c^2}{2r_2^2} - \rho g z_2 + \rho g z_1$$

$$P_2 - P_1 = \frac{\rho v_1^2}{2} - \frac{\rho v_2^2}{2} - \rho g z_2 + \rho g z_1$$

$$\frac{P_2 - P_1}{\rho g} = \frac{v_1^2}{2g} - \frac{v_2^2}{2g} - z_2 + z_1$$

$$\Rightarrow \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2 = \frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1$$

Forced vortex motion

$$dP = \frac{\rho v^2}{r} . dr - \rho g dz$$

for forced vortex $v = r\omega$

$$dP = \frac{\rho\, r^2 \omega^2 dr}{r} - \rho g dz$$

$$\int_1^2 dP = \int_1^2 \rho \omega^2 r dr - \int_1^2 \rho g dz$$

$$P_2 - P_1 = \frac{\rho \omega^2 (r_2^2 - r_1^2)}{2} - \rho g (z_2 - z_1)$$

for the points on surface ($P_1 = P_2 = P_{atm}$)

$$0 = \frac{\cancel{\rho} \omega^2}{2} (r_2^2 - r_1^2) - \cancel{\rho} g (z_2 - z_1)$$

$$\Rightarrow \boxed{z_2 - z_1 = \frac{\omega^2}{2g} (r_2^2 - r_1^2)}$$

when both point on surface of parabola

$\Rightarrow$ at $r_1 = 0$
$r_2 = R$

$z_2 - z_1 = H =$ height of Parabola

$$\boxed{H = \frac{\omega^2 R^2}{2g}} \Rightarrow \boxed{H = \frac{V^2}{2g}}$$

here $V = R\omega$

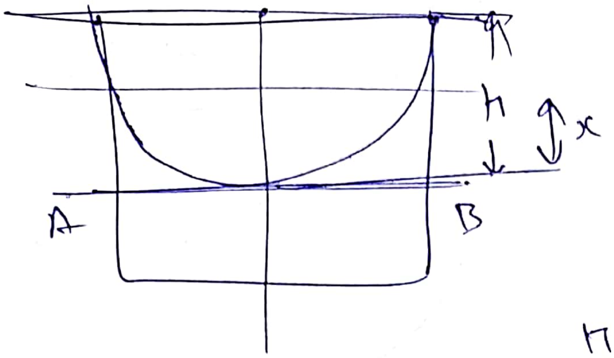
$$\omega = \frac{2\pi N}{60}$$

$\Rightarrow$ Volume of Parabola formed is

$$\boxed{Vol. = \frac{\pi R^2 H}{2}}$$

Ch-6

Q 1



$$z_2 - z_1 = \frac{\omega^2}{2g}(r_2^2 - r_1^2)$$

$$z_2 - z_1 = \frac{h}{2}$$

$$\frac{H}{2} =$$

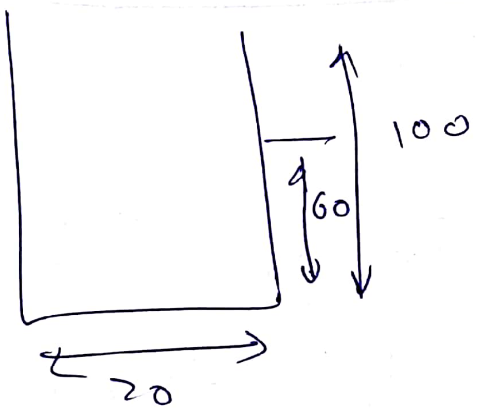
Volume above AB $= \frac{1}{2}\pi r^2 h$

initial vol. = Volume above AB

$$\frac{1}{2}\pi r^2 h = \pi r^2 x$$

$$\Rightarrow x = \frac{h}{2}$$

Q2

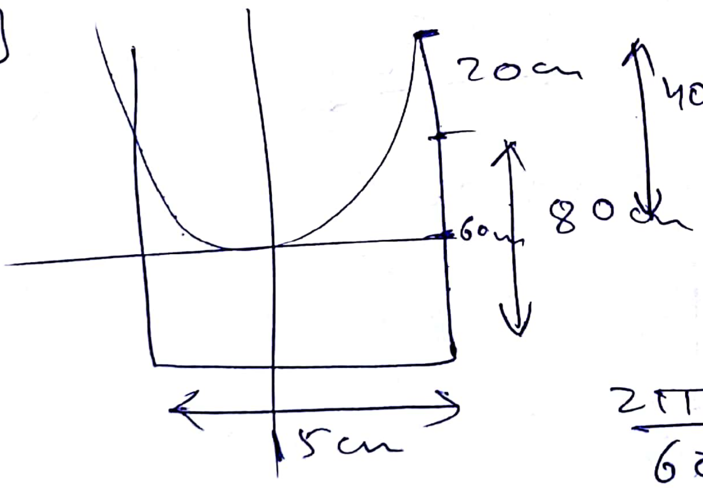


$$H = \frac{R^2 \omega^2}{2g}$$

$= .5033$ m

$= 50.3$ cm

Q3



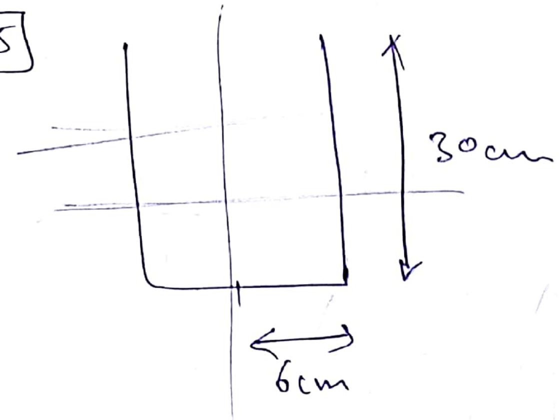
$$\Rightarrow 0.40 = \left(\frac{.15}{2}\right)^2 \times \frac{\omega^2}{2g}$$

$$\Rightarrow \omega^2 = 1395.2$$

$$\frac{2\pi N}{60} = \omega = 37.35 \text{ rad/s}$$

$$\Rightarrow N = 356.68 \text{ rpm}$$

Q5



$$H_1 = \frac{(.06)^2 \times \left(\frac{2\pi \times 300}{60}\right)^2}{2g}$$

$$= 0.18109 \text{ m}$$

$$= 18.109 \text{ cm}$$

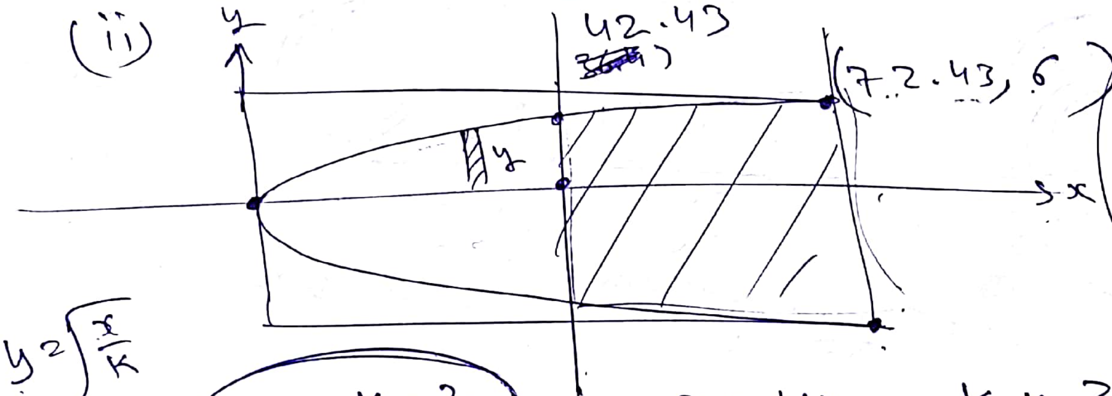
total vol. $= \pi \times (.06)^2 \times (.3)$

$$= \underline{3392.92 \text{ cm}^3}$$

(i) water left $= 3392.92 - \frac{\pi}{2} R^2 \times 18.109$

$$= \underline{2368.88 \text{ cm}^3}$$

(ii)



$\int_{a}^{b} \pi y^2 \, dx$

$y = \sqrt{\frac{x}{K}}$

$x = Ky^2$

$72.43 = K \times 36$

$$\Rightarrow \boxed{K = \frac{72.43}{36}}$$

$$\text{Vol} = \int_{42.43}^{72.43} \pi \times \frac{x}{K} \, dx = \frac{\pi x^2}{2K}$$

$$= \frac{\pi}{2K}\left[(72.43)^2 - (42.43)^2\right]$$

$$= \cancel{3059.67} \text{ cm}^3$$

$$\underline{2690.258}$$

Vol. left $= 3392.92 - 3059.67$

$$= \cancel{3333.1258} \text{ cm}^3$$

$$\underline{702.661}$$

~~Q8~~ Ch – 6
Objective

Q2 $\frac{1}{2}$

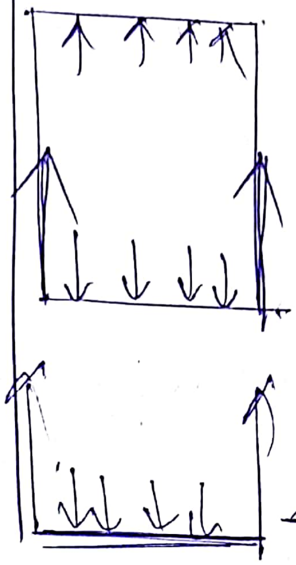
Q3 zero

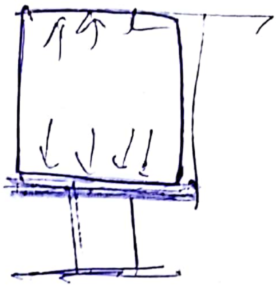
Q6 1000

Q1 obj.

