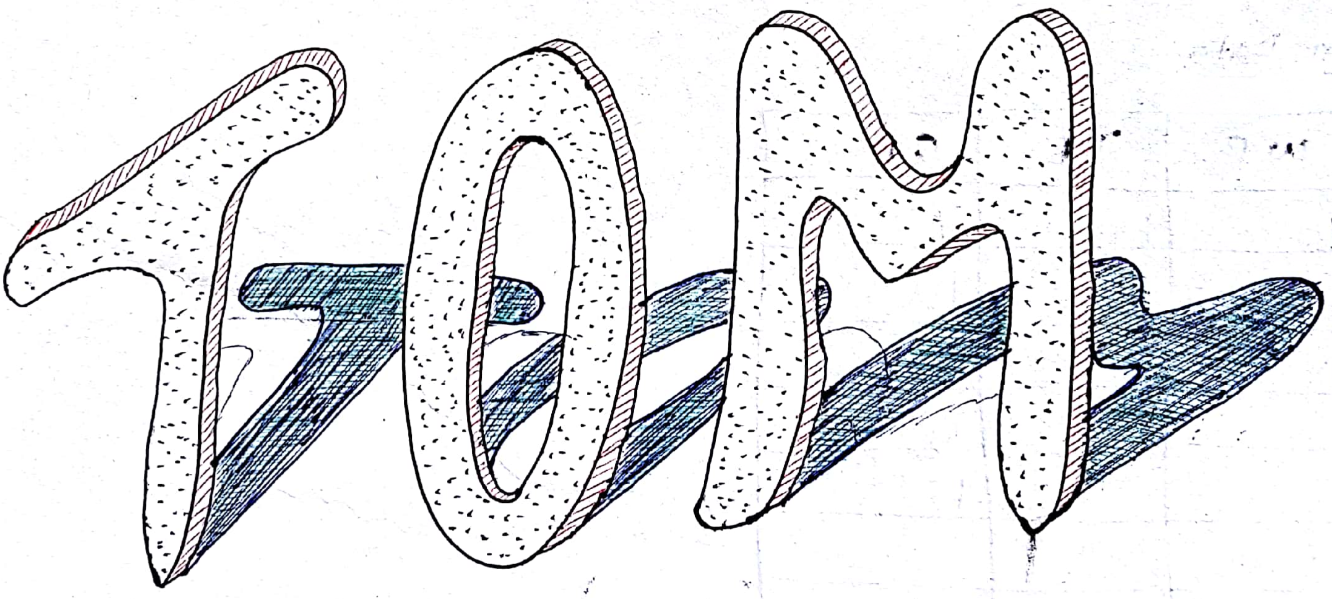


Mechanical Engineering

↓

Engineering of mechanics

Study of motion

Study of motion without consideration of forces

study of motion with consideration of forces

Kinematics

m/s $\quad \vec{v} = \frac{d\vec{s}}{dt}$

m/s² $\quad \vec{a} = \frac{d\vec{v}}{dt}$

m/s³ $\quad \vec{J} = \frac{d\vec{a}}{dt}$

Dynamic

or

Kinetics

Newton's 2nd Law

$$\vec{F}_{ext.} = \frac{d(m\vec{v})}{dt}$$

mass

eg:- Dynamic viscosity

$\mu \Rightarrow \frac{N\text{-}S}{m^2}$

Theory of machine = Kinematic of m/c + Dynamic of machine + Vibration

- Simple mechanism
- motion analysis
  - → Velocity analysis
  - → Acceleration Analysis
- Gears
- Gear Trains
- Flywheels
- Governors
- Balancing
- Mechanical Vibration
- Cams & followers
- Gyroscope

Book :- SS Ratan for questions only

CHAPTER-1

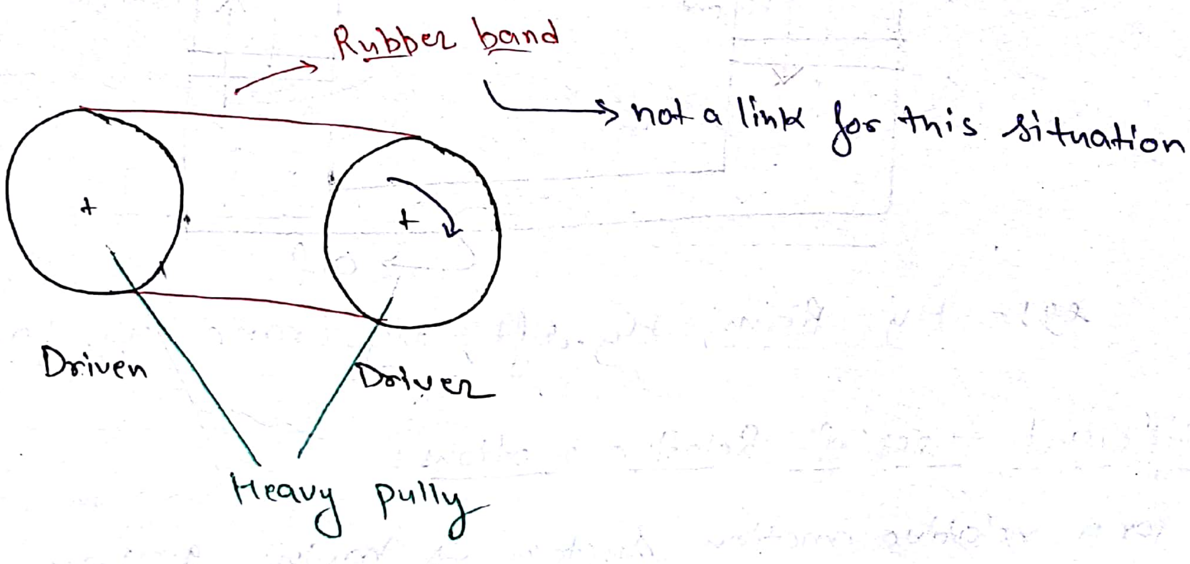
SIMPLE MECHANISMS

⇒ Kinematic link or element

"Every part of the m/c which is having some relative motion w.r.t. some other part is known as Kinematic link or element."

It is neccessary for the link to be resistant body so that it is capable of transmitting power & motion from one element to the other element.

eg:-



Types of Links

1. Rigid link

Deformations are negligible [microscopic]

for eg:- Crank
- Connecting Rod
- Piston
- Cylinder

2. Flexible Link

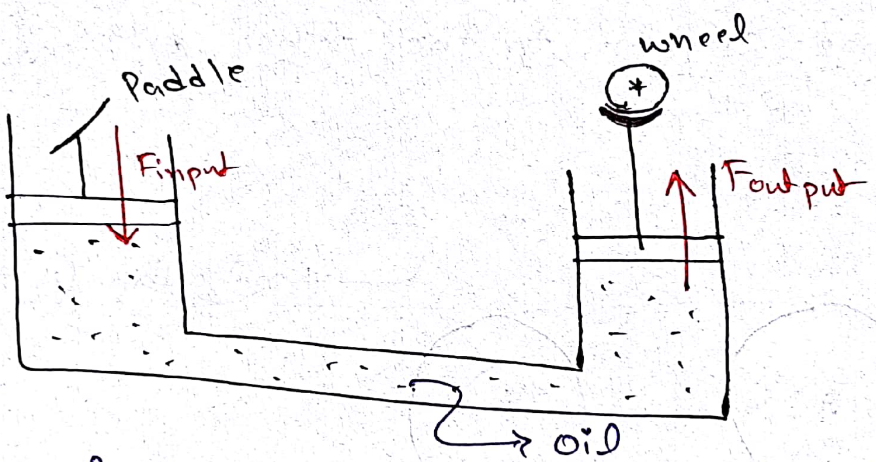
Deformations are there (not negligible) But are in permissible limits.

eg:- Belt, rope, chain Drives

3. Fluid Links

When the power is transmitted because of fluid pressure.

eg:- Hydraulic Brakes

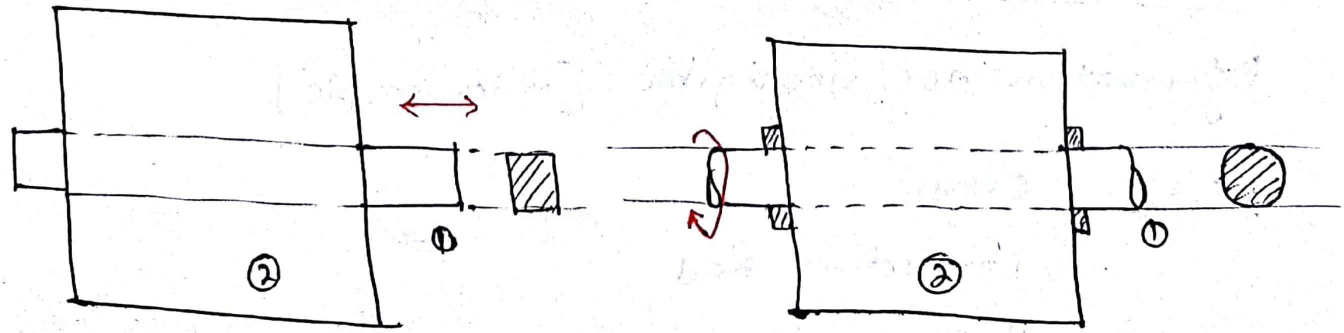


eg:- Hy. Ram, Hy. lift, Hy. crane, Hy. jack.

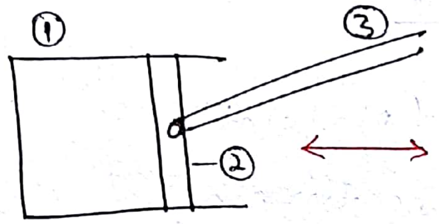
Different types of Relative motions:

for a relative motion system is having 2 links.

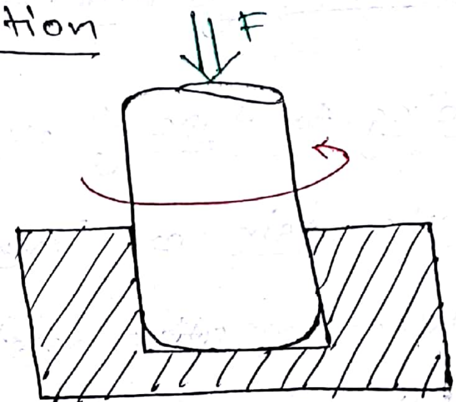
1. Completly Self constraint motion ⇒ Desired motion



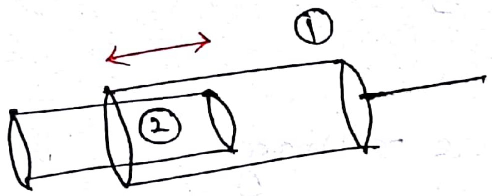
2. Successfully (→ Forcefully) constraint motion (⇒ Desired motion)



Piston - cylinder

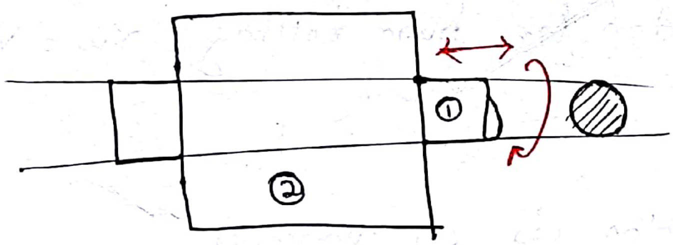


Shaft in footstep bearing



Syringe

3. Incompletely constraint motion (unconstraint motion)



Kinematic Pair

"Any connection b/w the two links is always a joint or a pair but this pair will only be a Kinematic pair if the relative motion b/w the links is a constrained motion."

- Classification of Kinematic Pairs

A) According to type of relative motion

1. Turning pair or Revolving pair (pin joint)

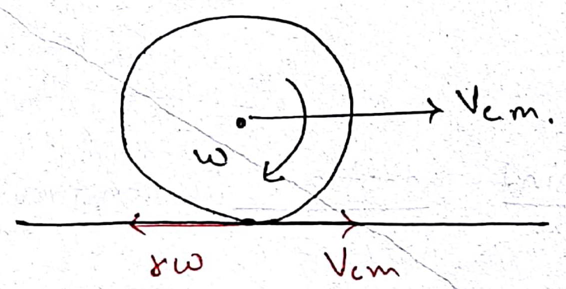
   when relative motion is pure turning

2. Sliding pair (prizmatic Pair)

   when motion is pure sliding.

3. Rolling pair

   when relative motion is pure rolling



condition for pure rolling $r\omega = V_{c.m.}$

4. Screw Pair

   when the relative motion is on threads

   eg:- nut - bolt.

5. Spherical Pair

   when the relative motion is 3-D rotation (spherical motion)

B) According to type of motion

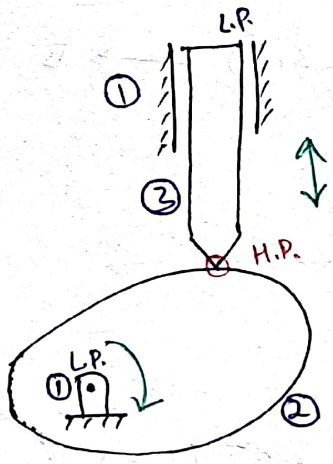
1. Lower Pair (L.P.) : Surface contact

2. Higher Pair (H.P.) : Line/Point constact

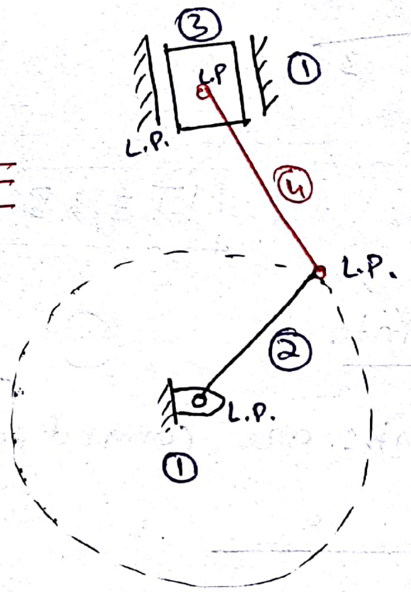
3. Wrapping Pair : when one link is wrapped over the other.

eg:- Belt - pulley.

1 HP ≡ 2 LP



≡



c) According to type of closure:

- Self closed Pair (closed Pair) : Permanent contact

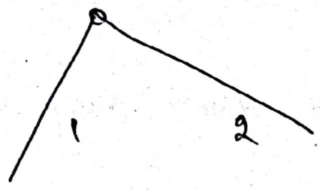
- Forced closed Pair (open pair) : Forceful contact

eg:- H.P. in Cam & follower
- Door closures
- Automatic clutch operating system.

Type of Joint.

- **Binary joint**

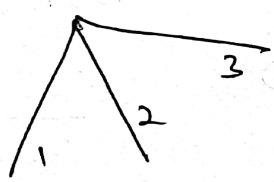
where two link are connected.



- **Ternary joint**

where 3 links are connected

(1,2) ⟶ B
(2,3) ⟶ B
(3,1) ⟶ B

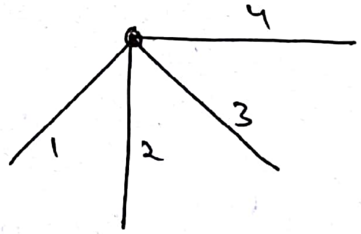


$\boxed{1T \equiv 2B}$

- **Quaternary joint.**

where 4 links are connected

(1,2) (1,3) (1,4)
(2,3) (2,4)
(3,4)



$\boxed{1Q \equiv 3B}$

Kinematic Chain

" if all the links are connected in such a way such that 1st link is connected to the last link in order to get the close chain. and if all the relative motions in this close chain are constraint then such a chain is known as Kinematic chain."

Kinematic chain

(constraint chain)

To use this chain one link should be fixed.

what?

mechanism

concept (Science)

↳ which can give you desired o/p w.r.t. given input.

utilize it.

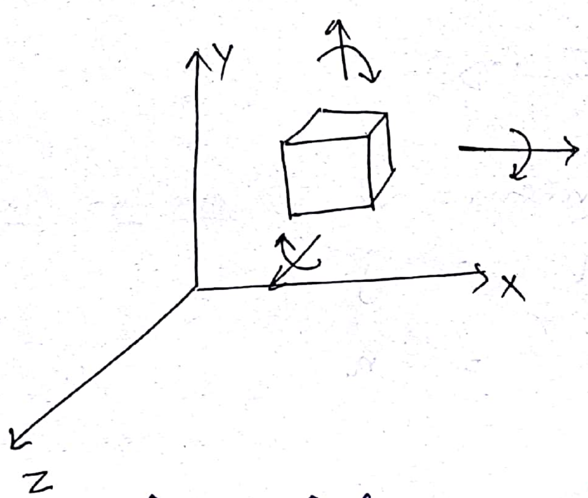
machine

How?

Product (Engineering)

Degrees of freedom (or mobility)

" The minimum number of independent variables required to define the position or motion of the system is known as Degrees of freedom of system."



3-D Space system

max. number of motions

$= 3T + 3R$

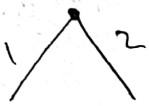
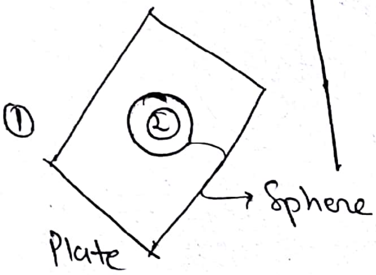
$= 6$

Degree of freedom = $\boxed{D.O.F = 6 - \text{Restraints}}$

L.P. → 1 D.O.F

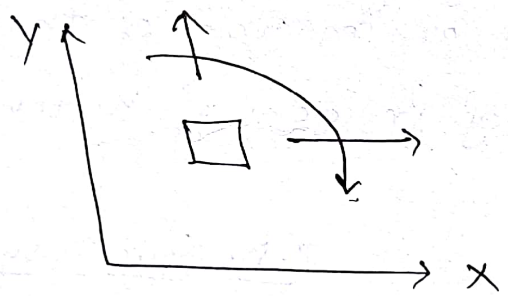
H.P. → 2 D.O.F

Spherical Pair → D.O.F = 3

Pair	Restraints	D.O.F.
 Turning Pair	$3T + 2R = 5$	$6-5=1$
	$1T = 1$	$6-1=5$

Aim :- To find the D.O.F. of 2D- planner mechanism

2D System



max. motion = 3

no. of links in a mechanism = l one link fixed

no. of Binary joint = j

no. of Higher Pair = h

$$\boxed{F = 3(l-1) - 2j - h}$$

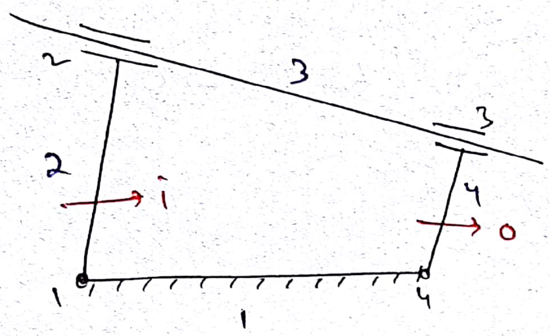
→ Kutzback's eqn

$F = 3(l-1) - 2j - h - F_r$

F_r → no of those motion which are not the part of mechanism.

Example

1.



$l = 4$

$j = 4$

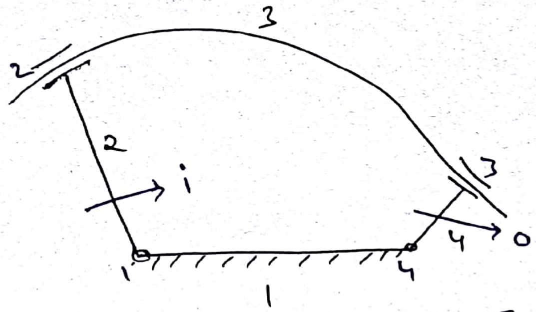
$h = 0$

$F_r = 1$

$F = 3(4-1) - 2\times4 - 0 - 1$

$= 9 - 8 - 1 = 0$

2.



$l = 4$

$j = 4$

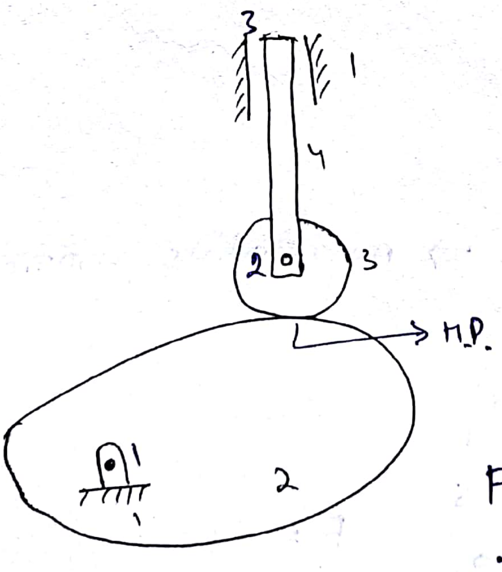
$h = 0$

$F_r = 0$

$F = 3(4-1) - 2\times4 - 0 - 0$

$F = 1$

3.



$l = 4$

$j = 3$

$h = 1$

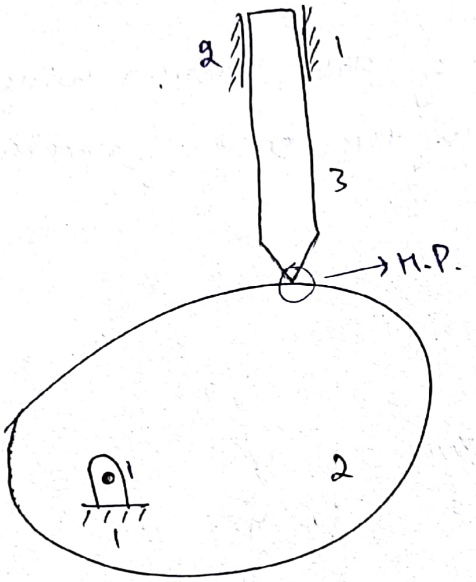
$F_r = 1$

$F = 3(4-1) - 2\times3 - 1 - 1$

$= 9 - 6 - 1 - 1$

$= 1$

4.



$l = 3$
$j = 2$
$h = 1$
$F_r = 0$

$$F = 3(3-1) - 2 \times 2 - 1 - 0$$
$$= 6 - 4 - 1$$
$$= 1$$

If $F = 0$

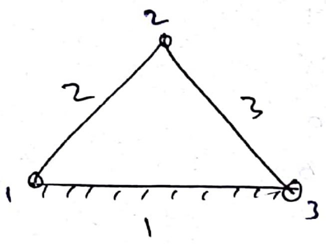
⇓

no relative motion

⇓

frame | structure

eg:



$l = 3$
$j = 3$
$h = 0$

$F = 0$

If $F < 0$ $(-1, -2, \ldots)$

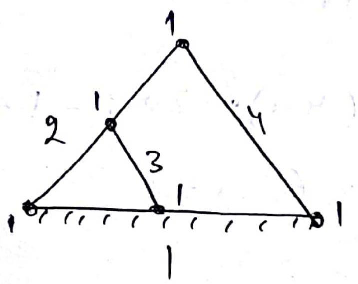
⇓

Super structure

⇓

Indeterminate structure ⇒ no relative motion

eg:-



$l = 4$
$j = 5$
$h = 0$

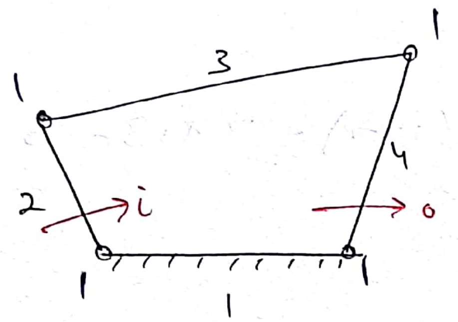
$F = -1$

If $F = 1$

⇓

Kinematic chain

eg:-



$l = 4$
$j = 4$
$h = 0$

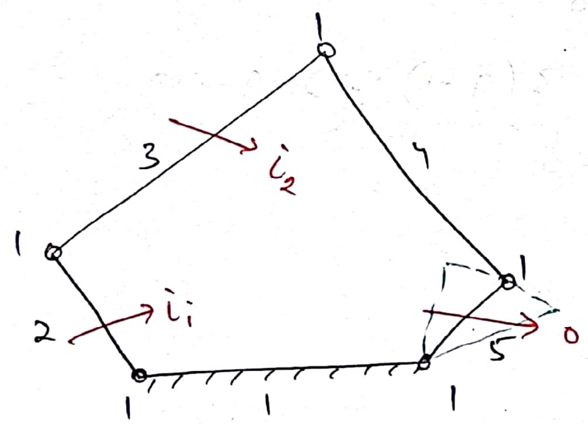
$F = 1$

If $F > 1$ $(2, 3, 4, \ldots)$

⇓

Unconstraint chain

eg:-



$l = 5$
$j = 5$
$h = 0$

$F = 2$

* D.O.F. is the no. of inputs required to get constrained output in any chain.

Alternative way:-

1. $\left(j + \frac{h}{2}\right) = \left(\frac{3l}{2} - 2\right)$ ⇒ Kinematic chain

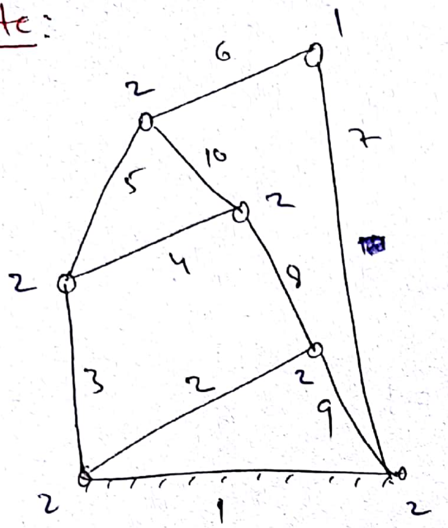
2. $\left(j + \frac{h}{2}\right) > \left(\frac{3l}{2} - 2\right)$ ⇒ Frame | Structure

3. $\left(j + \frac{h}{2}\right) < \left(\frac{3l}{2} - 2\right)$ ⇒ unconstraint chain

1/6/16

Note:

a)



$l = \cancel{7}\ 10$

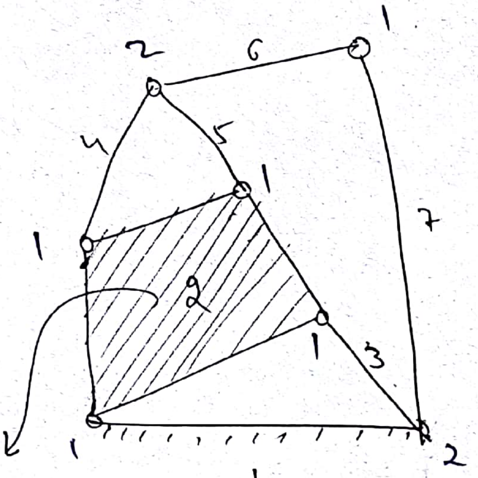
$j = 13$

$h = 0$

$$F = 3\times(10-1) - 2\times 13 - 0$$

$$= 1$$

b)



Single plate

$l = 7$

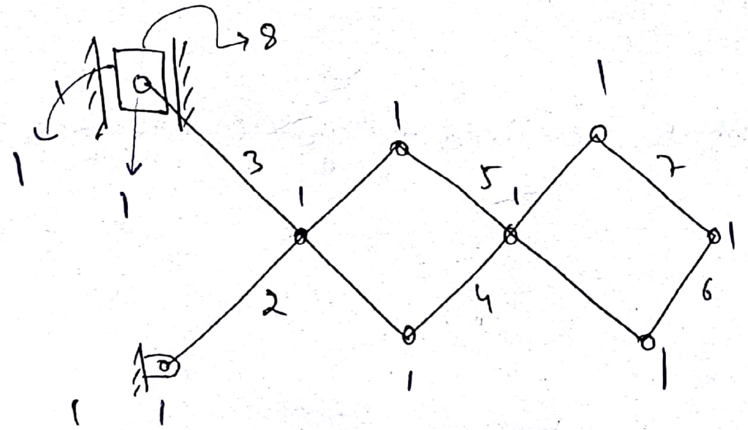
$j = 9$

$h = 0$

$$F = 3(7-1) - 2\times 9 - 0$$

$$F = 0$$

c)



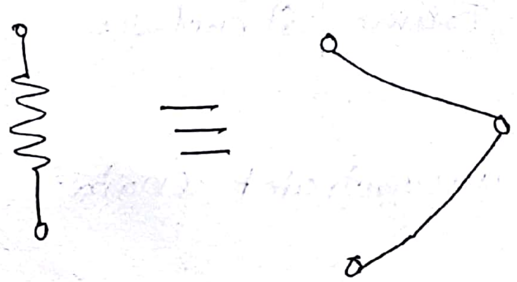
$l = 8$

$j = 10$

$h = 0$ $\quad F = 3(8-1) - 2\times 10 - 0$

$F = 1$

d) Spring as a link.



Grubler's equ^n

For those mechanism in which

$F = 1$

$h = 0$

Applying Kutzback's equ^n

$F = 3(l-1) - 2j - h$

$1 = 3(l-1) - 2j - 0$

$\Rightarrow \boxed{3l - 2j - 4 = 0}$ Grubler's equ^n

$\Downarrow$

$3l \rightarrow$ always even

$l \rightarrow$ always even

So, $(l)_{mini.} = 4$

$\Downarrow$

First mechanism in Lower Pair.

→ Simple mechanism

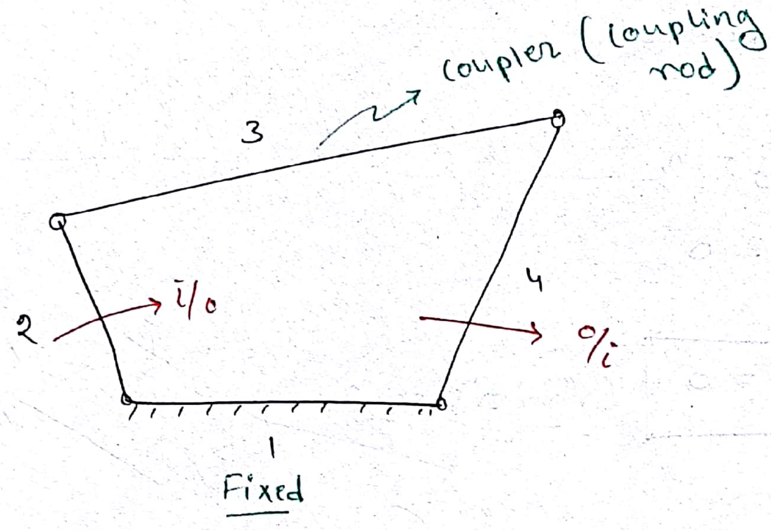
- → Four bar mechanism
- → Single slider crank mechanism
- → Double slider crank mechanism

A) Four Bar Mechanism :

[Quadric Cycle mechanism]

↓

4 links + 4 TP



Best Position ⇒ fixed ⇒ bcz it governs both → i, → o

Input / output

→ Complete rotation ($360°$) → crank

→ Partial rotation ($< 360°$) → rocker/lever

Inversions

mechanisms which are obtained by fixing one by one different different links.

1. Double - crank mechanism
2. Crank - rocker mechanism
3. Double - rocker mechanism.

★ If no. of links = l

no. of inversions $\leq l$

Grashof's Law:

" For the continus relative motion b/w the number of links in a mechanism. The sumation of shortest & longest length should not be greater than the submission of lengths of other two links."

$$\boxed{s + l \leq p + q}$$

Best position → fixed (bcz it governs $\angle$ i o

Best link for rotation → smallest.