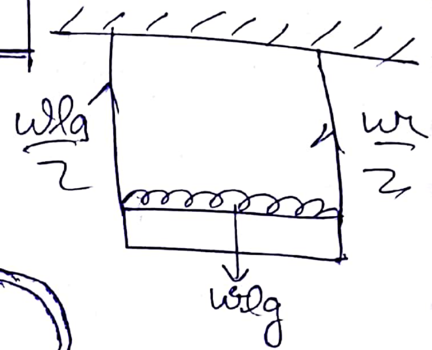
$$z_2 - z_1 = \frac{\omega^2}{2g}(r_2^2 - r_1^2)$$

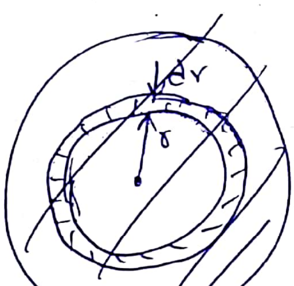
$$= \frac{\omega^2 l^2}{2g}$$

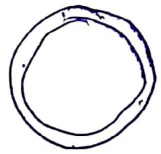




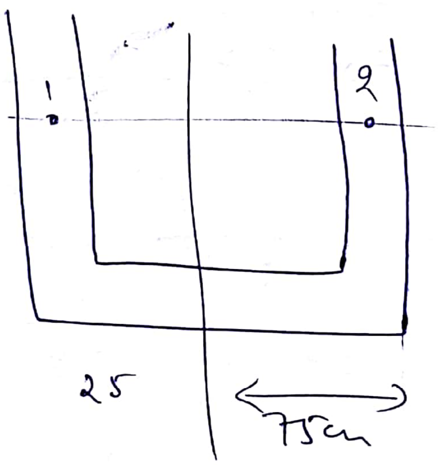
Pressure diff =>







$\cancel{dF = \frac{mv^2}{r} =}$ $\cancel{m = 2\pi r\, dr \times \rho t}$



$$N = \frac{60}{\pi} \text{ rpm}$$

$$\omega = \frac{2\pi\, 60}{60 \times \pi} = 2 \text{ rad/s}$$

$P_2 < P_1$ z

$$z_2 - z_1 = \frac{\omega^2}{2g}(r_2^2 - r_1^2)$$

$$= \frac{2 \times 2}{2 \times 9.81} \times \left((.75)^2 - (.25)^2\right)$$

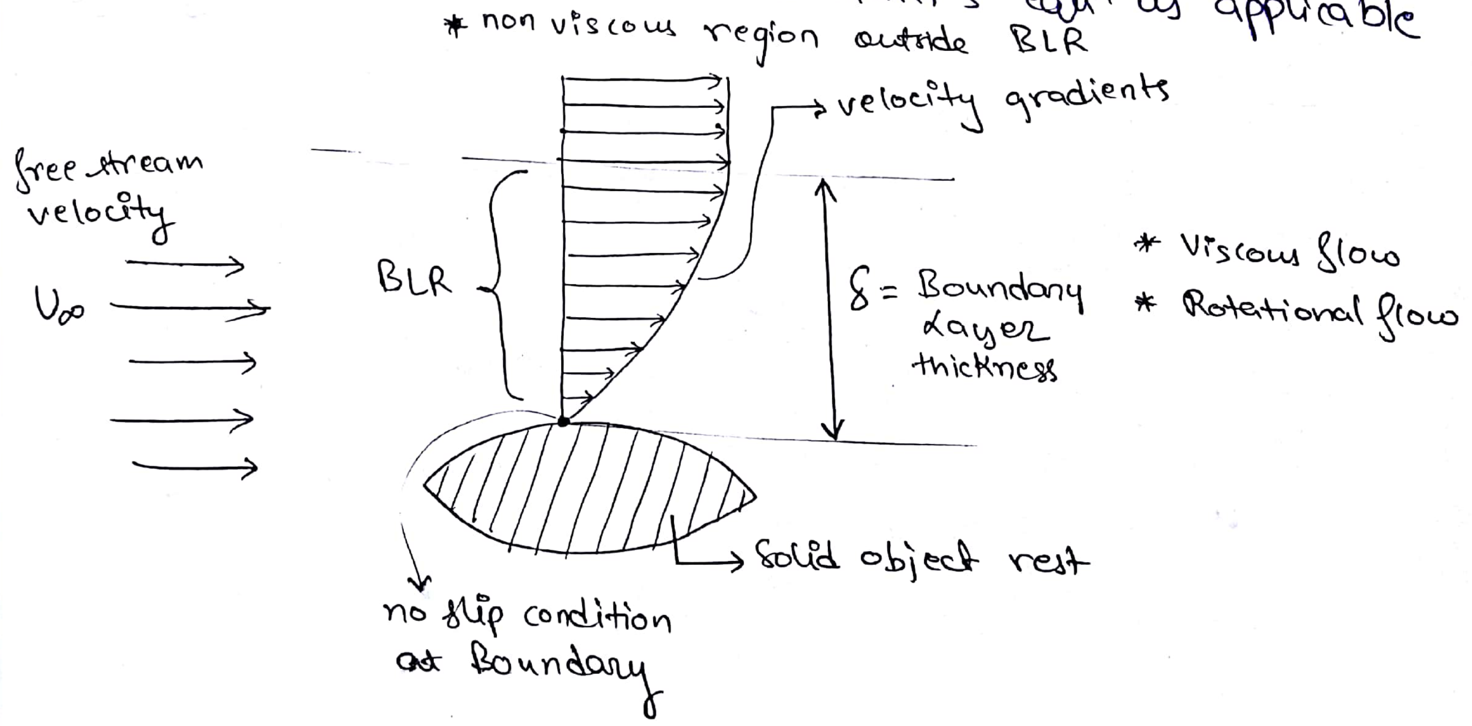
$$= 0.1019 \text{ m}$$

$$= 10.19 \text{ cm}$$

CHAPTER - 7

BOUNDARY LAYER THEORY (BLT)

- Boundary layer theory was invented by/developed by Prandtl in the year 1904. when a real fluid flows past a solid object, the velocity of the fluid on the surface of the object will be same as that of the surface.
- If the object is at rest, the fluid will also have zero velocity bcz of no slip condition at the boundary.
- Away from the boundary the velocity increases gradually and reaches free stream velocity at some distance and hence there is a small region close the boundary where velocity gradients exist. And this region is known as boundary layer region. (BLR)
- The flow in BLR is viscous and rotational.
- Outside the boundary layer region the flow is non viscous and hence the beornaulli's eqn is applicable



CHAPTER – 7

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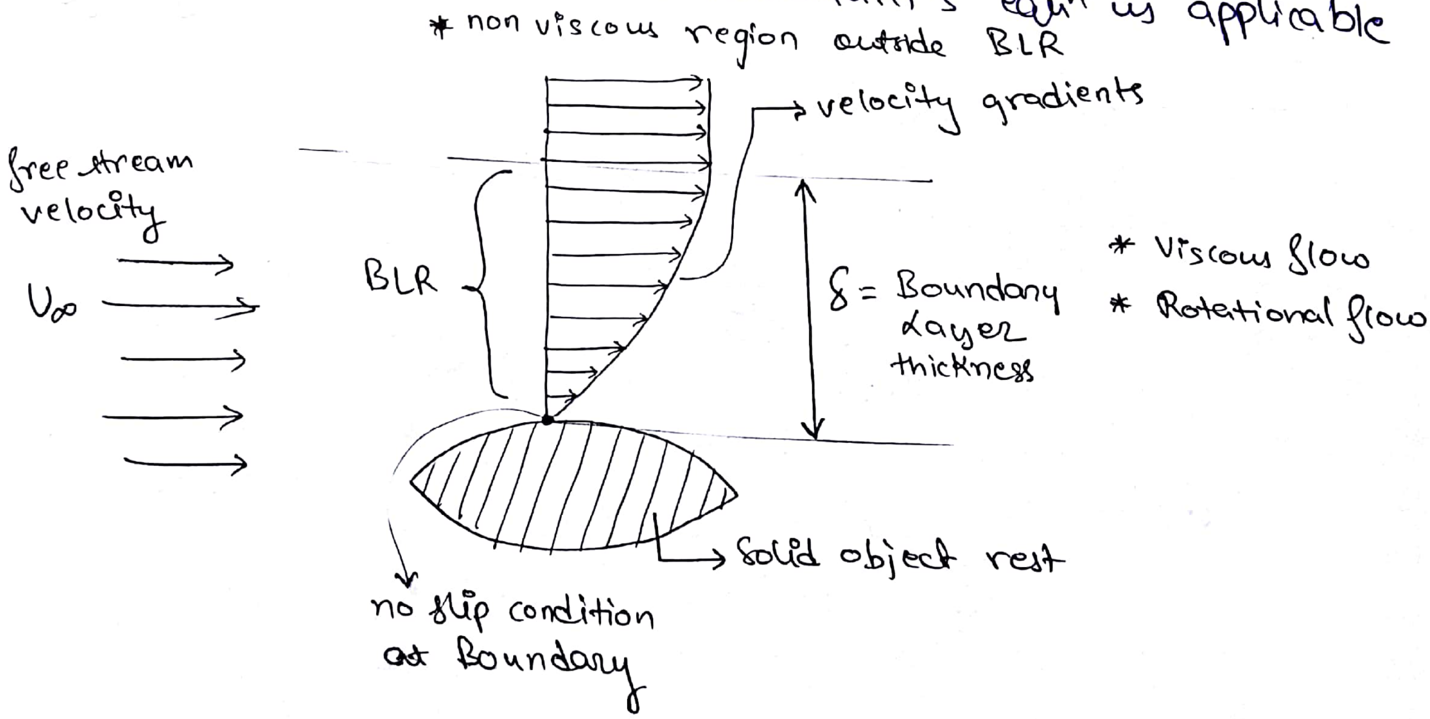
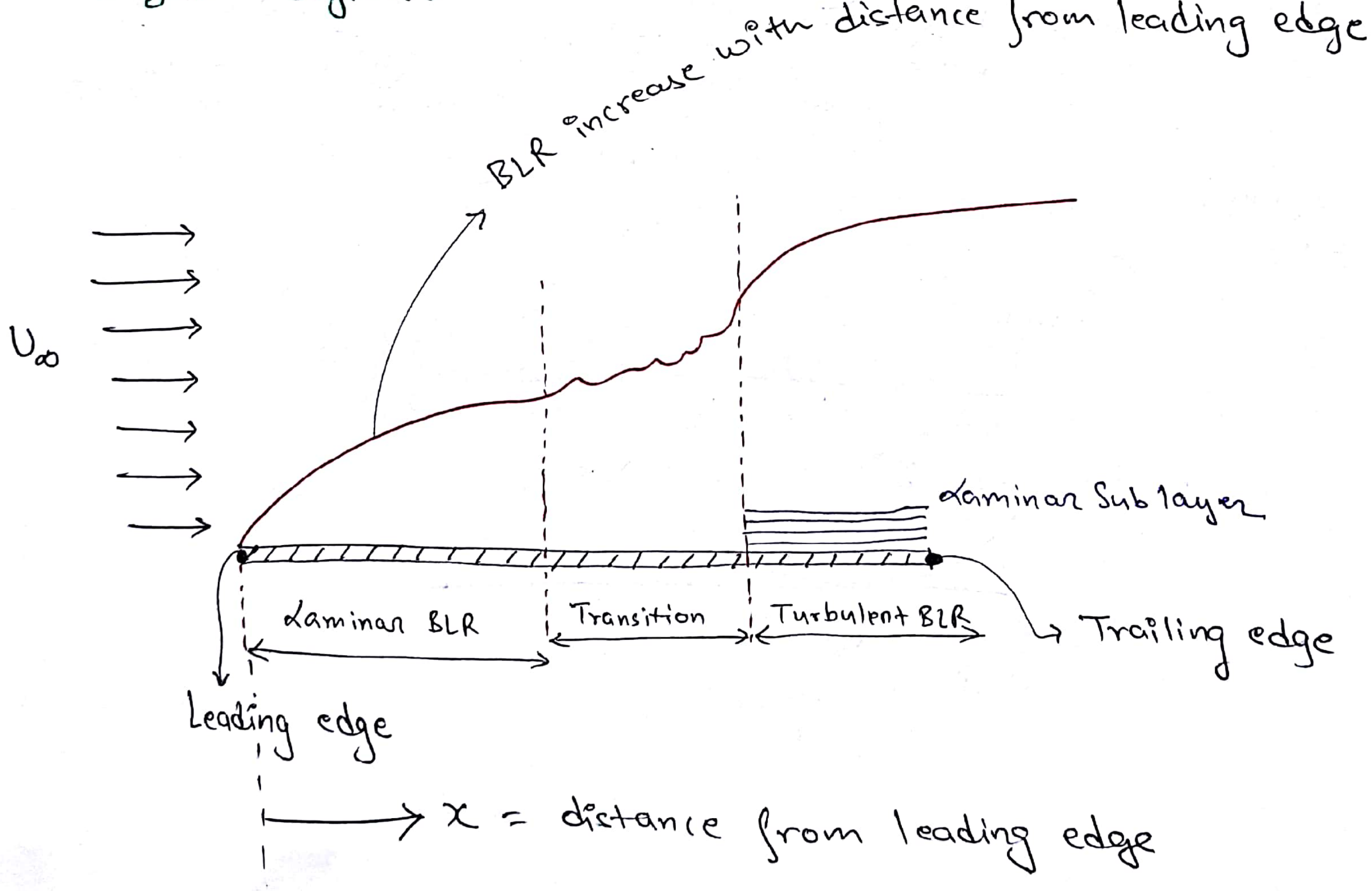


Growth of Boundary layer over a flat plate

- when a real fluid flows past a flat plate the velocity of the fluid on the plate will be same as that of the plate.
- if the plate is at rest, the fluid will also have zero velocity and the boundary layer thickness (BLT) grows with distance from the leading edge.
- Upto a certain distance from the boundary the flow is laminar in the Boundary layer region.
- As the laminar BL grows in instability the boundary layer changes from laminar to turbulent through transition.
- It is found that even in turbulent boundary layer region there is a small region close to the boundary where the flow is still laminar and this region is known as laminar sub-layer.
- Laminar sublayer ~~sub~~ exists in turbulent boundary layer region.



Boundary conditions

at $y = 0$, $u = 0$

$y = \delta$, $u = U_\infty$

$y = \delta$, $\frac{du}{dy} = 0$

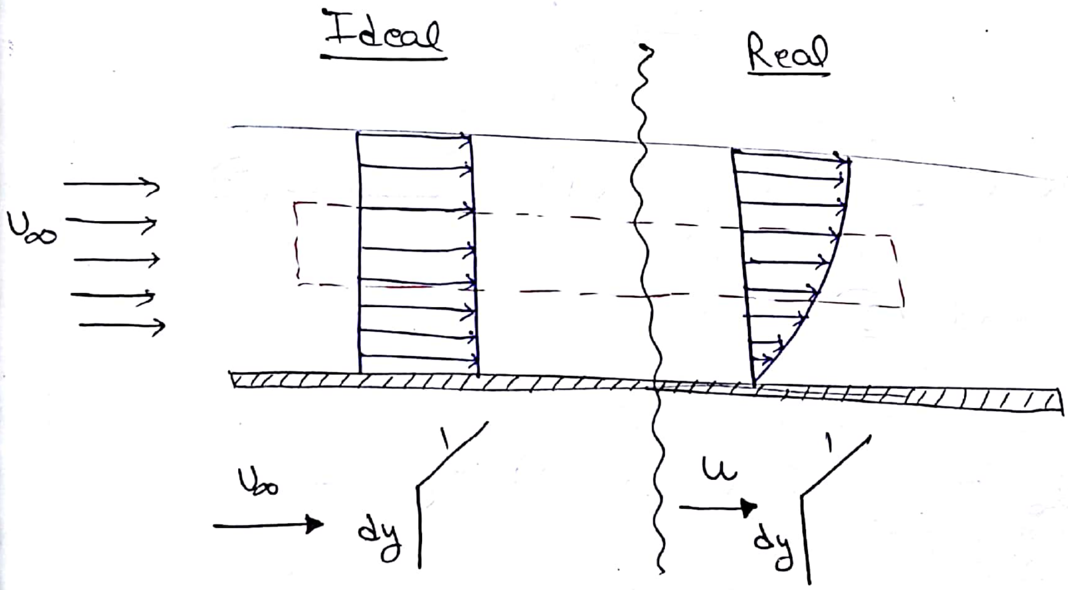
$x = 0$, $\delta = 0$

Boundary layer thickness (δ)

BLT (δ) is the vertical distance from the boundary in which velocity gradients exist. or it is also defined as the vertical distance from boundary upto the point where velocity becomes 99% of free stream velocity.