⇒ $s + l < p + q$
Law satisfied

1. s fixed ⟶ Double crank mechanism

2. s adjusent to fixed → crank rocker mechanism

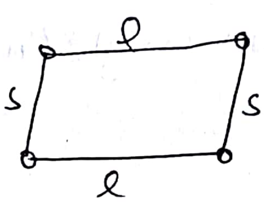
3. s coupler ⟶ Double rocker mechanism

⇒ $s + l = p + q$ & no pair of equal lengths
Law satisfied

All results are same as $s + l < p + q$

⇒ $s + l = p + q$ and have pair of equal lengths

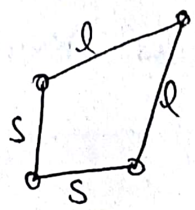
Parallelogram linkage



s fixed ⇒ Double crank

l fixed ⇒ Double crank

Deltoid linkage



s fixed ⇒ Double crank

l fixed ⇒ Crank-rocker

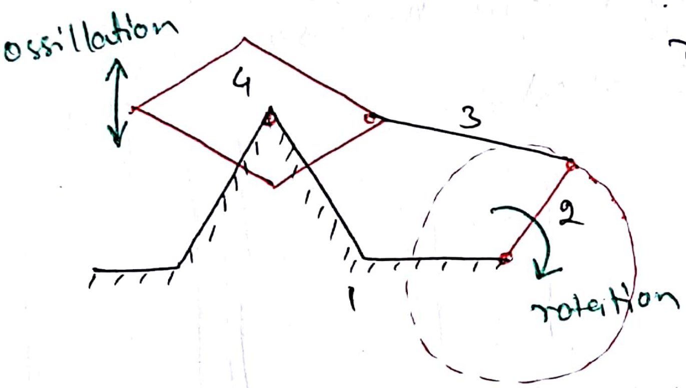
$\Rightarrow \quad s + l > p + q$

Law not satisfied

$\Rightarrow$ Always double rocker

Some Practicle Applications of 4-bar mechanism

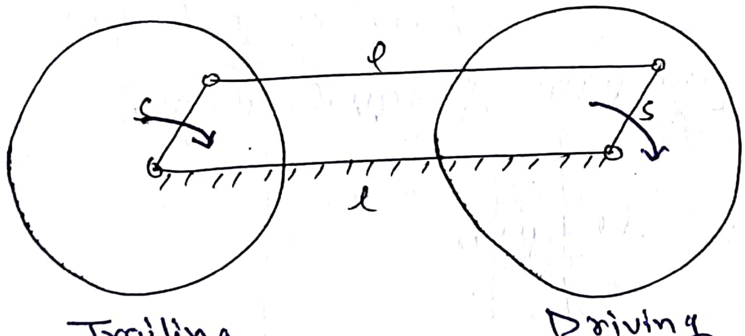
1. Beam Engine [James watt]



rotation $\longleftrightarrow$ ossillation

crank-rocker mechanism

2. Coupling Rod of locomotive

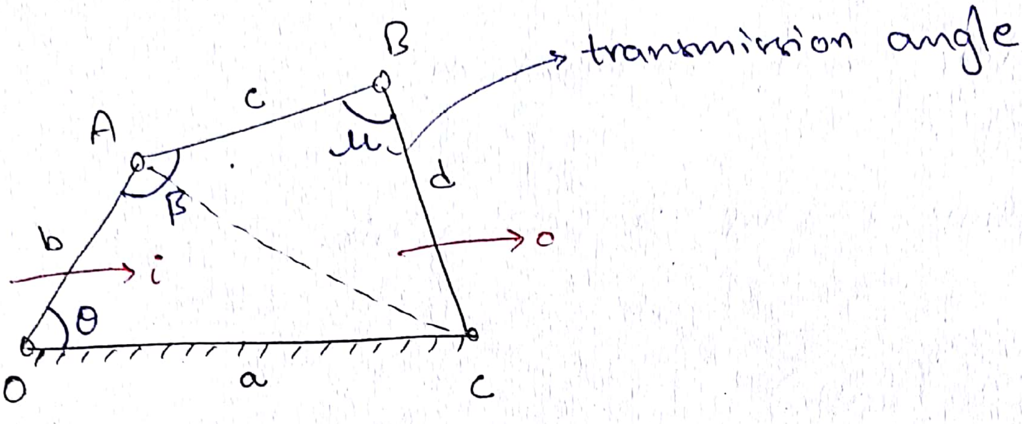


Parallelogram linkage

Double-crank mechanism

Transmission angle in 4-Bar mechanism: (μ)

The angle b/w the coupler link & output link in 4-bar mechanism is known as Transmission angle (μ).



$$AC^2 = a^2 + b^2 - 2ab\cos\theta = c^2 + d^2 - 2cd\cos(\mu)$$

Differentiating both the side.

$$-2ab(-\sin\theta)d\theta = -2cd(-\sin\mu)\,d\mu$$

$$\frac{d\mu}{d\theta} = \frac{ab}{cd}\,\frac{\sin\theta}{\sin\mu}$$

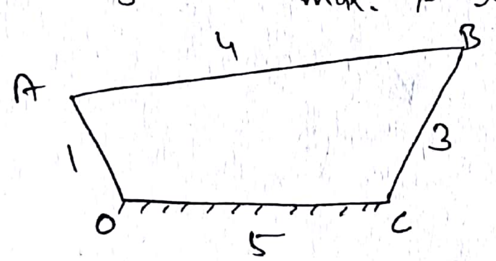
for μ to be maximum / minimum

$$\frac{d\mu}{d\theta} = 0 \Rightarrow \sin\theta = 0$$

$$\Rightarrow \theta = 0,\ 180^\circ$$

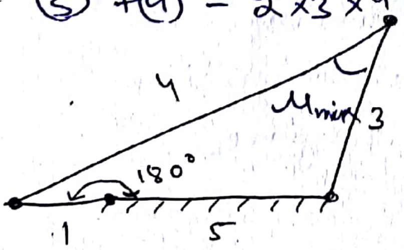
minimum　　maximum

Example　find μ_{max} & μ_{min}



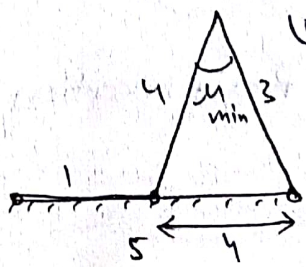
Solⁿ　for $\mu_{max} \Rightarrow \theta = 180^\circ$

$$(6)^2 = (3)^2 + (4)^2 - 2\times3\times4\cos(\mu_{max})$$



for $\mu_{min} \Rightarrow \theta = 0^\circ$

$$(4)^2 = (4)^2 + (3)^2 - 2\times4\times3\times\cos(\mu_{min})$$



B) Single slider crank mechanism:

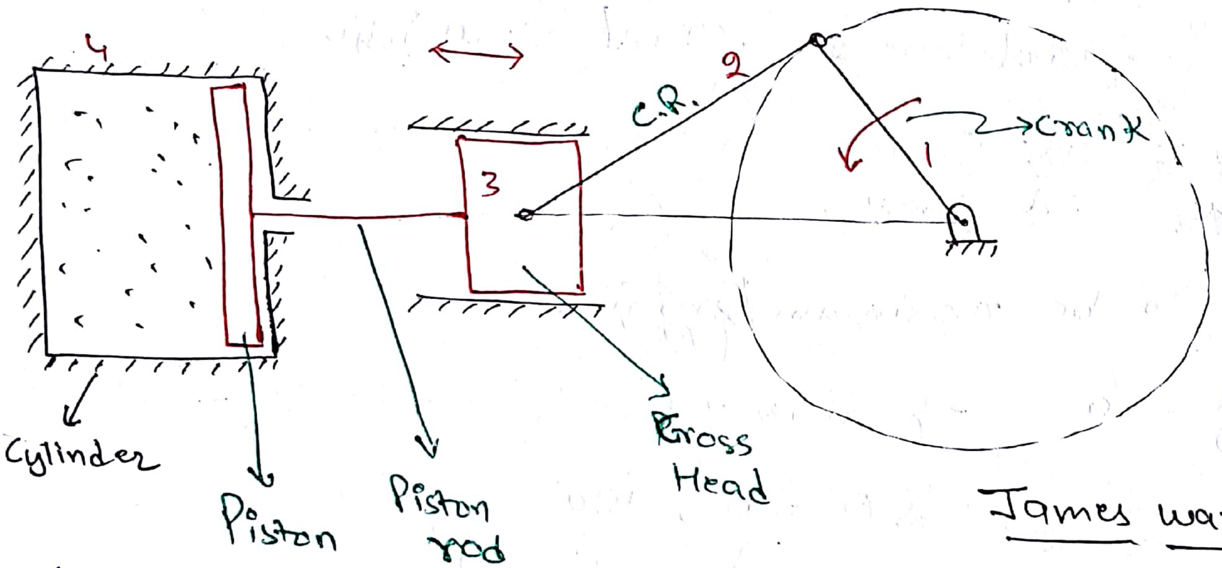
⇓

4 link + 3 T.P. + 1 sliding pair.

★ Ist inversion (Basic)

- Cylinder is fixed.

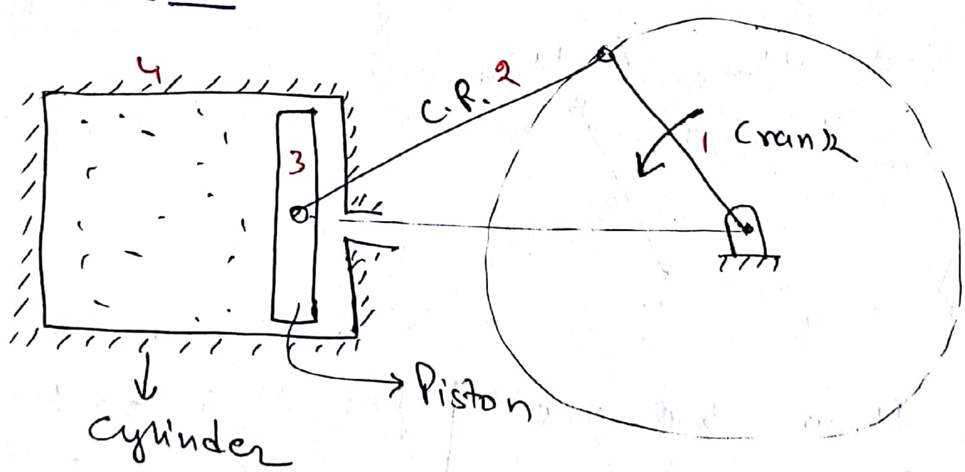
rotation ⟷ Reciprocation



External combustion Engine

James watt's machine

- Cylinder is fixed



Rotation ⟷ Reciprocation

Internal combustion engine

* II inversion

↳ Crank is fixed

Whitworth QRM | Rotary IC engine (Gnome Engine)

* III inversion

⟶ connecting rod fixed

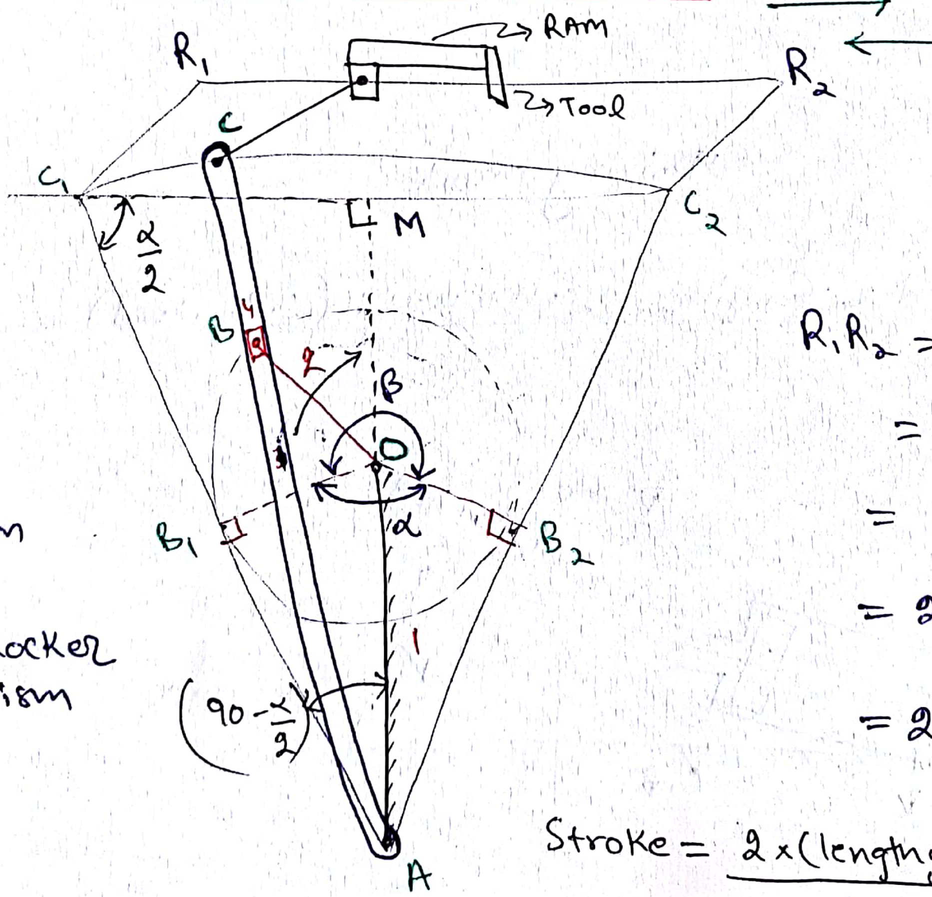
Crank & slotted lever QRM mechanism | Oscillating cylinder engine mechanism

* IV inversion

⟶ Piston / slider is fixed

[Handpump / Pendulum pump]

Crank & slotted lever QRMM:



Rotation ↓ oscillation

Crank-Rocker mechanism

Stroke ⇓ R_1R_2

$$R_1R_2 = C_1C_2$$
$$= 2(C_1M)$$
$$= 2\,AC_1\cos\left(\frac{\alpha}{2}\right)$$
$$= 2\,AC_1 \times \frac{OB_1}{OA}$$
$$= \frac{2(AC)(OB)}{OA}$$

$$\text{Stroke} = \frac{2\times(\text{length of slotted bar})(\text{length of crank})}{\text{length of connecting rod}}$$

$\beta \rightarrow$ Cutting angle

$\alpha \rightarrow$ Return angle

$\therefore \quad \alpha + \beta = 360^\circ$

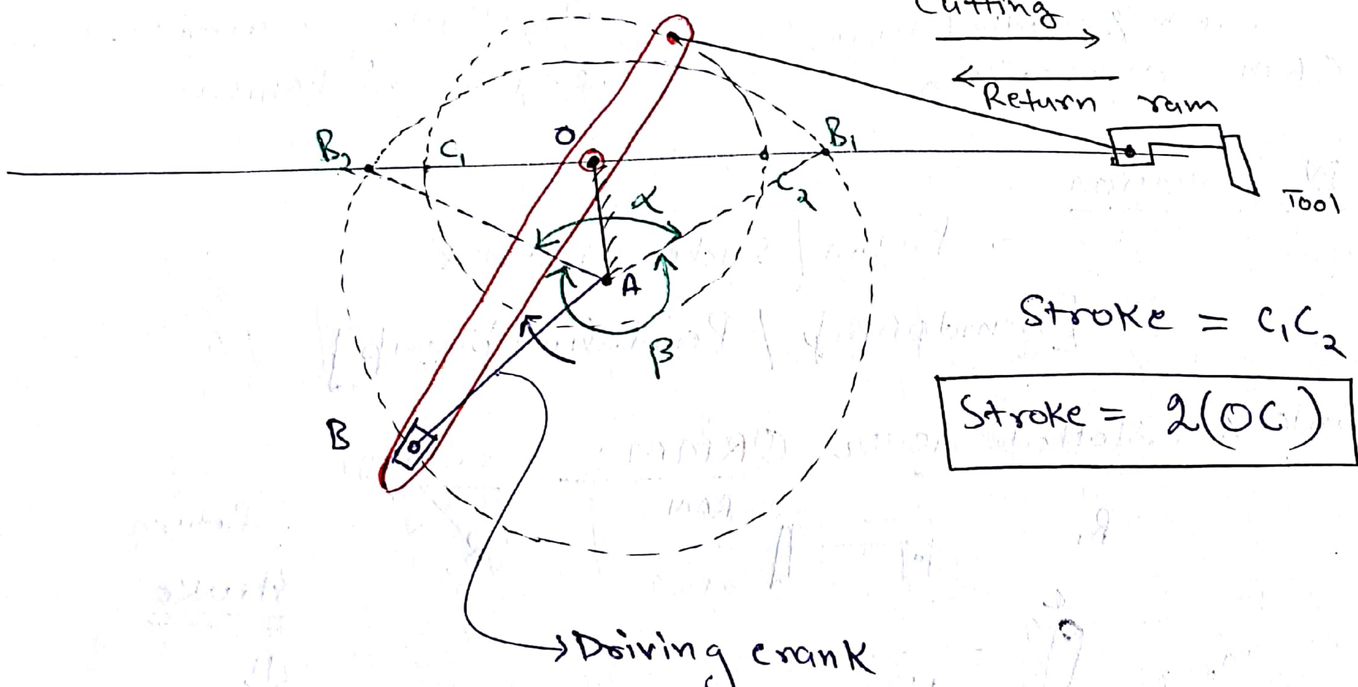
$\alpha < \beta \rightarrow$ QRMM

Quick Return Ratio $= \dfrac{\text{time cutting}}{\text{time Return}} = \dfrac{\beta}{\alpha} > 1$