for numericals , at $y = \delta$; $u = U_\infty$

Displacement thickness (δ^*)



$$\dot{m}_{ideal} = \rho . dy \times 1 . U_\infty$$
$$= \rho U_\infty dy$$

$$\dot{m}_{real} = \rho . dy \times 1 . u$$
$$= \rho u dy$$

$$\dot{m}_{ideal} > \dot{m}_{real}$$

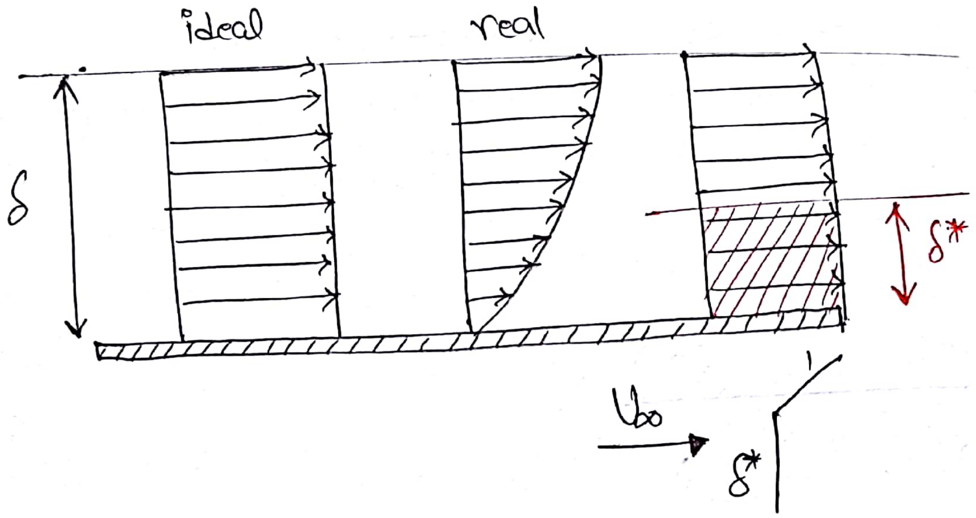
as $U_\infty > u$

loss of $\dot{m}$ in $dy = \dot{m}_{ideal} - \dot{m}_{real}$

$= \rho U_\infty dy - \rho u dy$

loss of $\dot{m}$ in $dy = \rho(U_\infty - u) dy$

total loss of $\dot{m}$ in $\delta = \int_0^\delta \rho(U_\infty - u) dy$ —— (A)



$\dot{m}$ through $\delta^* = \rho \delta^* . U_\infty \times 1$ —— (B)

Equating (A) and (B)

$$\rho \delta^* U_\infty = \int_0^\delta \rho (U_\infty - u) dy$$

$$\Rightarrow \boxed{\delta^* = \int_0^\delta \left(1 - \frac{u}{U_\infty}\right) dy}$$

⇓

Displacement thickness.

it is the vertical distance by which the solid boundary must be shifted in case of ideal fluid to compensate for the loss in mass flow rate due to BLR.

Momentum thickness $(\delta^{**})(\theta)$

It is the vertical distance by which the solid boundary must be shifted in case of ideal fluid to compensate the loss of momentum due to BLR.

$$\boxed{\theta = \delta^{**} = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) dy}$$

→ Velocity profile

Energy thickness (δ_e)

It is the vertical distance by which the solid boundary must be shifted in case of ideal fluid to compensate the loss of K.E. due to BLR

$$\boxed{\delta_e = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u^2}{U_\infty^2}\right) dy}$$

Ch-9 Q1

$$\delta^* = \int_0^\delta \left(1 - \frac{y}{\delta}\right) dy$$

$$= \left(y - \frac{y^2}{2\delta}\right)_0^\delta = \delta - \frac{\delta^2}{2\delta} = \frac{\delta}{2}$$

$$\delta^{**} = \int_0^\delta \left(\frac{y}{\delta} - \frac{y^2}{\delta^2}\right) dy$$

$$= \frac{y^2}{2\delta} - \frac{y^3}{3\delta^2} = \frac{\delta}{2} - \frac{\delta}{3} = \frac{\delta}{6}$$

Shape factor (H) it is defined as the ratio of displacement thickness to momentum thickness.

$$\boxed{H = \frac{\delta^*}{\delta^{**}}}$$

$\delta > \delta^* > \delta^{**}$

Ch-9
Q2

$$\tau = \tau_0 \left(1 - \frac{y}{\delta}\right)$$

$$\tau = \mu \left(\frac{du}{dy}\right)$$

$$\mu \frac{du}{dy} = \tau_0 \left(1 - \frac{y}{\delta}\right)$$

$$du = \frac{\tau_0}{\mu}\left(1 - \frac{y}{\delta}\right) dy$$

$$u = \frac{\tau_0}{\mu}\left(y - \frac{y^2}{2\delta}\right) + c$$

at $y = 0$ $u = 0$

$\Rightarrow c = 0$

$$u = \frac{\tau_0}{\mu}\left(y - \frac{y^2}{2\delta}\right)$$

at $y = \delta$
$u = U_\infty$

$$U_\infty = \frac{\tau_0}{\mu}\left(\delta - \frac{\delta^2}{2\delta}\right) \Rightarrow U_\infty = \frac{\tau_0}{\mu} \times \frac{\delta}{2}$$

$$\frac{u}{U_\infty} = \frac{\cancel{\frac{\tau_0}{\mu}}\left(y - \frac{y^2}{2\delta}\right)}{\cancel{\frac{\tau_0}{\mu}} \times \frac{\delta}{2}} \Rightarrow \frac{u}{U_\infty} = \frac{2y}{\delta} - \frac{y^2}{\delta^2}$$

$$\delta^* = \int_0^\delta \left(1 - \frac{2y}{\delta} + \frac{y^2}{\delta^2}\right) dy$$

$$= y - \frac{2y^2}{2\delta} + \frac{y^3}{3\delta^2}\Big]_0^\delta$$

$$= \cancel{\delta} - \cancel{\delta} + \frac{\delta}{3} = \frac{\delta}{3}$$

Flow over flat plate

- Von-Karman momentum integral equation

$$\boxed{\frac{\tau_0}{\rho V_\infty^2} = \frac{d\theta}{dx}}$$

valid for all profiles

τ_0 = wall shear stress
$\theta = \delta^{**}$ = momentum thickness
x = distance from leading edge
U_∞ = free stream velocity

- Reynold's no. $Re = \frac{\rho U_\infty l}{\mu}$

$Re < 5\times10^5 \longrightarrow$ Laminar flow

$Re > 5\times10^5 \longrightarrow$ turbulent flow [transition length is very small.

- Coefficient of Drag (C_D)

$$\boxed{F_D = \frac{C_D}{2}\rho A V_\infty^2}$$

$$\boxed{(\tau_0)_x = \frac{C_{fx}}{2} \times \rho V_\infty^2}$$

C_{fx} = skin friction coefficient

Ques 3 for the velocity Profile $\frac{u}{U_\infty} = \frac{3y}{2\delta} - \frac{y^3}{2\delta^3}$

find 1) boundary Layer thickness (δ)
2) shear ~~surface on~~ stress on surface of plate
3) Drag force
4) Avg. Drag coefficient in terms of Re.

Solu^n 1) using $\frac{\tau_0}{\rho U_\infty^2} = \frac{d\theta}{dx}$

Step-1 find θ

$$\theta = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right)dy$$

$$= \int_0^{\delta} \left(\frac{3y}{2\delta} - \frac{y^3}{2\delta^3}\right)\left(1 - \frac{3y}{2\delta} + \frac{y^3}{2\delta^3}\right) dy$$

$$= \int_0^{\delta} \left(\frac{3y}{2\delta} - \frac{9y^2}{4\delta^2} + \frac{\cancel{3}6y^4}{4\delta^4} - \frac{y^3}{2\delta^3} + \cancel{\frac{3y^4}{4\delta^4}} - \frac{y^6}{4\delta^6}\right) dy$$

$$= \frac{3\delta}{4} - \frac{\cancel{9}3}{4\,\cancel{12}}\delta + \frac{6}{20}\delta - \frac{\delta}{8} - \frac{\delta}{28}$$

$$= \left(\frac{210 - 210 + 84 - 35 - 10}{280}\right)\delta$$

$$= \frac{39}{280}\delta$$

$$\boxed{\theta = \frac{39}{280}\delta}$$

$$\Rightarrow \frac{\tau_0}{\rho U_\infty^2} = \frac{d}{dx}\left(\frac{39\delta}{280}\right) \Rightarrow \tau_0 = \frac{39}{280} \times \rho U_\infty^2 \frac{d\delta}{dx} \Rightarrow (A)$$

$$\tau_0 = \mu \left.\frac{du}{dy}\right|_{y=0}$$

$$\tau_0 = \mu \frac{d}{dy}\left[U_\infty\left(\frac{3y}{2\delta} - \frac{y^3}{2\delta^3}\right)\right]_{y=0}$$

$$\tau_0 = \mu U_\infty \left(\frac{3}{2\delta} - \frac{3y^2}{2\delta^3}\right)_{y=0}$$

$$\boxed{\tau_0 = \frac{3\mu U_\infty}{2\delta}} \Rightarrow (B)$$

equating (A) & (B)

$$\frac{39}{280}\rho U_\infty^2 \frac{d\delta}{dx} = \frac{3\mu U_\infty}{2\delta}$$

$$\delta\, d\delta = \frac{140}{13}\frac{\mu}{\rho U_\infty}\,.dx$$

$$\frac{\delta^2}{2} = \frac{140}{13} \frac{\mu}{\rho U_\infty} \cdot x + c$$

at $x = 0$ $\quad \delta = 0$ $\quad \Rightarrow \quad c = 0$

$$\delta^2 = \frac{280}{13} \frac{\mu x}{\rho U_\infty} \times \frac{x}{x} = \frac{280}{13} \frac{x^2}{Re_x}$$

$$\Rightarrow \boxed{\delta = \frac{4.64 x}{\sqrt{Re_x}}} \Rightarrow \boxed{\delta \propto x^{1/2}}$$

Note: from the above we can conclude that the boundary layer thickness δ increases with distance from the leading edge and the variation is given by $\boxed{\delta \propto x^{1/2}}$

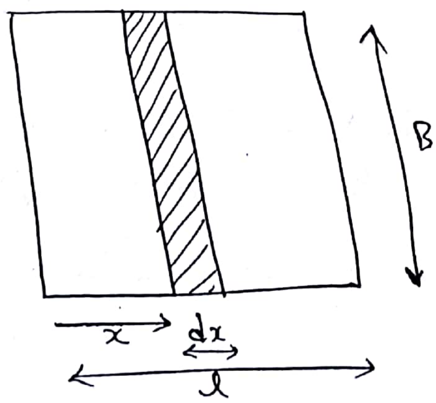
2) $\quad \tau_0 = \frac{3 \mu U_\infty}{2\delta} \quad ; \quad \delta = \frac{4.64 x}{\sqrt{Re_x}}$

$$\Rightarrow \tau_0 = \frac{3 \mu U_\infty \sqrt{Re_x}}{2 \times 4.64 x}$$

$$\tau_0 = \frac{0.323 \mu U_\infty \sqrt{Re_x}}{x} \Rightarrow \boxed{\tau_0 \propto x^{-1/2}}$$

$$F_D = \tau_0 \times A \Rightarrow \boxed{F_D \propto \tau_0 \propto x^{-1/2}}$$

3)



$$dF = \tau_0 B \, dx$$

$$\int dF = \int_0^l \frac{0.323 \mu U_\infty}{x} \cdot \frac{\sqrt{\rho U_\infty x}}{\sqrt{\mu}} \cdot B \cdot dx$$

$$F_D = \frac{.323 \mu U_\infty \times \sqrt{\rho U_\infty}}{\sqrt{\mu}} \cdot \int_0^l \frac{\sqrt{x}}{x} dx$$

$$F_D = \frac{0.646\, \mu U_\infty \sqrt{\rho U_\infty l}}{\sqrt{\mu}} \cdot B$$

$$\Rightarrow \quad F_D = .646\, \mu U_\infty \sqrt{Re_l} \cdot B = \frac{1}{2} C_D \times \rho A U_\infty^2$$

$$\Rightarrow \quad C_D = \frac{0.646\, \mu U_\infty \sqrt{Re_l} \cdot B}{\frac{1}{2} \times \rho \times B \times l \times U_\infty^2}$$

$$C_D = \frac{1.29\, \mu \sqrt{Re_l}}{\rho l U_\infty} \Rightarrow \boxed{C_D = \frac{1.29}{\sqrt{Re_l}}}$$

Ques: for velocity profile $\frac{u}{U_\infty} = \frac{2y}{\delta} - \frac{y^2}{\delta^2}$.

find :- 1) δ, 2) τ_0, 3) F_D, 4) $(C_D)_{avg}$

Soln

$$\tau_0 = \mu \left.\frac{du}{dy}\right|_{y=0} = \mu \times \frac{2 U_\infty}{\delta}$$

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U_\infty}\right) dy = \int_0^\delta \left(1 - \frac{2y}{\delta} + \frac{y^2}{\delta^2}\right) dy$$

$$= \delta - \delta + \frac{\delta}{3} = \frac{\delta}{3}$$

$$\theta = \delta^{**} = \int_0^\delta \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\left(1 - \frac{2y}{\delta} + \frac{y^2}{\delta^2}\right) dy$$

$$= \frac{2y}{\delta} - \frac{5y^2}{\delta^2} + \frac{2y^3}{\delta^3} - \cancel{\frac{y^2}{\delta^2}} + \cancel{\frac{2y^3}{\delta^3}} - \frac{y^4}{\delta^4}$$

$$= \delta - \frac{5}{3}\delta + \delta - \frac{\delta}{5} = \frac{15 - 25 + 15 - 3}{15}\delta$$

$$\boxed{\theta = \frac{2}{15}\delta}$$

$$\frac{\tau_0}{\rho U_\infty^2} = \frac{d\theta}{dx} \Rightarrow \int \tau_0\, dx = \int \rho U_\infty^2\, d\theta$$

$$\cancel{\tau_0 x = \rho U_\infty^2 \times \frac{2}{15}\delta + \cancel{c}}$$

$$\cancel{\Rightarrow \delta = \frac{7.5\, \tau_0\, x}{\rho U_\infty^2} = \frac{7.5\, \mu \times 2\, \cancel{U_\infty}}{\rho U_\infty^{\cancel{2}}} \frac{}{\delta} \times x}$$

$$\cancel{\frac{\delta^2}{\cancel{x}} = \frac{15\, \mu}{\rho U_\infty x} x^2 \Rightarrow \delta = \frac{\sqrt{15}\, x}{\sqrt{Re_x}}}$$

$$\theta = \frac{2}{15}\delta$$

$$d\theta = \frac{2}{15}\, d\delta$$

$$\mu\, 2 \frac{\cancel{U_\infty}}{\delta}\, dx = \rho U_\infty^{\cancel{2}} \int \frac{2\, d\delta}{15}$$

$$\mu\, 30\, x = \rho U_\infty \times \cancel{2} \frac{\delta^2}{\cancel{2}} + \cancel{c}$$

$$\boxed{\delta = \frac{\sqrt{30}\, x}{\sqrt{Re_x}}}$$

2) $$\tau_0 = \mu \times 2 \frac{U_\infty}{\delta} = \frac{2\mu U_\infty}{\sqrt{30} \times x} \times \sqrt{Re_x}$$

3)

$$dF = \frac{2}{\sqrt{30}} \mu U_\infty \times \frac{\sqrt{\rho U_\infty \cancel{x}}}{\sqrt{x}\ \sqrt{\mu}} \times B\, dx$$

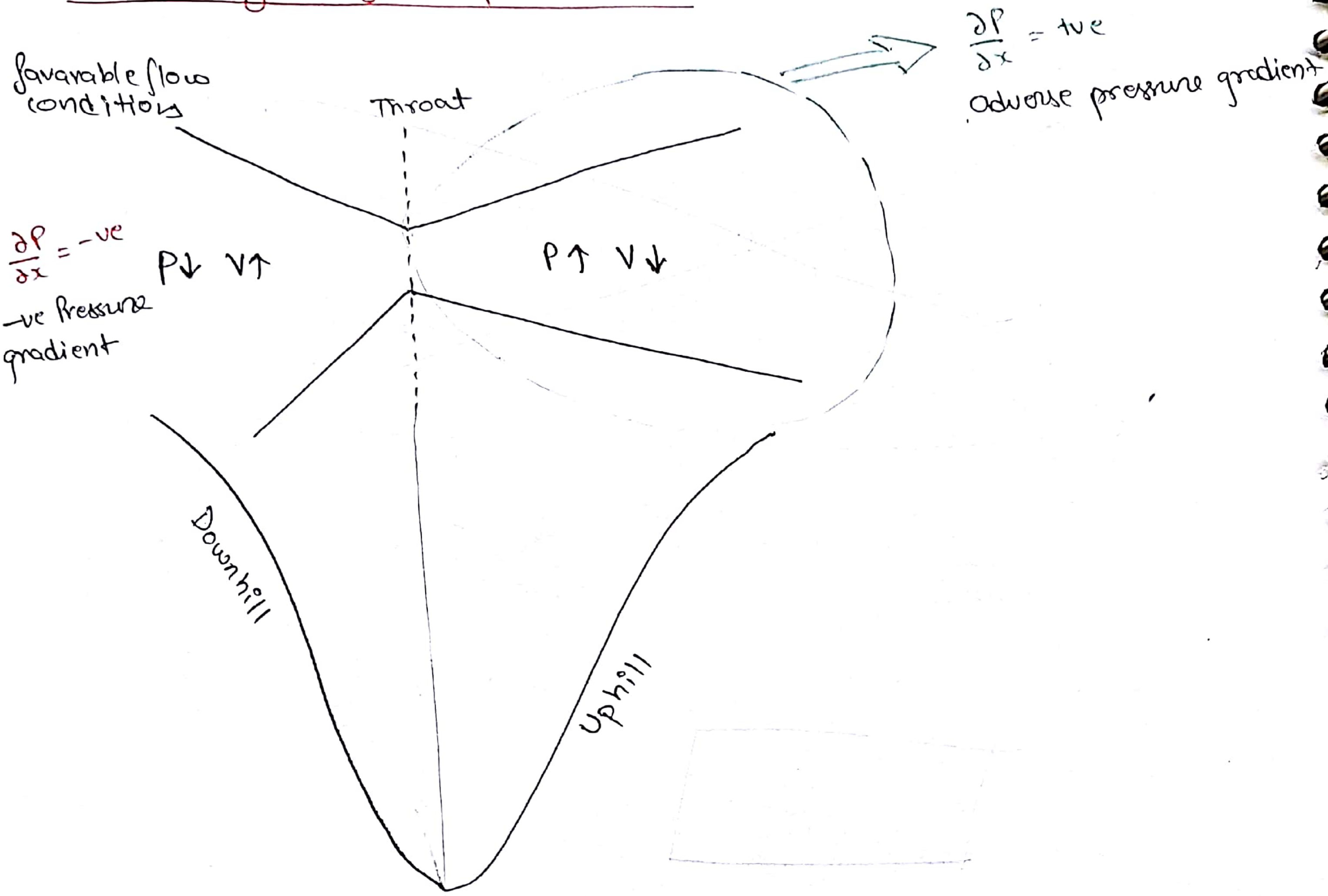
$$F_D = \frac{4}{\sqrt{30}} \mu U_\infty \times \sqrt{\frac{\rho U_\infty \ell}{\mu}} \times B$$