Whitworth QRMM:

Rotation $\rightarrow$ Rotation

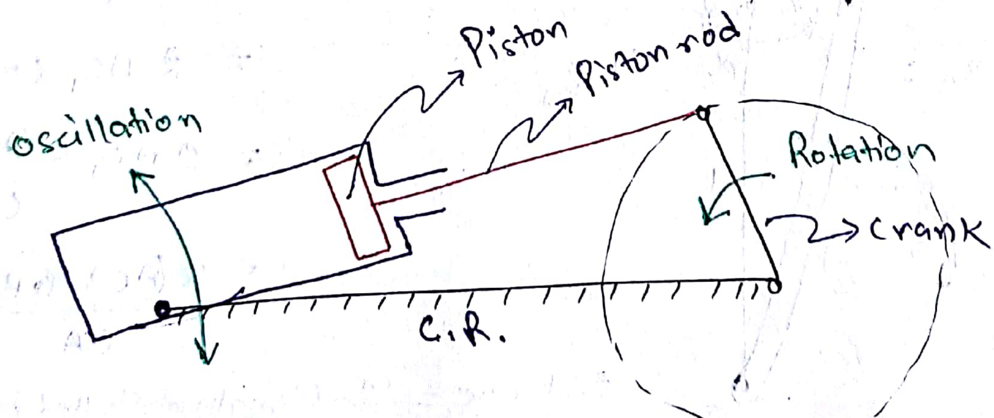
Double - crank mechanism



Stroke $= C_1C_2$

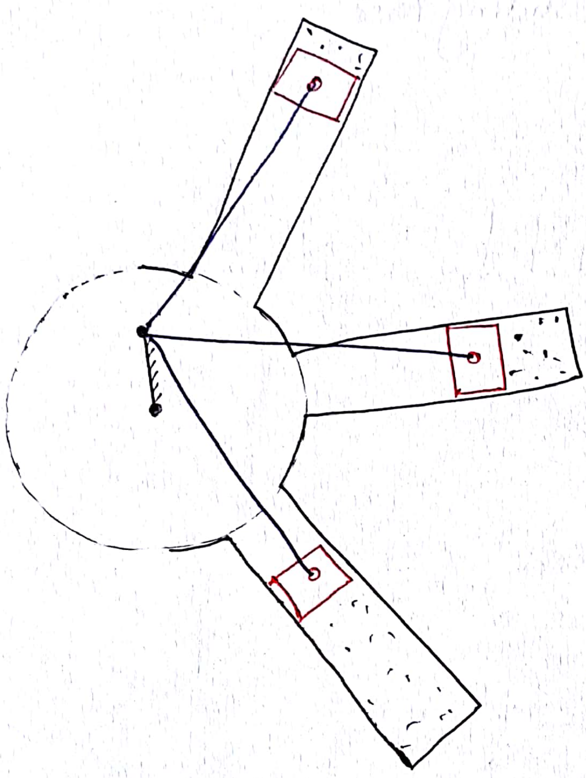
$\boxed{\text{Stroke} = 2(OC)}$

Oscillating Cylinder Engine mechanism: (C.R. fixed)



Rotary I.C. engine [crank fixed]

GNOME Engine



when combustion takes place inside the cylinder

⇓

Input force comes on Piston

⇓

This force is transfered to C.R.

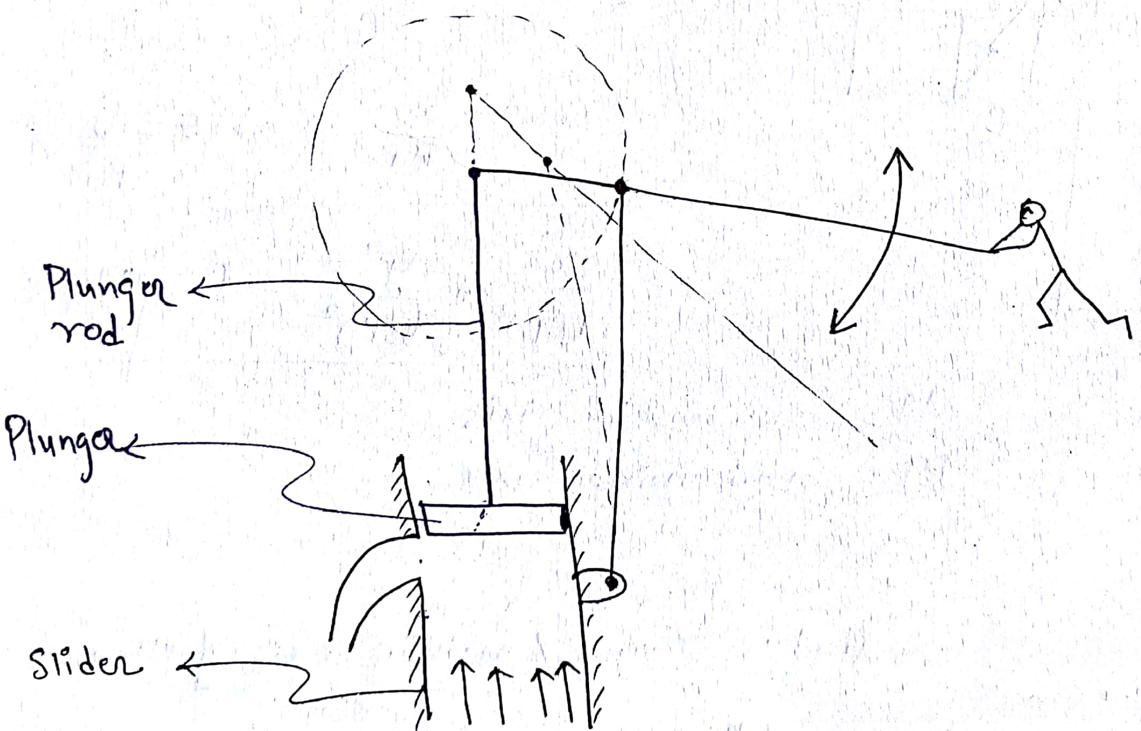
⇓

C.R. & Piston Both rotates

⇓

Cylinder block Rotates (output)

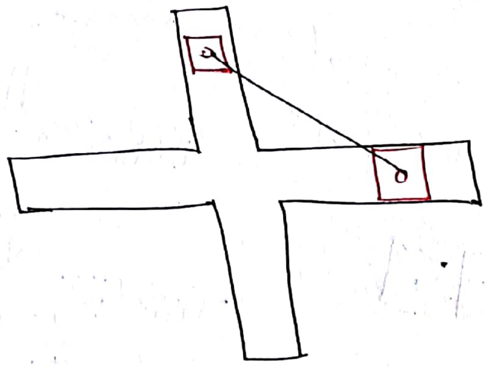
Hand pump (Slider fixed)



c) Double slider crank chain :

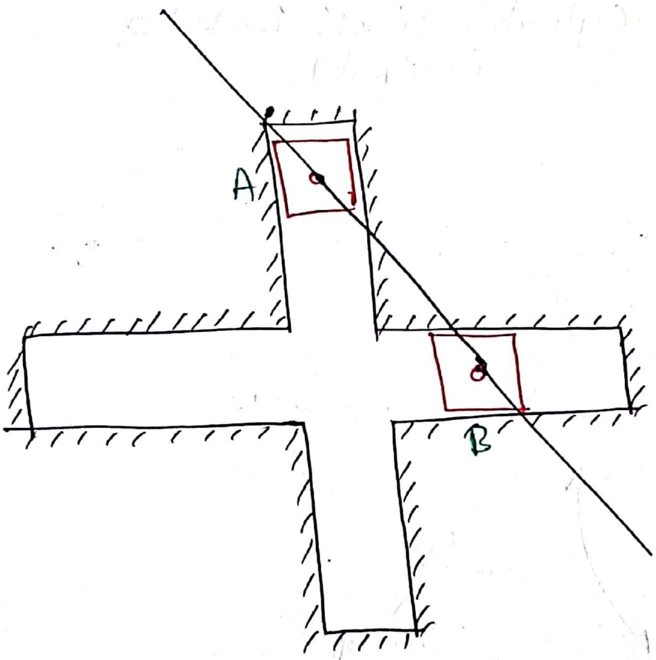
⇓

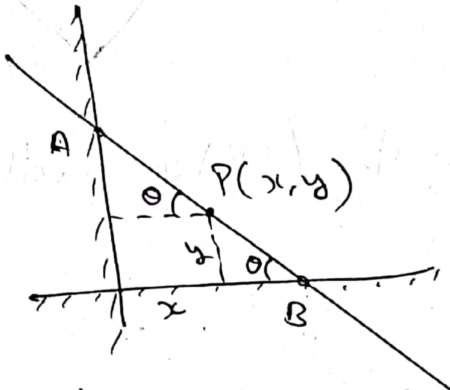
4 links + 2TP + 2 sliding Pair



I inversion [slotted bar fixed]

↳ Elliptical Trammels





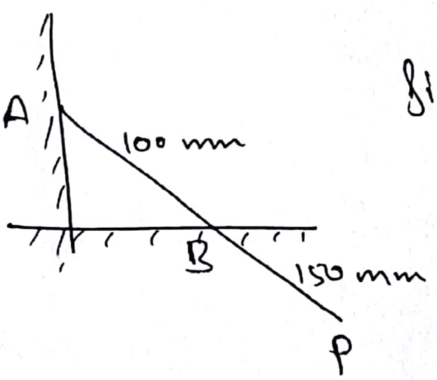
$\cos\theta = \frac{x}{AP}$; $\sin\theta = \frac{y}{BP}$

$$\boxed{\frac{x^2}{(AP)^2} + \frac{y^2}{(BP)^2} = 1}$$ ellipse

AP = Semi major axis

BP = Semi minor axis

Ques:



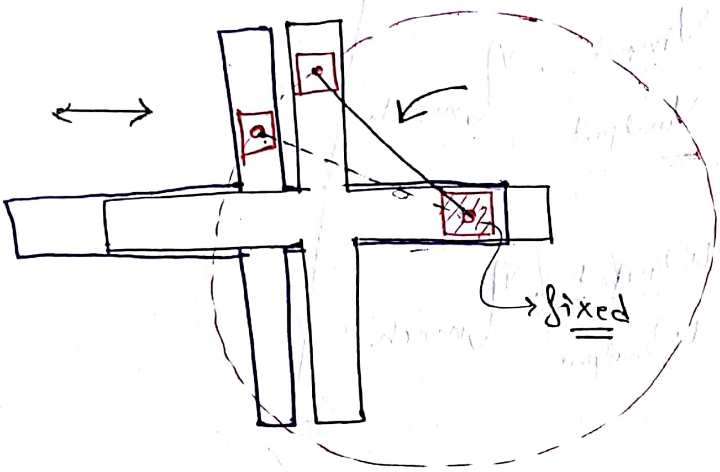
find :- Major & minor exis of ellipse

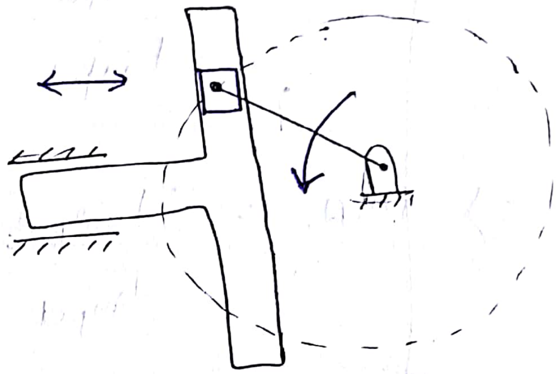
Soluⁿ: major axis = 500 mm

minor axis = 300 mm

II inversion [one of slider is fixed]

↳ Scotch-yoke Mechanism :



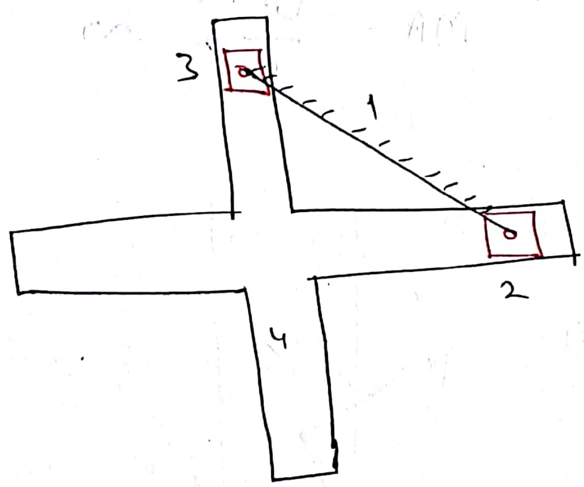


III inversion [connecting link is fixed]

↳ Oldham's coupling

⇓

Used to connect the shafts having lateral misalignment



maximum velocity of slider in plate

$= r \cdot \omega_{driver}$

$= \left(\text{Distance b/w Shaft}\right) \times \left(\omega_{Driver}\right)$

Mechanical Advantage

$$MA = \frac{F_{output}}{F_{input}}$$

Or

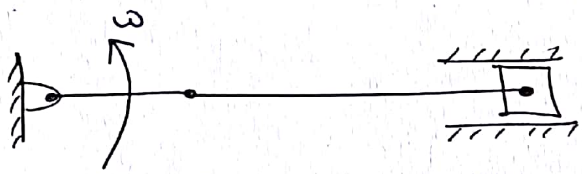
$$MA = \frac{T_{output}}{T_{input}}$$

$$\eta_{mechanism} = \frac{P_{output}}{P_{input}} = \frac{F_{output} \cdot V_{output}}{F_{input} \cdot V_{input}} = \frac{T_{output} \cdot \omega_{output}}{T_{input} \cdot \omega_{input}}$$

$$\Rightarrow \boxed{M.A. = \frac{F_{output}}{F_{input}} = \frac{V_{input}}{V_{output}} \cdot \eta_{mech.}}$$

$$\Rightarrow \boxed{M.A. = \frac{T_{output}}{T_{input}} = \frac{\omega_{input}}{\omega_{output}} \cdot \eta_{mech.}}$$

Ques: Find M.A.



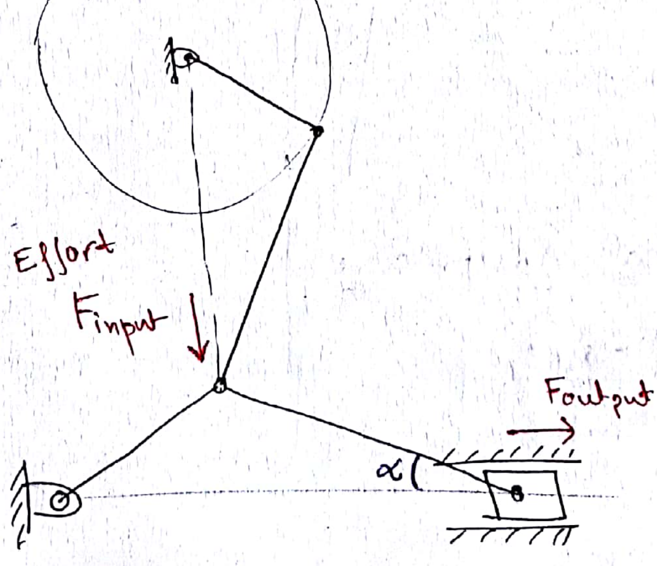
Solu^n

$\omega_{input} = \omega$

$\omega_{output} = 0$

$MA = \frac{\omega}{0} = \infty$

Toggle mechanism



$\tan\alpha = \frac{F_{input}}{F_{output}}$

$F_{output} = \frac{F_{input}}{\tan\alpha}$

As $\alpha \to 0$

$\tan\alpha \to 0$

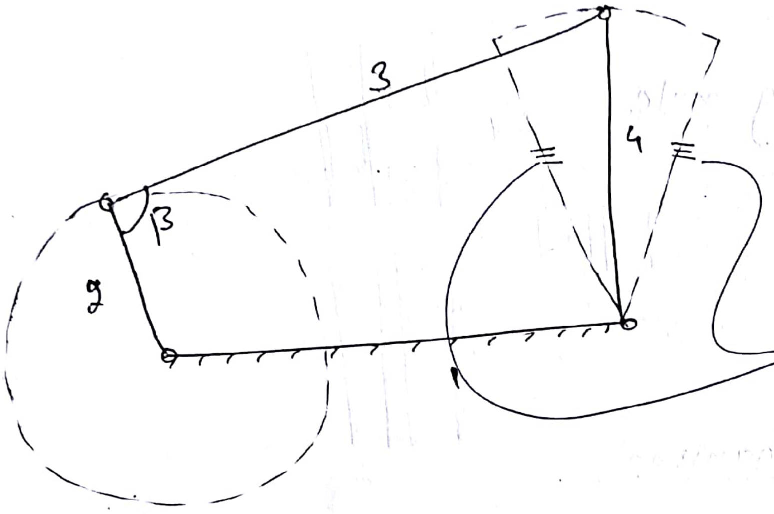
$\Rightarrow F_{out} \to \infty$

$\boxed{MA = \infty}$

2/6/16

Toggle position in 4 Bar mechanism

Extreme positions of output link in a 4 bar crank rocker mechanism.