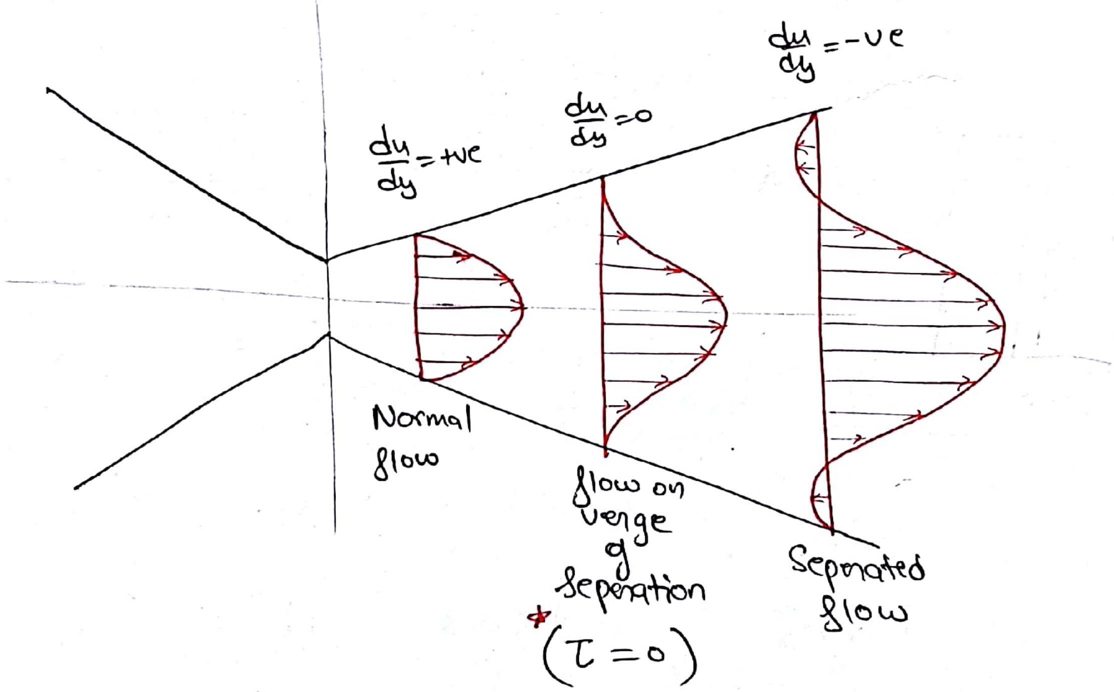
$$F_D = 0.7303\ \mu U_\infty B \times \sqrt{Re_\ell}$$

$$\cancel{B}\ell \times \frac{1}{2} C_D \rho \cancel{A} U_\infty^{\cancel{2}} = 0.7303\ \mu \cancel{U_\infty} \cancel{B} \times \sqrt{Re_\ell}$$

$$C_D = \frac{1.46}{\sqrt{Re_\ell}}$$

Boundary layer seperation





- When the fluid flows in the converging passage the velocity increases and the pressure decreases and hence the fluid flows under -ve pressure gradients. And this flow is also known as accelerating flow and hence the boundary layer thickness decreases and therefore -ve pressure gradients are also known as favorable pressure gradients.

- when the fluid flows through diverging section, the pressure increases & velocity decreases and the fluid flows under +ve pressure gradients. [adverse pressure gradients]
- If the angle of divergence is large then the retardation of fluid particals will be more and at some point the momentum of fluid partical may not support the flow and the flow will seperate from its boundary and reverses its flow direction & this phenomenon is known as boundary layer seperation.
- The main reason of boundary layer seperation are +ve pressure gradient (adverse pressure gradients)

Methods to control Boundary Layer Seperation

1) Accelerating the flow in the BLR
2) Sucing the fluid in BLR
3) Streamlining the body
4) Keeping the divergence angle to a minimum.

Blausius Equations [applied when velocity profile is not given]

	Laminar	Turbulent
δ	$\frac{5x}{\sqrt{Re_x}}$	$\frac{0.371x}{(Re_x)^{1/5}}$
C_D	$\frac{1.328}{\sqrt{Re_l}}$	$\frac{0.074}{(Re_l)^{1/5}}$
C_{fx}	$\frac{0.664}{\sqrt{Re_x}}$	$\frac{0.058}{(Re_x)^{1/5}}$

Ch-9 Obj. 01

$\rho = 1.2$

$V = 10 m/s$

$\delta^* = 0.5 mm$

Mass loss $= \rho \delta^* \times U_\infty$

$= 6 \times 10^{-3}$ Kg/m

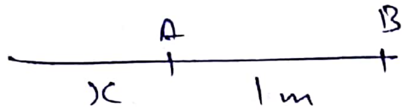
Q2

$$5\times10^{5} = \frac{10\times l}{15\times10^{-6}}$$

$$5\times \cancel{10^{5}}\times10^{-\cancel{6}}\times15 = 10l$$

$$75\times10^{-2} = l \Rightarrow l = .75\ \text{m}$$

Q4



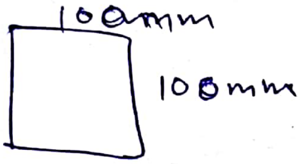
$$\frac{\delta_B}{\delta_A} = \sqrt{\frac{x_B}{x_A}}$$

$$\frac{9}{4} = \frac{x+1}{x}$$

$$9x = 4x+4$$

$$5x = 4 \Rightarrow x = .8\ \text{m}$$

Q7



$V = 10$ m/s δ = negli. at inlet

$\delta^* = 5$ mm at exit

$$10\times10\times10 = 9\times9\times V \Rightarrow V = 12.345\ \text{m/s}$$

Q8

$$\tau_0 = \frac{3}{2}\frac{\mu U_\infty}{\delta} \qquad \delta = \frac{4.64\times1}{\sqrt{\frac{\rho V l}{\mu}}}$$

$$= \frac{\frac{3}{2}\times1.5\times10^{-5}\times\cancel{2}\times5}{\delta}$$

$$\delta = \frac{4.64\times1}{\sqrt{\frac{2\times1}{1.5\times10^{-5}}}} = 365.14$$

$$= 3.54\times10^{-3}\times1.23$$

$$= 4.36\times10^{-3}$$

$$= 12.707\ \text{mm}$$

$\boxed{Q9}$ $\cancel{\frac{u}{U_\infty} = \left(\frac{y}{\delta}\right)}$ $\frac{\tau_o}{\rho U_\infty^2} = \frac{d\theta}{dx}$

$$\frac{u}{U_\infty} = \frac{y}{h}$$

$$\tau_o\,dx = dF_D = \rho U_\infty^2\,d\theta$$

$$\delta^{**} = \theta = \int_0^h \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right)dy$$

$$\int dF_D = \int \rho U_\infty^2 \frac{h}{6}$$

$$= \int_0^h \left(\frac{y}{h} - \frac{y^2}{h^2}\right)dy$$

$$F_D = \rho U_\infty^2 \frac{h}{6}$$

$$= \frac{y^2}{2h} - \frac{y^3}{3h^2} = \frac{h}{6}$$

$$\text{total} = \frac{\rho U_\infty^2 h}{3}$$

$\boxed{Q10}$

$$\dot{m}_{pq} = \rho \times \underbrace{U_\infty}_{10\ m/s} \times \underbrace{\delta}_{0.01\ m}$$

$$= 1 \times 10 \times 0.01 = 0.1\ kg/s$$

$$\dot{m}_{qr} = \frac{\dot{m}_{pq}}{2} = 0.05\ kg/s$$

$\boxed{Q11}$

$$\theta = \frac{\delta}{6} \qquad \frac{\tau_o}{\rho U_\infty^2} = \frac{d\theta}{dx}$$

$$F_D = \int \tau_o\,dx = \rho U_\infty^2 \times \int d\theta$$

$$= \rho U_\infty^2 \times \frac{\delta}{6}$$

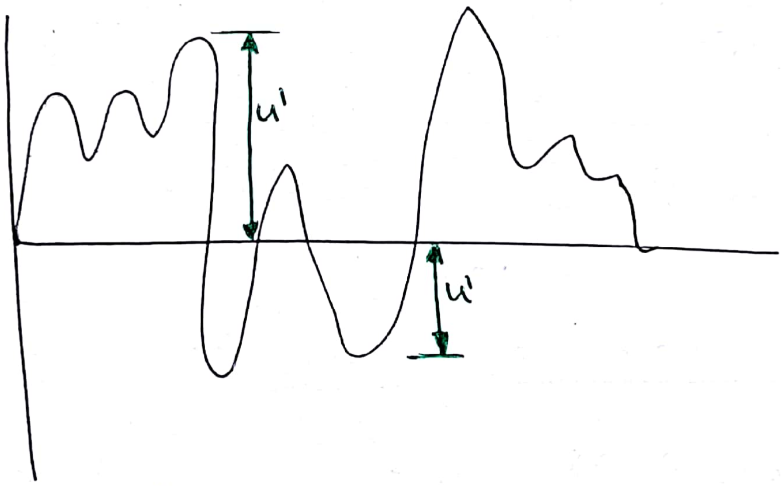
$$= \frac{1 \times 100 \times 0.01}{6} = \frac{1}{6}$$

$$= 0.1667\ N$$

CHAPTER - 8

TURBULENT FLOW [ELEMENTRY]

- In case of turbulent flow there is huge order intermixing of fluid Particals and due to this various properties of the fluid is going to change with space and time.
- For analysing turbulent flows any fluid parameter can be written as a combination of average component and fluctuating component.



$\bar{u}$ = mean or average component

Any Parameter = Average Component + fluctuating Component

$$u = u' + \bar{u}$$

$$P = P' + \bar{P}$$

- A flow is said to be a turbulent flow in pipe if the reynold's no. is > 4000
- In case of turbulent flow, τ is combination of laminar shear stress near the boundary and turbulent shear stress away from boundary & hence τ is combination of $\tau_{laminar}$ & $\tau_{turbulent}$.

$$\boxed{\tau_{\substack{total\\turbulent}} = \tau_{laminar} + \tau_{turbulent}}$$

$\tau_{laminar}$ → in region close to boundary

$\tau_{turbulent}$ → in region away from boundary

Shear stress in turbulent flow

$$\tau_{\substack{total\\turbulent}} = \tau_{laminar} + \left(\tau_{turbulent}\right)$$ needed to be calculated

1. According to Boussinesq

$$\boxed{\tau_{turb.} = \eta \frac{d\bar{u}}{dy}}$$

η = eddy viscosity

2. According to Reynold's

$$\boxed{\tau_{turb.} = \rho u' v'}$$

u' & v' are the fluctuating components in x & y dirn.

3. According to Prandtl

$$\boxed{u' = v' = l \frac{du}{dy}}$$

l = Prandtl's mixing length

$l = Ky$

$K = 0.4$ for pipe flow

y = distance from pipe wall

$$\boxed{\tau_{turb} = \rho l^2 \left(\frac{du}{dy}\right)^2}$$

$$\tau_{total} = \tau_{laminar} + \tau_{turb.}$$ → negligible

$$\tau_{turb} = \rho l^2 \left(\frac{du}{dy}\right)^2$$

Velocity distribution

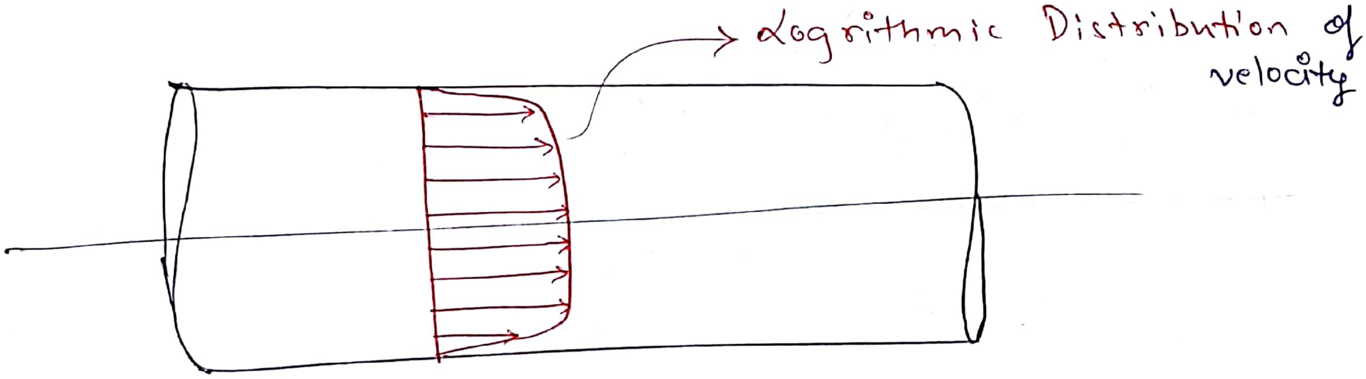
$$\tau = \rho l^2 \left(\frac{du}{dy}\right)^2 \Rightarrow \left(\frac{du}{dy}\right)^2 = \frac{\tau}{\rho l^2}$$

$l = Ky \quad \& \quad K = 0.4$

$$\frac{du}{dy} = \sqrt{\frac{\tau}{\rho}} \times \frac{1}{l} = \frac{V^*}{Ky}$$

$$du = 2.5\, V^* \frac{dy}{y}$$

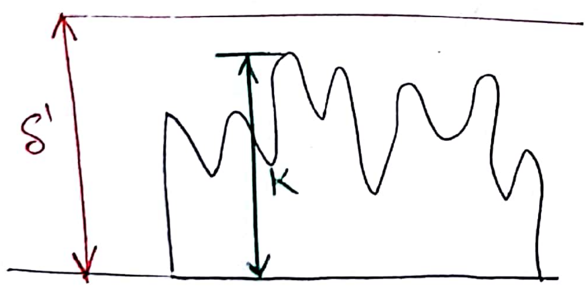
$$\boxed{u = 2.5\, V^* \ln y + C}$$

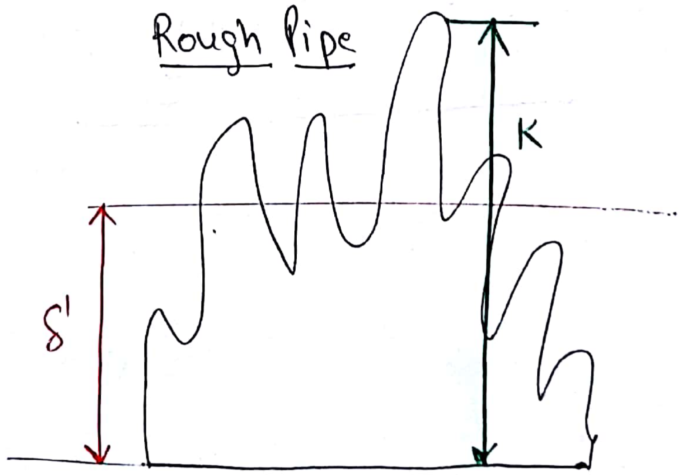


<u>Note:</u> The velocity Profile in case of turbulent flow is logarithmic

Hydrodynamically Smooth & Rough Pipes

<u>Smooth pipe</u> <u>Rough Pipe</u>





$K <<< \delta'$ $K >>> \delta'$

K = average height of Roughness
δ' = height of Laminar Sublayer

According to Nikuradse

$$\boxed{\delta' = \frac{11.6\,\nu}{V^*}}$$

$$V^* = \sqrt{\frac{\tau}{\rho}} = V \cdot \sqrt{\frac{f}{8}}$$

- Nikuradse's conditions

$\Rightarrow$ $\frac{K}{\delta'} < 0.25 \longrightarrow$ Smooth

$0.25 < \frac{K}{\delta'} < 6 \longrightarrow$ Transition

$\frac{K}{\delta'} > 6 \longrightarrow$ ~~turbulent~~ Rough

- Reynold's condition

$$Re = \frac{V^* K}{\nu}$$

$\Rightarrow$ $\frac{V^* K}{\nu} < 4 \longrightarrow$ Smooth

$4 < \frac{V^* K}{\nu} < 100 \longrightarrow$ Transition

$\frac{V^* K}{\nu} > 100 \longrightarrow$ Rough

- $u = 2.5\, V^* \ln y + c$

at $y = 0$, pipe wall
$u =$ undefined

at $y = y'$; $u = 0$

$y' =$ small distance from pipe wall where velocity becomes zero

- According to nikuradse

$$y' = \frac{\delta'}{107} \longrightarrow \text{smooth}$$

$$y' = \frac{K}{30} \longrightarrow \text{rough}$$

- $$\boxed{\frac{u-V}{V*} = 5.75 \log_{10}\left(\frac{y}{R}\right) + 3.75}$$

y = distance from pipe wall
R = radius of pipe
V = average velocity
V* = mean velocity