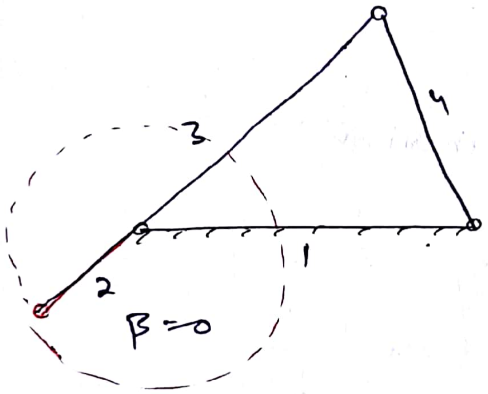


→ Extreme positions

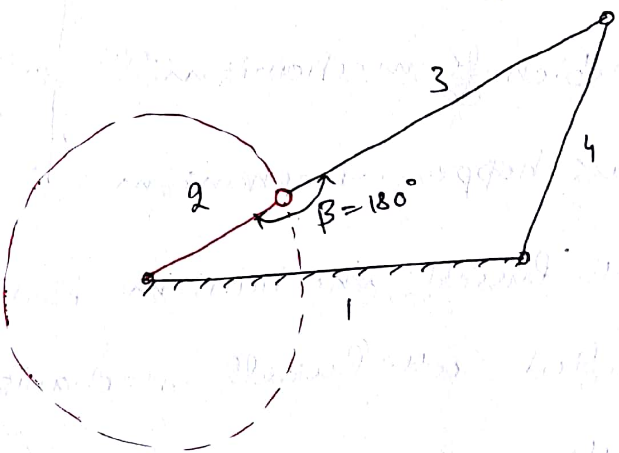
$\omega_{output} = 0$

$\Rightarrow MA = \infty$

Left Extreme position



Right Extreme position



Steering Gear mechanism

1. Acceramann Steering Gear
   - having only turning pair
   - Life is very high
   - exact at only 3 position
     - middle & extremes.
2. Davis steering Gear
   - Having both TP & SP. ⇒ life is very less
   - Exact at all position (follow of stearing law)
3. Rapson's slide
   - Used in ships → Cross slider crank mechanism.

Intermittent Motion Mechanisms:

Periodic motion with const. breaks at output w.r.t. given continous input

1. Geneva Mechanism

   Used in indexing of milling m/c

2. Ratched mechanism

   Used in Clocks.

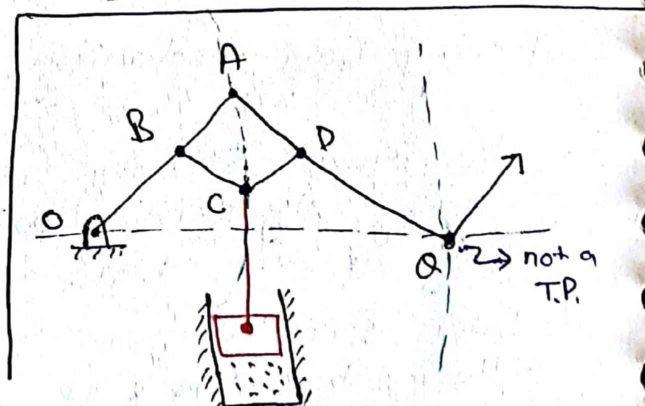
Straight line motion mechanism

- Exact st. line motion mechanism
- Approx. st. line motion mechanism

- Roberts mechanism
- Tchebicheffs mechanism
- Grass hopper mechanism

4 links
4 bar mechanism

- Scott-Russell mechanism 4 links
- Modified Scott-Russell mechanism 4 links
- Hart's mechanism - 6 links
- Watt's indicator mechanism 4 links
- Pantograph mechanism 4 links



Watt's indicator mechanism

Observations - Point C & Point Q both moves in approx st. line motion.

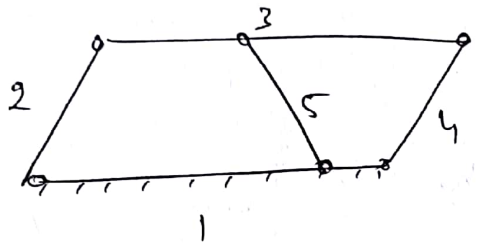
- There is no relative motion b/w link BC & CD

  BCD → one link

- link BCD] levers
link AQ] Double lever mechanism.

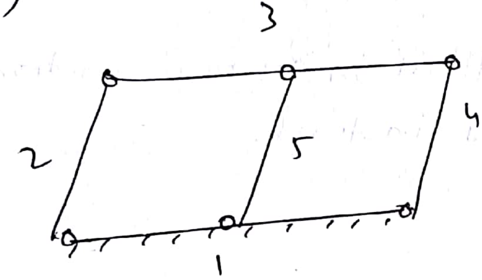
Note :-

1)



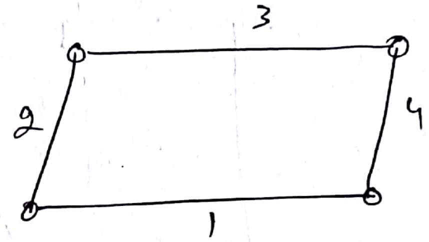
$l = 5$
$j = 6$
$\boxed{F = 0}$

2)



$l = 5$
$j = 6$
$F \neq 0$
$F = 1$

$\equiv$

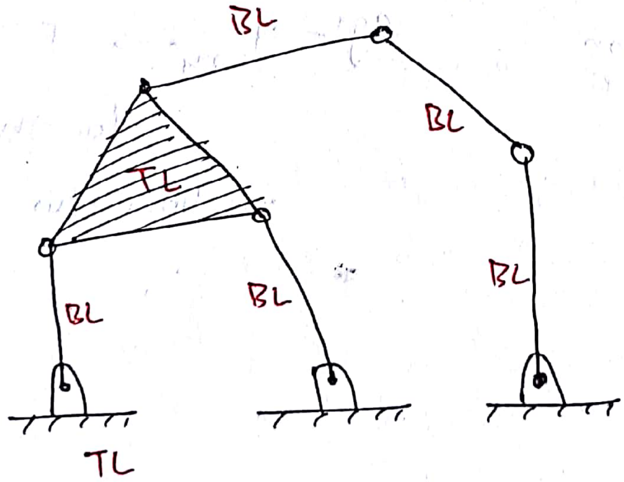


$\boxed{F = 1}$

3) Binary link → Connected at two places

Ternary link → connected at three places

Quatenary link → Connected at four Places



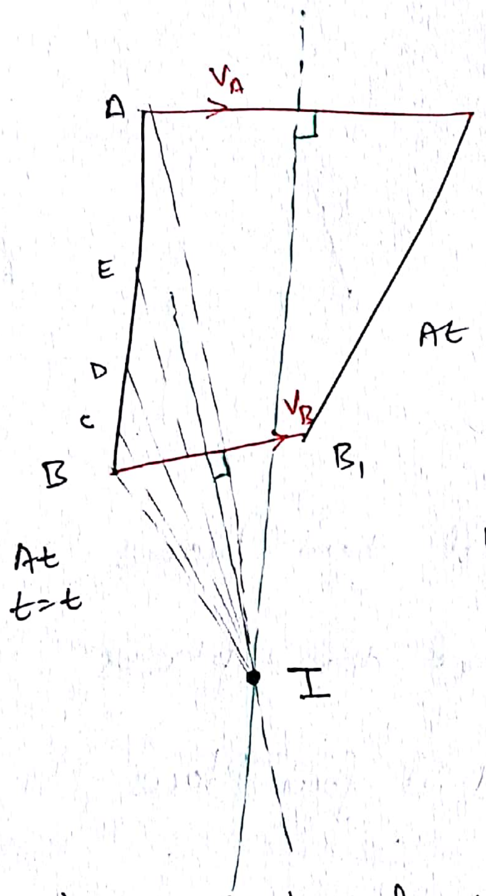
CHAPTER - 2

MOTION ANALYSIS

A) Velocity Analysis

★ Instantenous center method Approach

↳ By:- Arnold Kennedy



Link AB is in general motion at this instant.

At t= t+dt

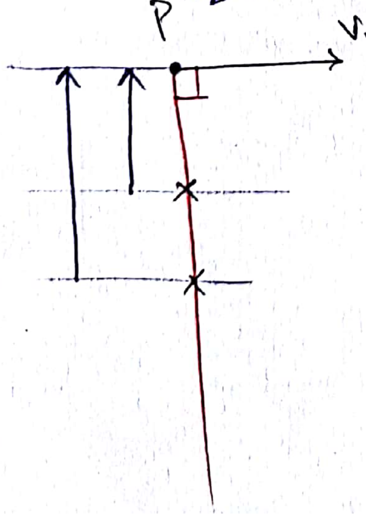
$$\omega_{AB} = \frac{V_A}{AI} = \frac{V_B}{BI} = \frac{V_C}{CI} \text{-----}$$

I = Defined for the relative motion b/w two links

Instantenous center of rotation

eg:- $I_{24} \rightarrow$ I - center for the relative motion b/w link 2&4

P → P is in general motion



In general when link moves, its I-center keeps on changing.

⇒ Locus of Ic of rotation for relative motion ⇒ Centrode

⇒ Locus of I-axis of rotation for relative motion ⇒ Axode

motion	Centrode	Axode
1. General motion	Curve	Curved surface
2. Pure rotation	Point	St. line
3. Pure translation	St. line	plane surface

"In general the motion of a link in a mechanism is neither pure translation nor pure rotation it is a combination of translation and rotation which we normally say the link is in General motion but any link at any instant can be assumed to be in pure rotation w.r.t. the point in the space known as Instantenous center of rotation. This point is also known as the virtual center."

⇒ no. of I-centers in a mechanism

If no. of links = l

no. of I center = ${}^{l}C_2$

for eg:- $l=5$; I-C = 10

I_{12} I_{13} I_{14} I_{15}

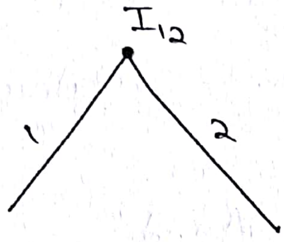
I_{23} I_{24} I_{25}

I_{34} I_{35}

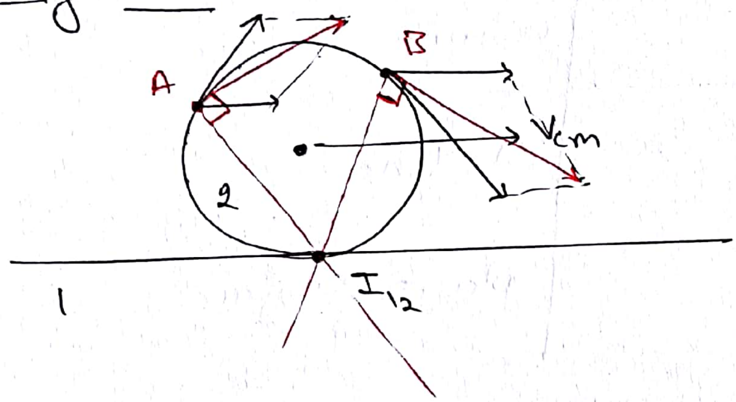
I_{45}

Basic I-center in mechanism

1. Turning Pair



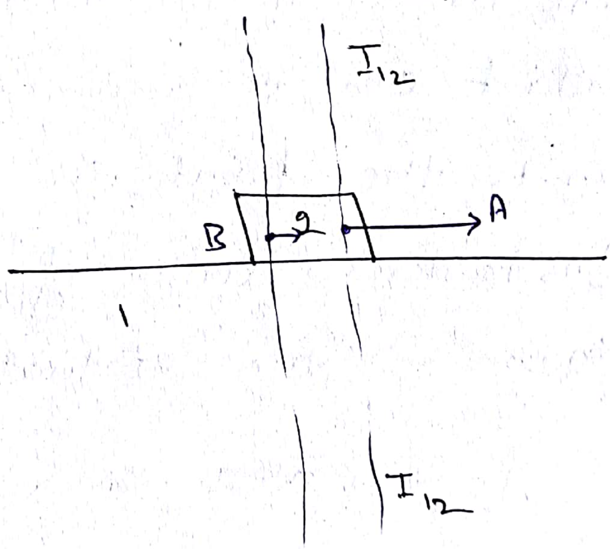
2. Rolling Pair



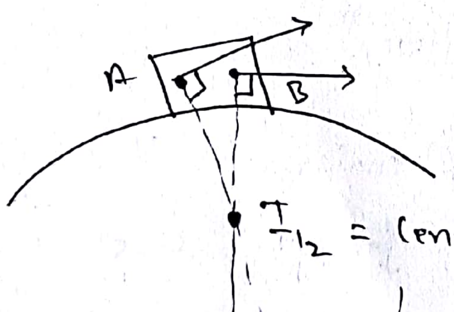
3. Sliding Pair

Curved Surface

Plain Surface

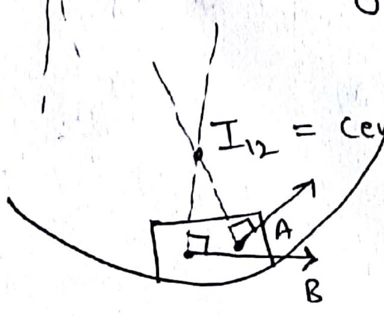


Convex



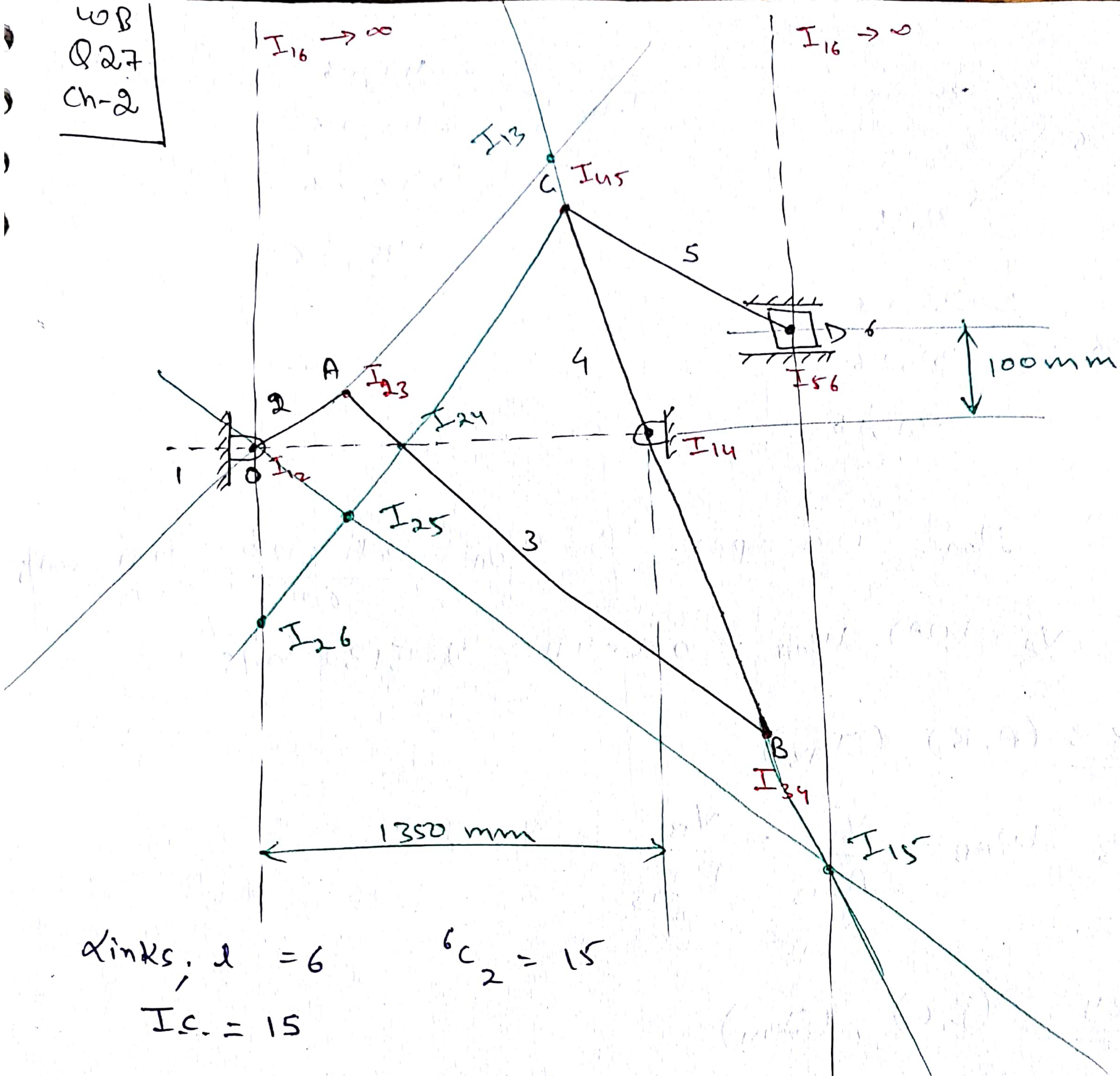
I_{12} = Center of curvature

Concave



I_{12} = center of curvature

WB
Q27
Ch-2



Links, $l = 6$ ${}^6C_2 = 15$

I.C. = 15

I_{12} I_{13} I_{14} I_{15} I_{16}

I_{23} I_{24} I_{25} I_{26}

I_{34} I_{35} I_{36}

I_{45} I_{46}

I_{56}

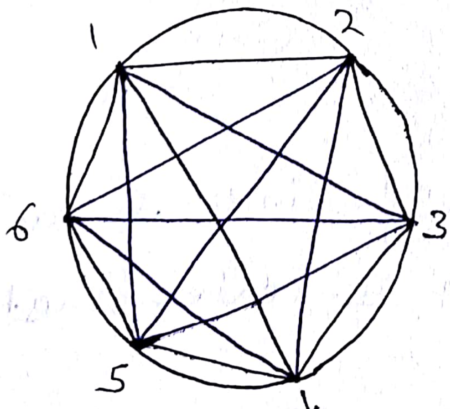
$I_{13} <$ 12, 23 / 14, 43

$I_{15} <$ 16, 65 / 14, 45

$I_{24} <$ 21, 14 / 23, 34

$I_{25} <$ 21, 15 / 24, 45

$I_{26} <$ 21, 16 / 25, 56



I_{35} — 34, 45 ; 32, 25 ; 31, 15

I_{46} — 41, 16 ; 42, 26 ; 43, 36 ; 45, 56

I_{36} — 35, 56 ; 32, 26 ; 31, 16

<u>given</u>

$N_{OA} = 120$ rpm $\Rightarrow \omega_{OA} = \frac{2\pi \times 120}{60} = 4\pi$ rad/s

$V_A = (OA) \cdot \omega_{OA} = 0.2 \times 4\pi = 2.5132$ m/s

<u>Link 3</u> (A, B), (I_{13})

$\omega_3 = \omega_{AB} = \frac{V_A}{AI_{13}} = \frac{V_B}{BI_{13}} \Rightarrow V_B = 3.2$ m/s (3.2)

$AI_{13} \Downarrow$ 8.4 cm, $BI_{13} \Downarrow$ 11 cm

$\omega_3 = 3.0648$ rad/s

<u>Link 4</u> (B, C), (I_{14})

$\omega_4 = \omega_{BC} = \frac{V_B}{BI_{14}} = \frac{V_c}{CI_{14}} \Rightarrow V_c = 1.6$ m/s

$BI_{14} \Downarrow$ 4 cm, $CI_{14} \Downarrow$ 2 cm

$\omega_4 = 8$ rad/s (8)

<u>Link 5</u> (C, D) (I_{15})

$\omega_5 = \omega_{CD} = \frac{V_c}{CI_{15}} = \frac{V_D}{DI_{15}} \Rightarrow V_D = 1.0738$ m/s (1.08)

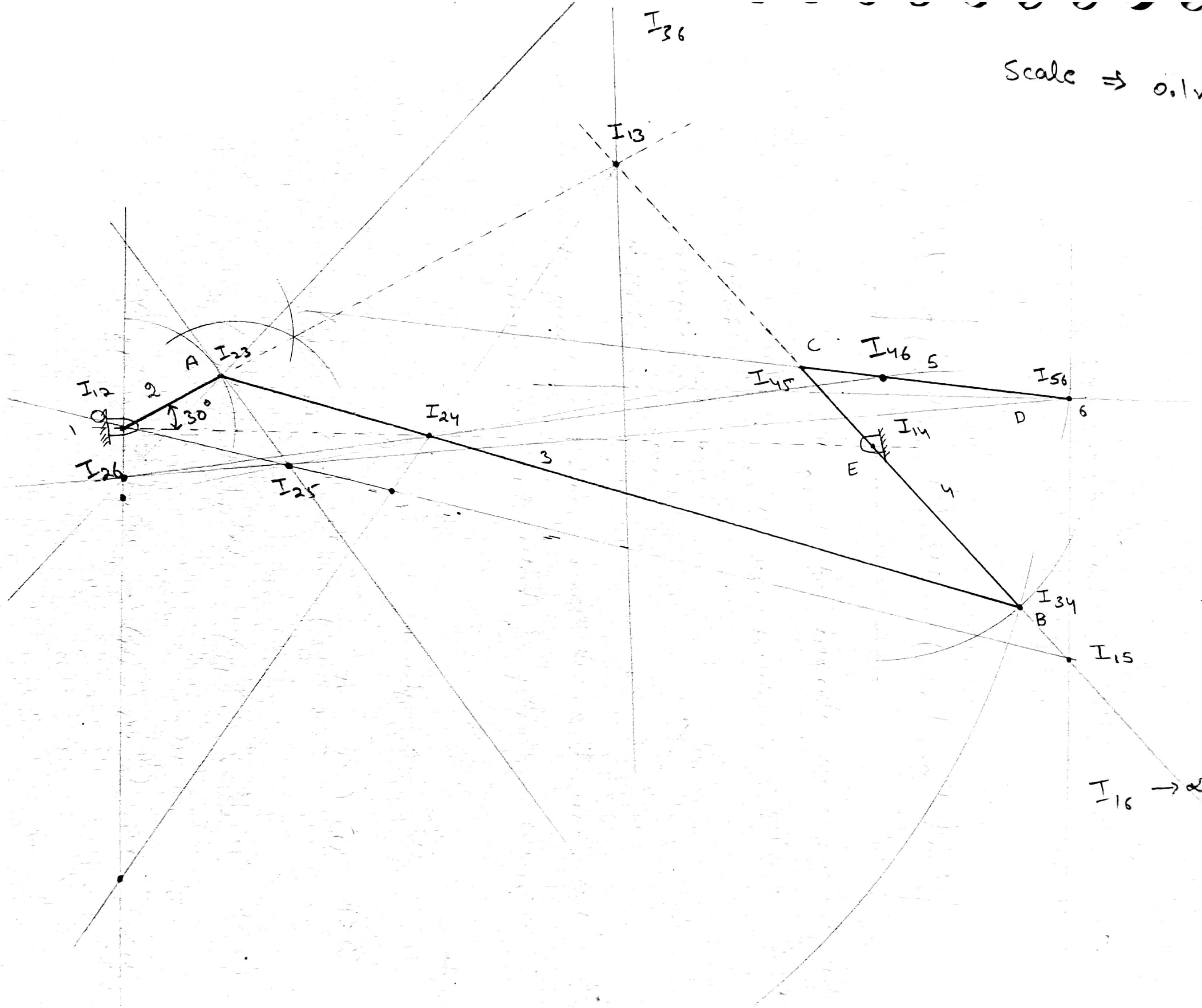
$CI_{15} \Downarrow$ 7.3 cm, $DI_{15} \Downarrow$ 4.9 cm

$\omega_5 = 2.19$ rad/s (2.16)

Kennedy's Theorem

For the relative motion b/w the no. of links in a mechanism, any three links, their I-center must lie on a straight line.

Scale ⇒ 0.1m = 1cm



Theorem of angular Velocities

Any IC

Can be treated on link m

Can be treated on link n

$$V_{I_{mn}} = \boxed{\omega_m(I_{mn}I_{1m}) = \omega_n(I_{mn}I_{1n})} = V_{I_{mn}}$$

This theorem is applied at I_{mn}

But total IC involved are I_{mn}, I_{1m}, I_{1n} } link 1, link m, link n

⇒ If I_{1m}, I_{1n} lies at same side of I_{mn} then direction of angular velocity of both link is same.

example

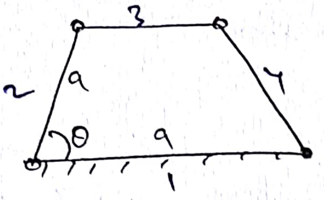
$\omega_2 \rightarrow$ given

(25) $\overset{C}{\omega_2}(I_{25}I_{12}) = \overset{AC}{\omega_5}(I_{25}I_{15})$

(24) $\overset{C}{\omega_2}(I_{24}I_{12}) = \overset{AC}{\omega_4}(I_{24}I_{14})$

(45) $\overset{AC}{\omega_4}(I_{45}I_{14}) = \overset{AC}{\omega_5}(I_{45}I_{15})$

Ques: 1

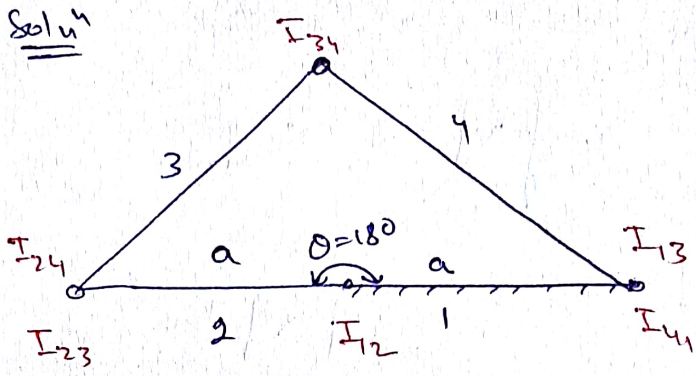


when $\theta = 180°$

$\omega_2 = 5$ rad/s (clock)

$\omega_3 = ?$

Sol^n



$\omega_2(I_{23}I_{12}) = \omega_3(I_{23}I_{13})$

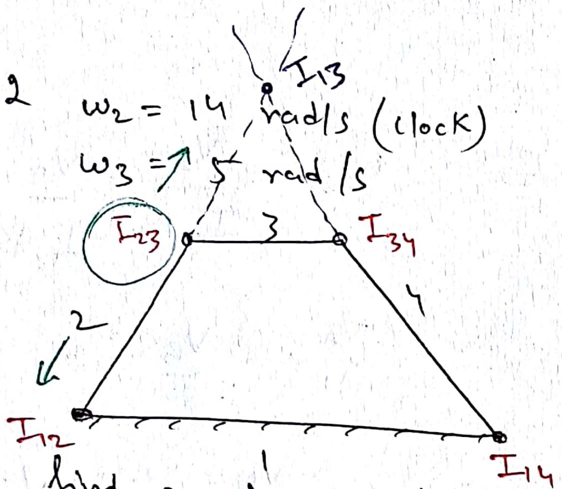
$5 \times a = \omega_3 \times 2a$

$\omega_3 = 2.5$ rad/s (clock)

Ques 2

$\omega_2 = 14$ rad/s (clock)

$\omega_3 = 5$ rad/s



find angular velocity of 2 w.r.t 3

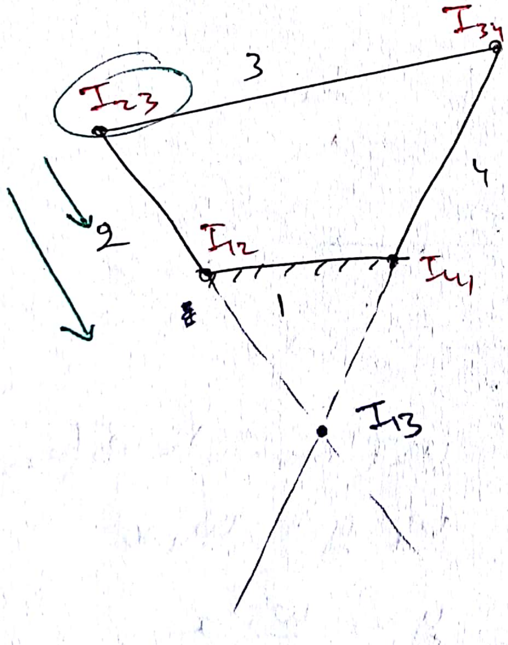
Sol^n

$\vec{\omega}_{23} = (\vec{\omega}_2 - \vec{\omega}_3)$

$= (14) - (-5)$

$= 19$ rad/s (clock)

Ques 3 find angular velocity of 2 w.r.t. 3



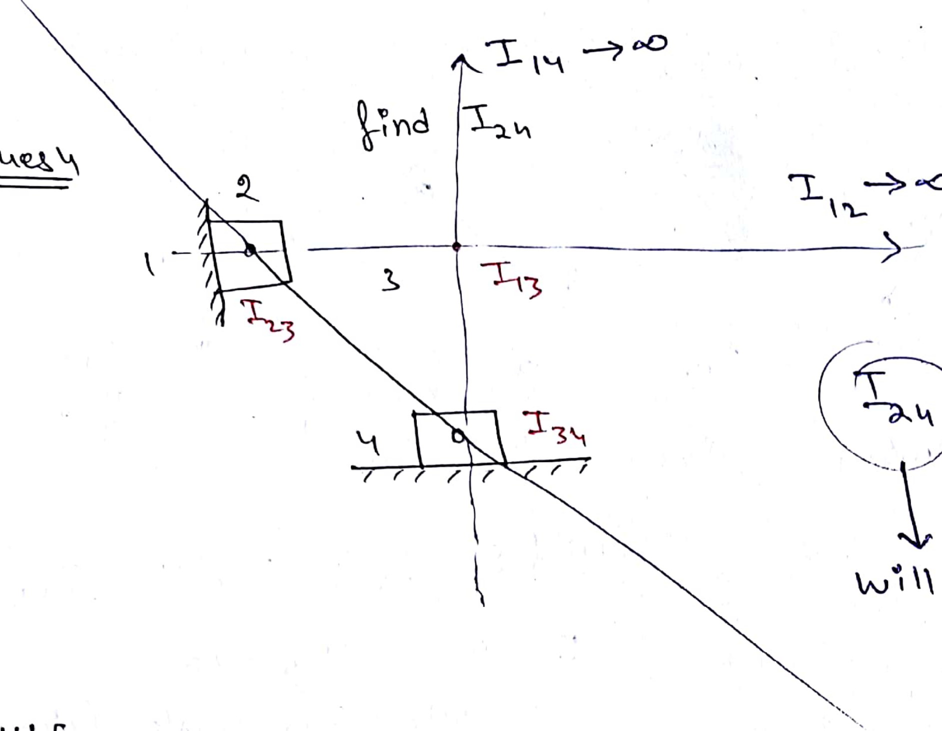
data to be same as Ques 2

Sol^n $\vec{\omega}_{23} = \vec{\omega}_2 - \vec{\omega}_3$

$= (14) - (5)$

$= 9$ rad/s

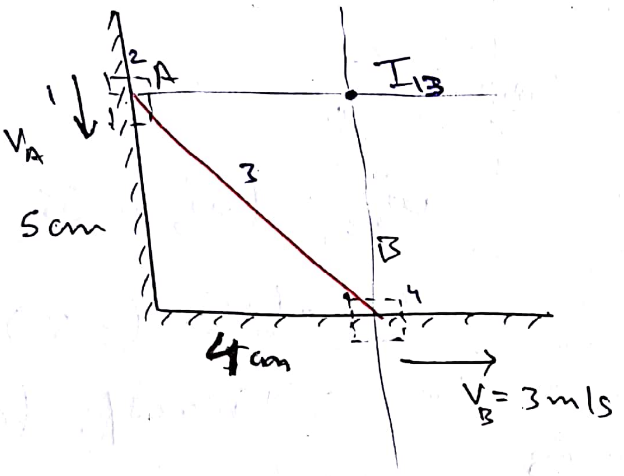
Ques 4



I_{24} < 23, 34 / 21, 14 (∞, ∞)