Ch-7
Q1

$$0 = 5.75 \ln_{10}\left(\frac{y}{R}\right) + 3.75$$

$$\ln_{10}\left(\frac{y}{R}\right) = \frac{-3.75}{5.75}$$

$$\ln\left(\frac{y}{R}\right) = \frac{-3.75}{5.75} \times \ln 10$$

$$\frac{y}{R} = 0.22275 \Rightarrow y = 0.22275 R$$

Q2

$$\frac{u-V}{V*} = 5.75 \ln_{10}\left(\frac{y}{R}\right) + 3.75$$

$u = u_{max}$ at $y = R$

$$\frac{u_{max} - V}{V*} = 3.75$$

$$\frac{\frac{u_{max}}{V} - \frac{V}{V}}{\cancel{V}\cdot\sqrt{\frac{f}{8}}} = 3.75$$

Pipe

$$\boxed{\frac{u_{max}}{V} = 1.33\sqrt{f} + 1}$$

$$\frac{u_{max}}{V} = 3.75 \times \frac{\sqrt{f}}{8} + 1 = 1.3258\sqrt{f} + 1$$

Friction factor in turbulent flow

Smooth

$$f = \frac{0.3164}{Re^{1/4}} \quad (Re < 10^5)$$

$$f = 0.0032 + \frac{0.221}{Re^{0.232}}$$

$Re \rightarrow 10^5$ to 4×10^7

Rough

$$\frac{1}{\sqrt{f}} = 2\ln_{10}\left(\frac{R}{K}\right) + 1.74$$

$$\boxed{\tau_0 = \frac{\rho f v^2}{8}}$$

valid for both - laminar - turb.

$\boxed{ch\text{-}7\ 03}$

$K = 0.48$ mm

$R = 0.3$ m

length $= 4.5$ m

$Q = 0.6\ m^3/s$

$\mu = 0.001\ NS/m^2$

$$P = \rho g Q H = 188.47\ W$$

$$H = \frac{f l v^2}{2gD} = 0.032\ m$$

$$V = \frac{0.6}{\pi R^2} = 2.122\ m/s$$

Rough

$$\frac{1}{\sqrt{f}} = 2\frac{\ln R/K}{\ln 10} + 1.74 = 7.33$$

$$\Rightarrow f = 0.0186$$

$\boxed{06}$

$V = 2.6$ m/s

$u_{max} = 3.17$

$$\frac{u_{max}}{V} = 1.33\sqrt{f} + 1$$

$$\frac{3.17}{2.6} = 1.33\sqrt{f} + 1 \Rightarrow f = 0.02717$$

Q7

$\rho = 10^3 \ kg/m^3$

$\nu = 10^{-6} \ m^2/s$

$K = 0.12 \ mm$

$\tau_o = 600 \ N/m^2$

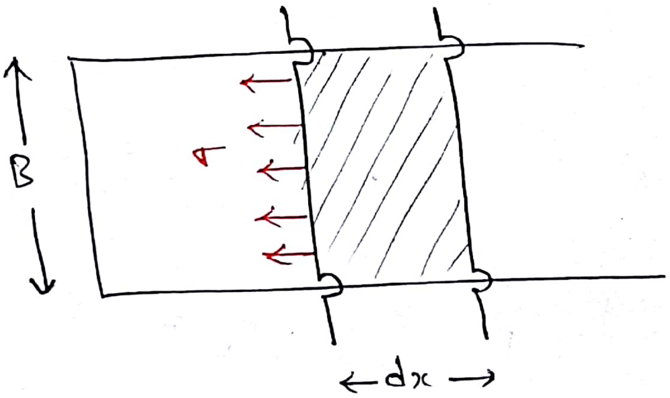
$\frac{K}{\delta'} = ? = 8.013$

$$\tau_o = \frac{\rho f v^2}{8}$$

$$V* = \sqrt{\frac{\tau_o}{\rho}}$$

$$\delta' = \frac{11.6 \times \nu}{V*} = 11.6 \nu \times \sqrt{\frac{\rho}{\tau_o}}$$

Work done in increasing the surface Area



$w =$ Force × Displacement

$= \sigma B dx$

$$\boxed{w = \sigma dA}$$

$$\boxed{\sigma = \frac{w}{dA}}$$

$dA =$ change in surface area

Note:
- Surface tension is also defined as the workdone per unit change in surface Area and this work provided from outside is stored in the fluid film in the form of energy.
- As surface tension always trys to minimise the surface area therefore for increasing the surface area work needs to be done from outside.

Momentum integral equation

$$\Sigma F = m \times a$$

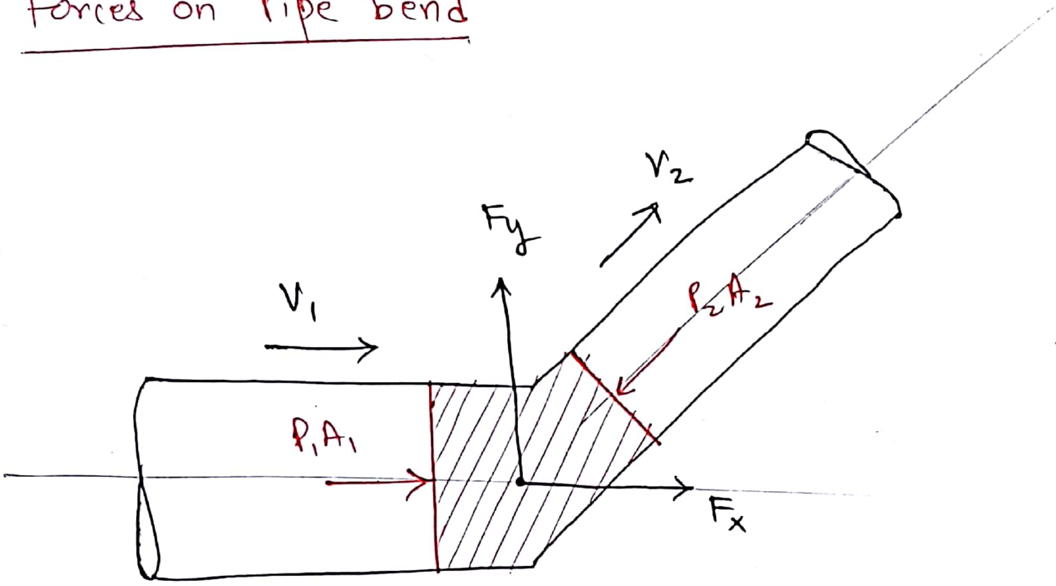
$$= m \times \frac{v-u}{t}$$

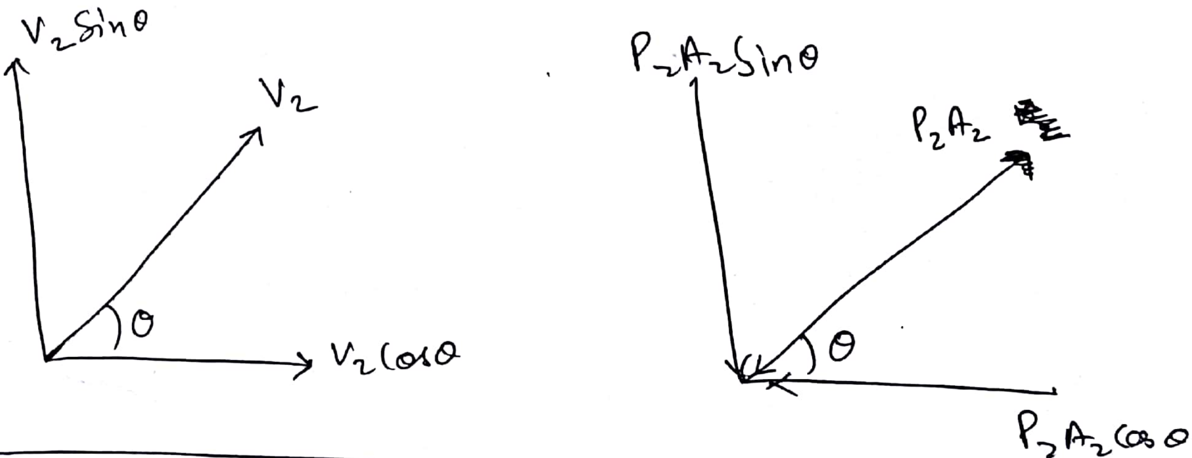
$$= \dot{m}\,(v-u)$$

$$= \rho A v\,(v-u)$$

$$\boxed{\Sigma F = \rho Q\,(v-u)}$$ momentum integral eqn

Forces on Pipe bend





$$\boxed{P_1A_1 - P_2A_2\cos\theta + F_x = \rho Q\,(V_2\cos\theta - V_1)}$$ in x-dirn

$$\boxed{-P_2A_2\sin\theta + F_y = \rho Q\,(V_2\sin\theta - 0)}$$ in y-dirn