I_{24} will be at ∞

Ques 5



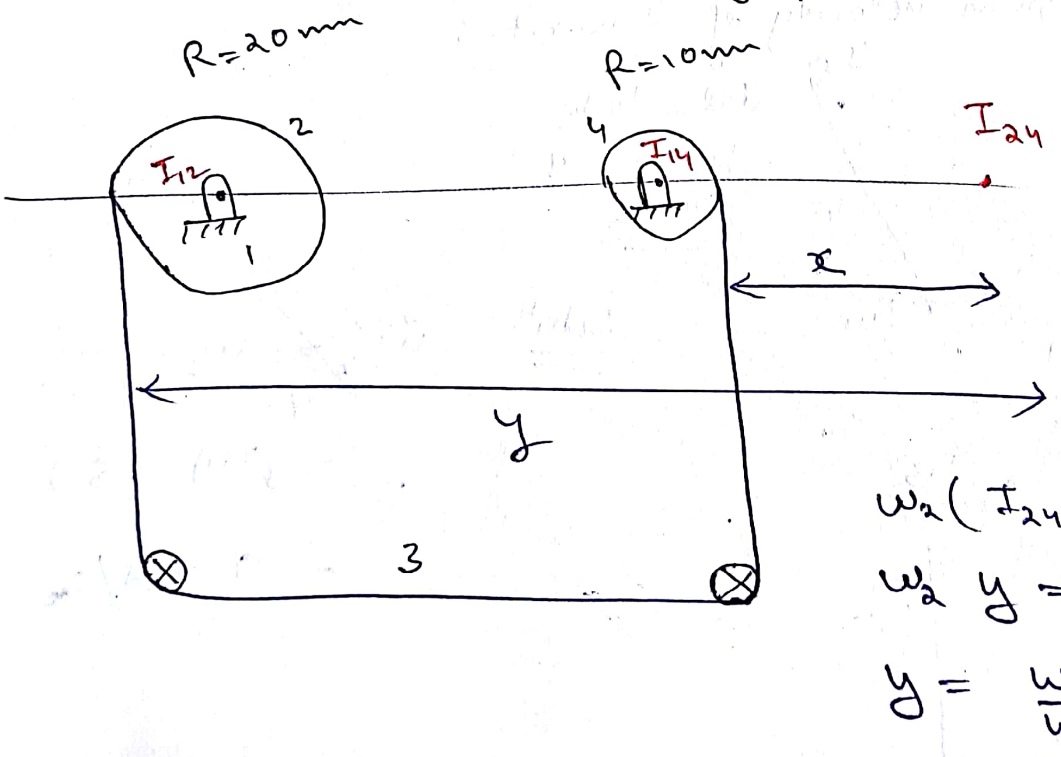
Soln

$$\omega_3 = \frac{V_A}{I_{13}A} = \frac{V_B}{I_{13}B}$$

$$\frac{V_A}{4} = \frac{V_B}{5}$$

$$\Rightarrow V_A = 2.4 \text{ m/s}$$

Ques 6 where is the I-center of spools.



$$\omega_2(I_{24}I_{12}) = \omega_4(I_{24}I_{14})$$

$$\omega_2\, y = \omega_4\, x$$

$$y = \frac{\omega_4}{\omega_2} x$$

$$y = \frac{R_2}{R_4} x = 2x$$

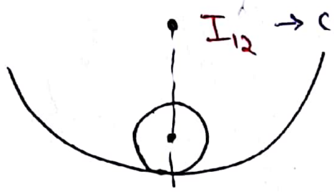
3/6/16

Note :

1.

Pure rolling

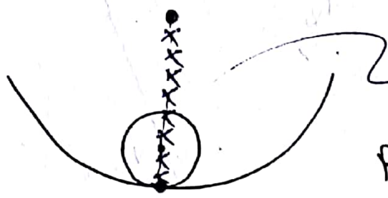
2.



→ center of curvature

Pure sliding

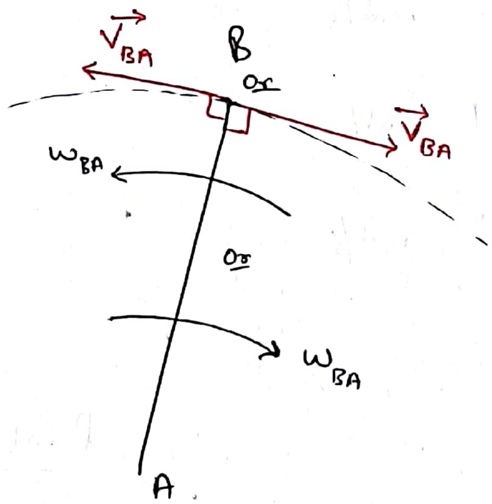
3.



→ In b/w two points

Rolling with sliding

B) Relative Velocity method :



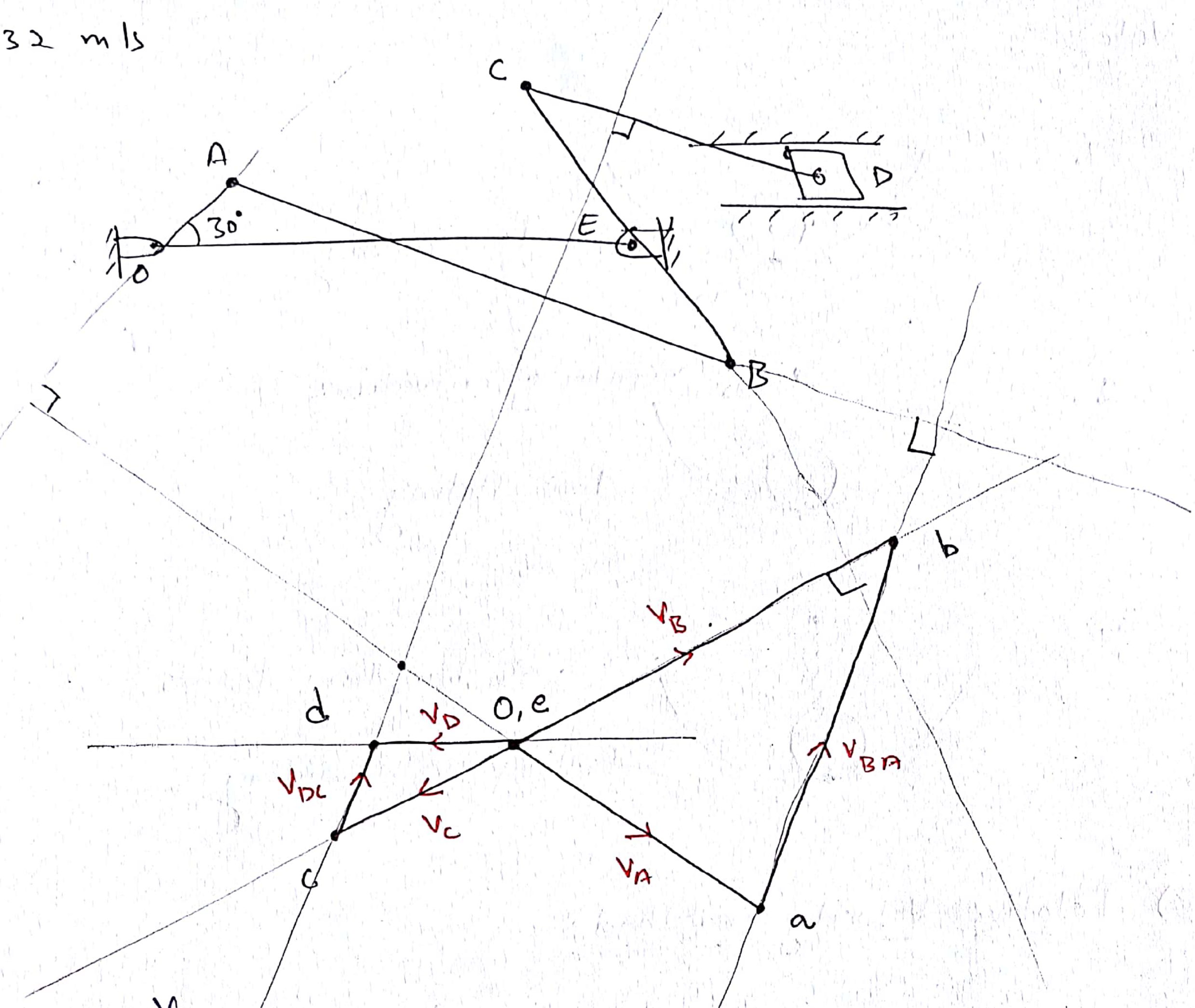
The velocity of point B w.r.t. point A will be in dir^n $\perp$ar to link AB

WB
Ch-2
Q27

Steps

Point	w.r.t.	Procedure
A	O	line ⊥ to link OA
B	A	line ⊥ to link AB
B	E	line ⊥ to link BE
C	$\frac{BC}{BE} = \frac{Bc}{BE}$	
D	C	line ⊥ar to link CD
D	fixed	line ∥ to motion of slider

$V_A = 2.5132$ m/s



$$\omega_{AB} = \frac{V_{AB}}{AB} = 2.97 \text{ rad/s}$$

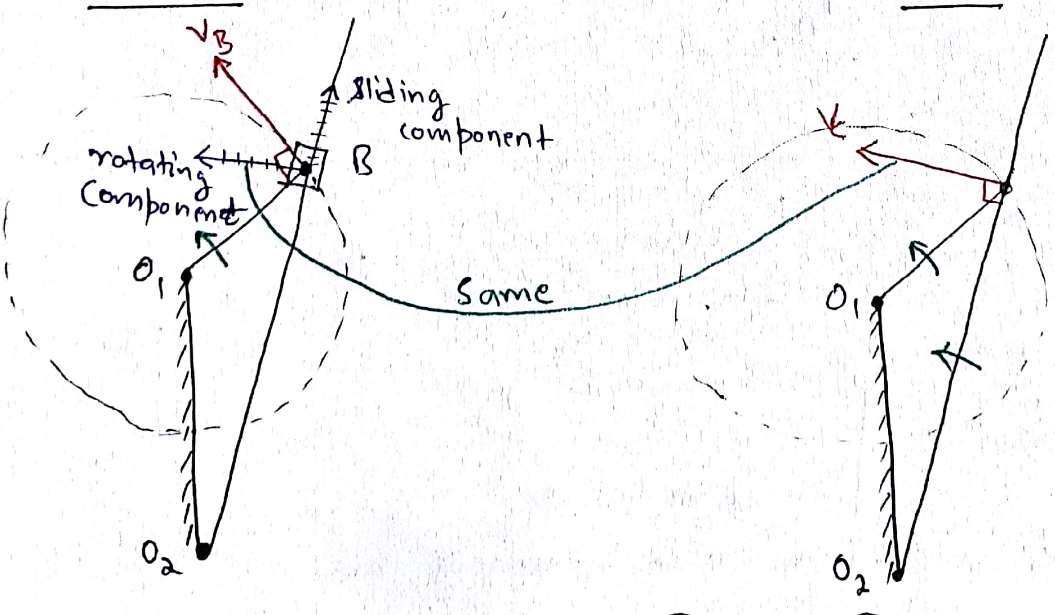
$$\omega_{BC} = \frac{V_{BC}}{BC} = 8.1 \text{ rad/s}$$

$$\omega_{CD} = \frac{V_{CD}}{CD} = 2.2 \text{ rad/s}$$

WB
Ch-2
Q26

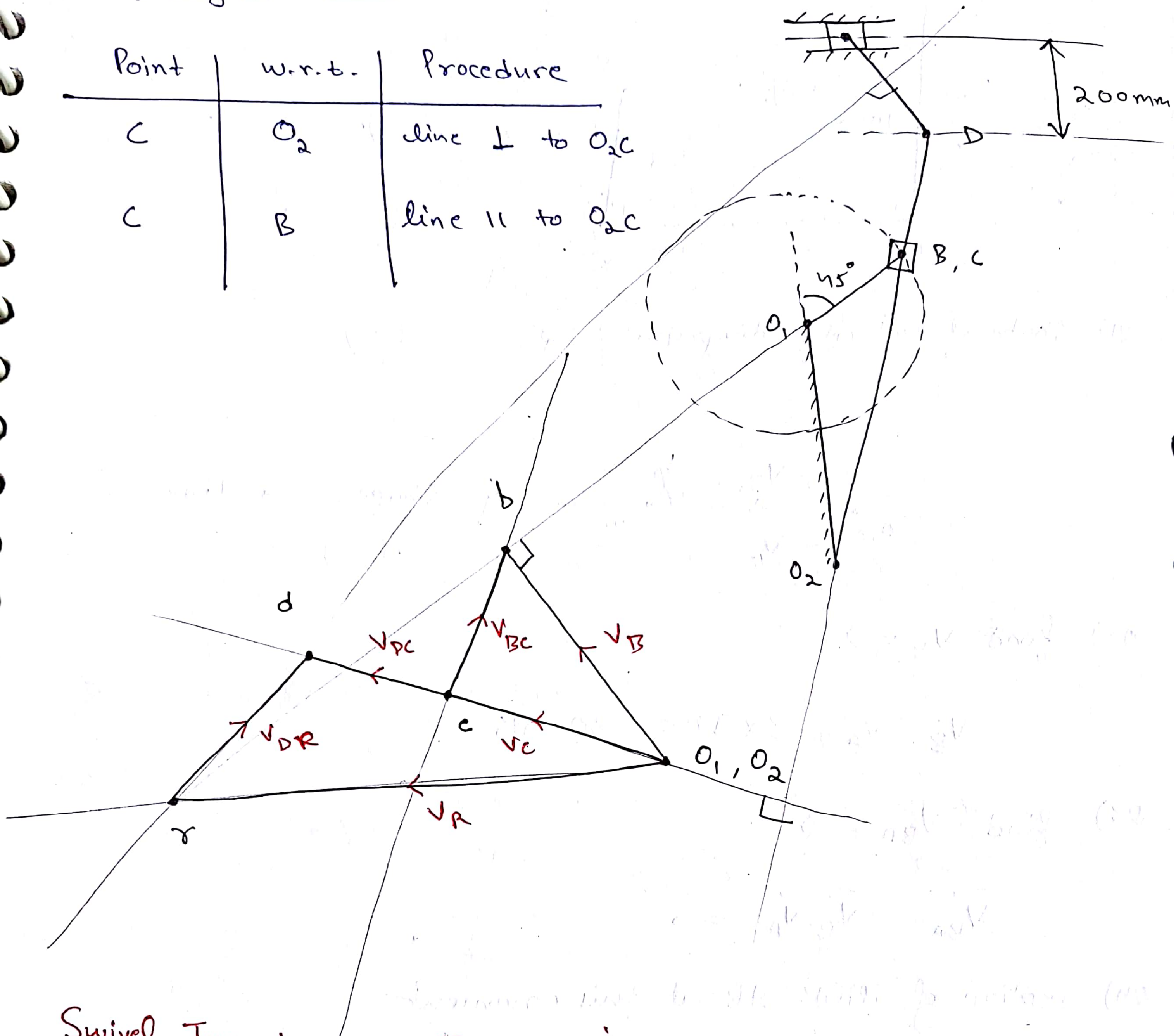
Point B

Point C

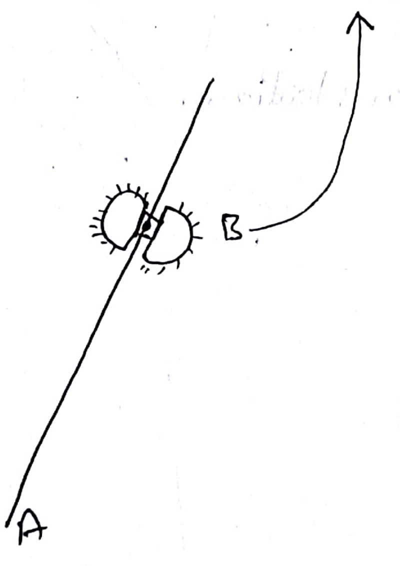


Steps for Poin 'c'

Point	w.r.t.	Procedure
C	O_2	line ⊥ to O_2C
C	B	line ∥ to O_2C

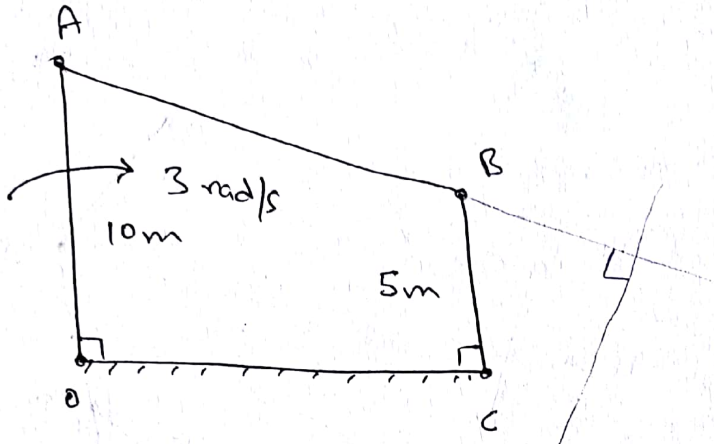


Swivel Trunnion or Trunnion joint

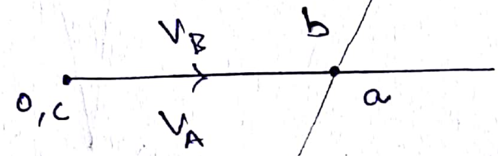


Point	w.r.t.	Procedure
B	A	line ⊥ to AB
B	fixed	line ∥ to AB

Ques



Q1) shape of velocity diagram = ?



Shape = St. line

Q2) find $V_B = ?$

$$V_B = V_A = 3 \times 10 = 30 \text{ m/s}$$

Q3) find $V_{BA} = ?$

$$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A = 0$$

Q4) motion of link AB at this moment

$$\omega_{BA} = \frac{V_{BA}}{BA} = \frac{0}{BA} = 0$$

motion is pure translation

Q5) find $\omega_{BC} = ?$

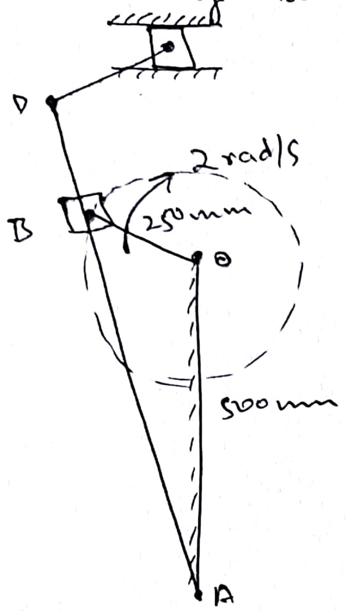
$$V_A = V_B$$

$$\omega_A(OA) = \omega_{BC}(BC)$$

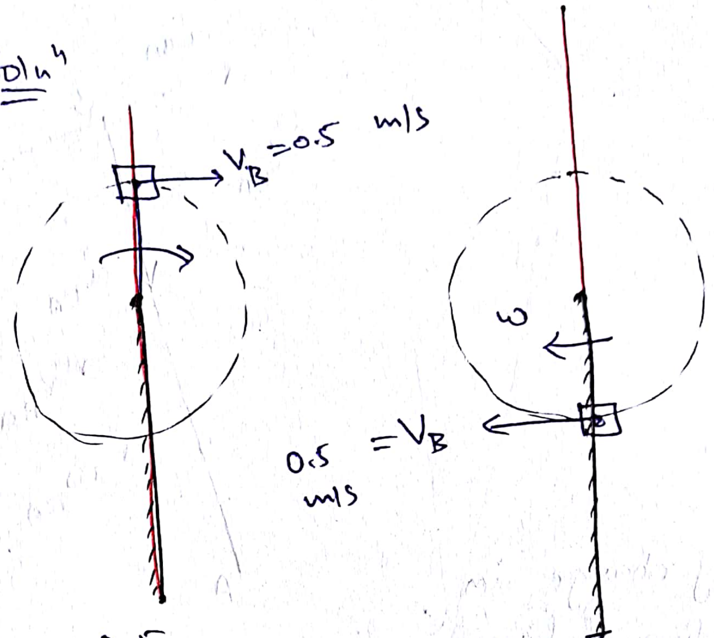
$$3 \times 10 = \omega \times 5$$

$$\omega_{BC} = 6 \text{ rad/s}$$

Ques: find angular velocity of slotted bar when velocity of Ram is maximum



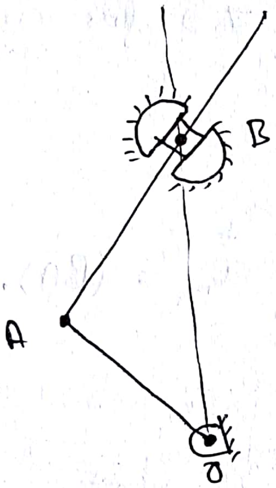
Soln



$$\omega_{SB} = \frac{0.5}{0.75} = \frac{2}{3} \text{ rad/s}$$

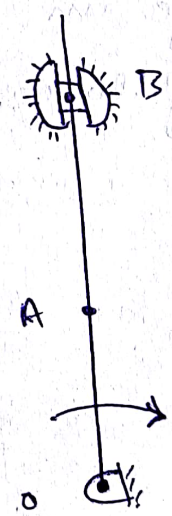
$$\omega_{SB} = \frac{0.5}{0.25} = 2 \text{ rad/s}$$

Ques:

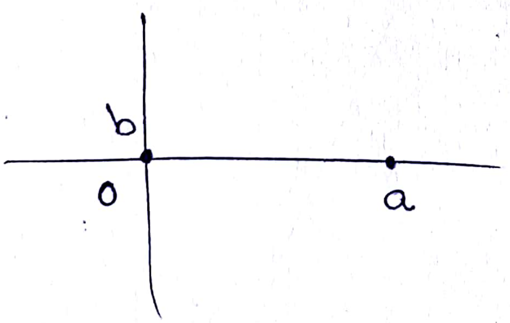


find angular velocity of Trunnion when OA & AB are vertical

Soln

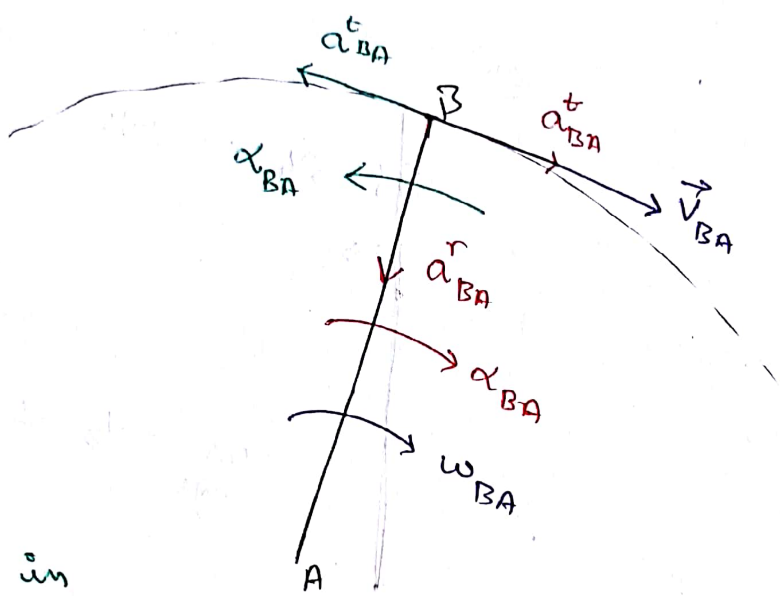


Velocity diagram



$$\omega_{Trunnion} = \frac{V_B}{\longrightarrow} = 0$$

Acceleration Analysis



Bcz of change in direction of linear velocity.

$$a^r_{BA} = \frac{V^2_{BA}}{BA}$$

$\Downarrow$

$\neq 0$ for circular motion

$$\vec{a}_{BA} = \frac{d\vec{V}_{BA}}{dt}$$

Bcz of change in magnitude of linear velocity.

$$a^t_{BA} = (BA).(\alpha_{BA})$$

$\Downarrow$

may or not — may be zero for circular motion

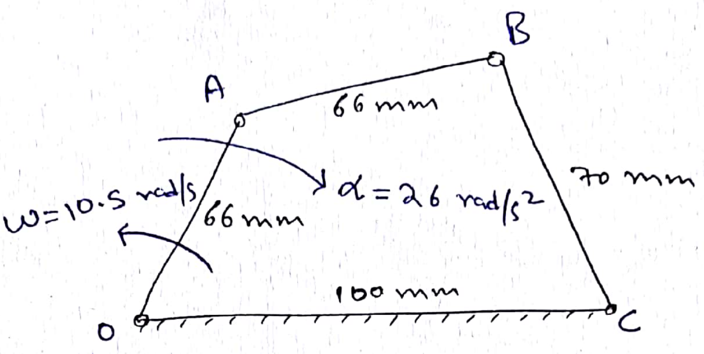
Ques find

$a_B = ?$

$a_{BA} = ?$

$\alpha_{BA} = ?$

$\alpha_{BC} = ?$



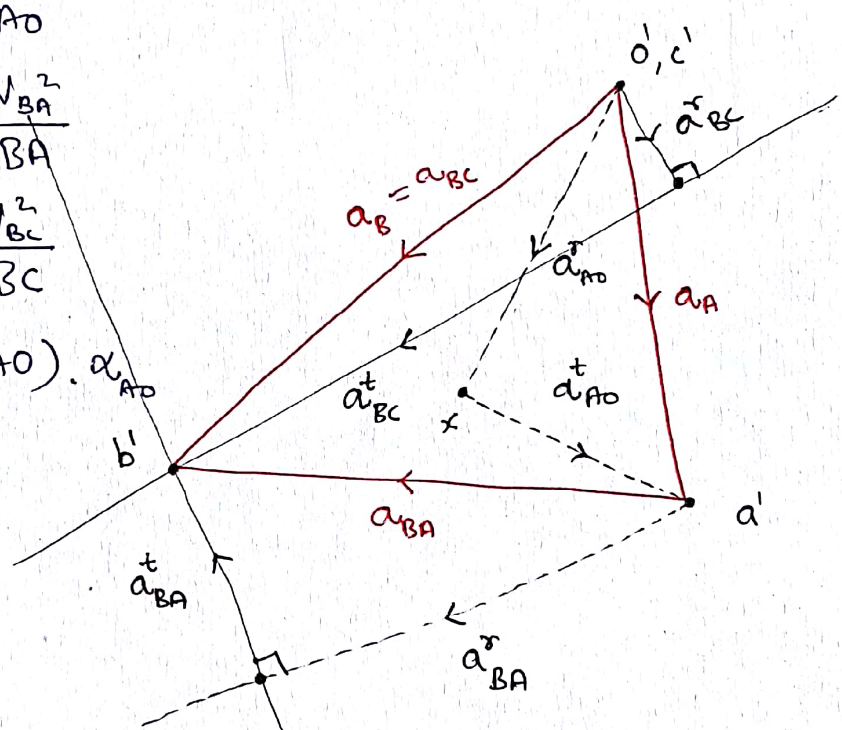
Sol^n After acceleration diagram

$$a^r_{AO} = \frac{V_{AO}^2}{AO}$$

$$a^r_{BA} = \frac{V_{BA}^2}{BA}$$

$$a^r_{BC} = \frac{V_{BC}^2}{BC}$$

$$a^t_{AO} = (AO).\alpha_{AO}$$



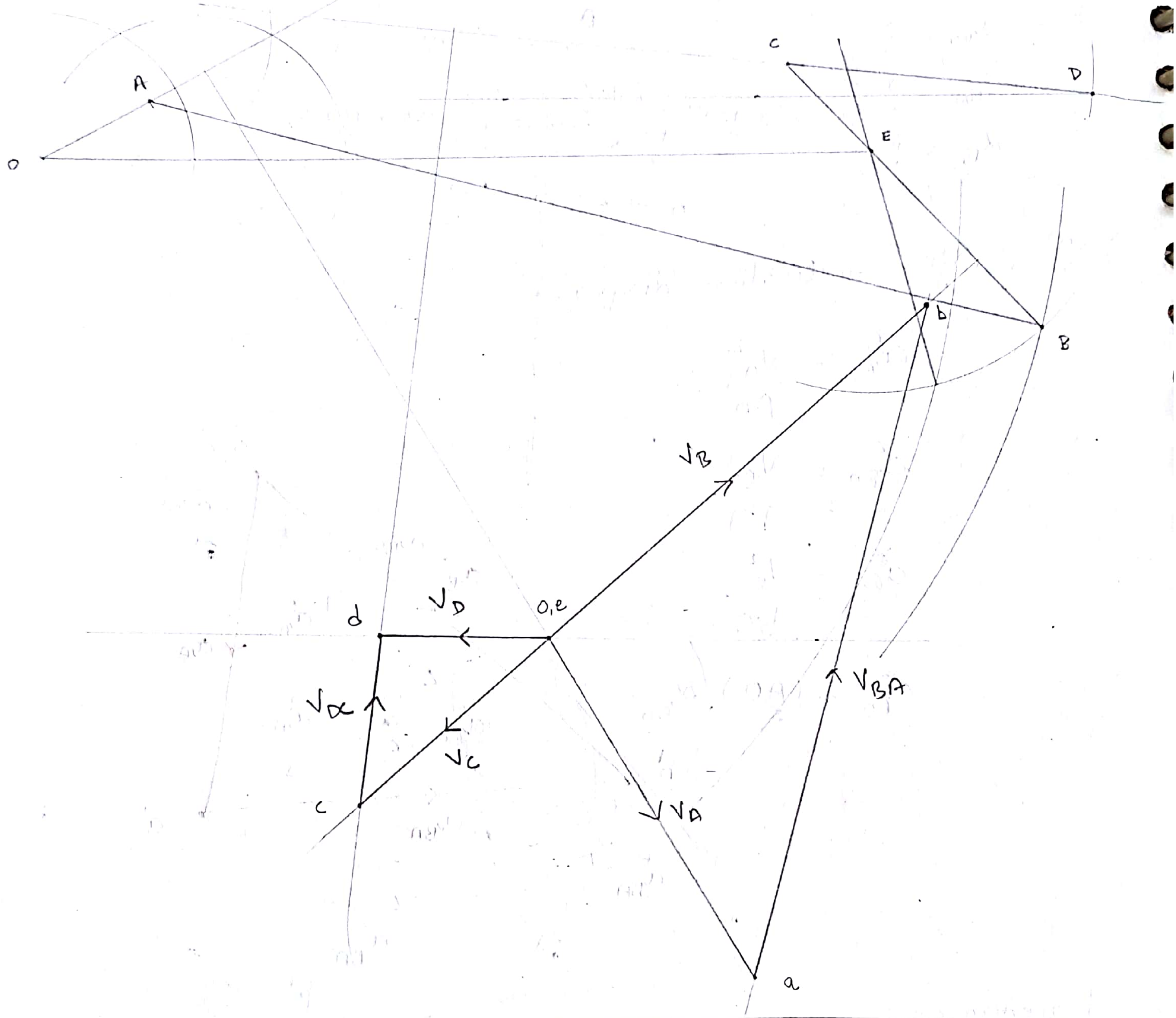
Procedure

Point	w.r.t.	Procedure	
A	O	$a^r_{AO} = \frac{V_{AO}^2}{AO}$ (Known)	$A \longrightarrow O$
		$a^t_{AO} = (AO)(\alpha_{AO})$ (Known)	Lar to AO
B	A	$a^r_{BA} = \frac{V_{BA}^2}{BA}$ (Known)	$B \longrightarrow A$
		$a^t_{BA} = (BA).(\alpha_{BA})$ (unknown)	
B	C	$a^r_{BC} = \frac{V_{BC}^2}{BC}$ (Known)	$B \longrightarrow C$
		$a^t_{BC} = (BC).(\alpha_{BC})$ (unknown)	

Q27
WB
Ch-2

Scale ⇒ 0.4 m/s ≡ 1cm [for velocity diagram]

Scale ⇒ 0.1 m ≡ 1cm [for mechanism diagram]



Rough

$V_B = \frac{8.1}{2.5} = 3.2 \text{ m/s}$

$V_D = \frac{2.75}{2.5} = 1.1 \text{ m/s}$

$V_C = \frac{4.05}{2.5} = 1.6 \text{ m/s}$

$V_{BA} = \frac{11.15}{2.5} = 4.46 \text{ m/s}$

$\omega_{BA} = \frac{V_{BA}}{BA} = \frac{4.46}{1.5} = 2.97 \text{ rad/s}$

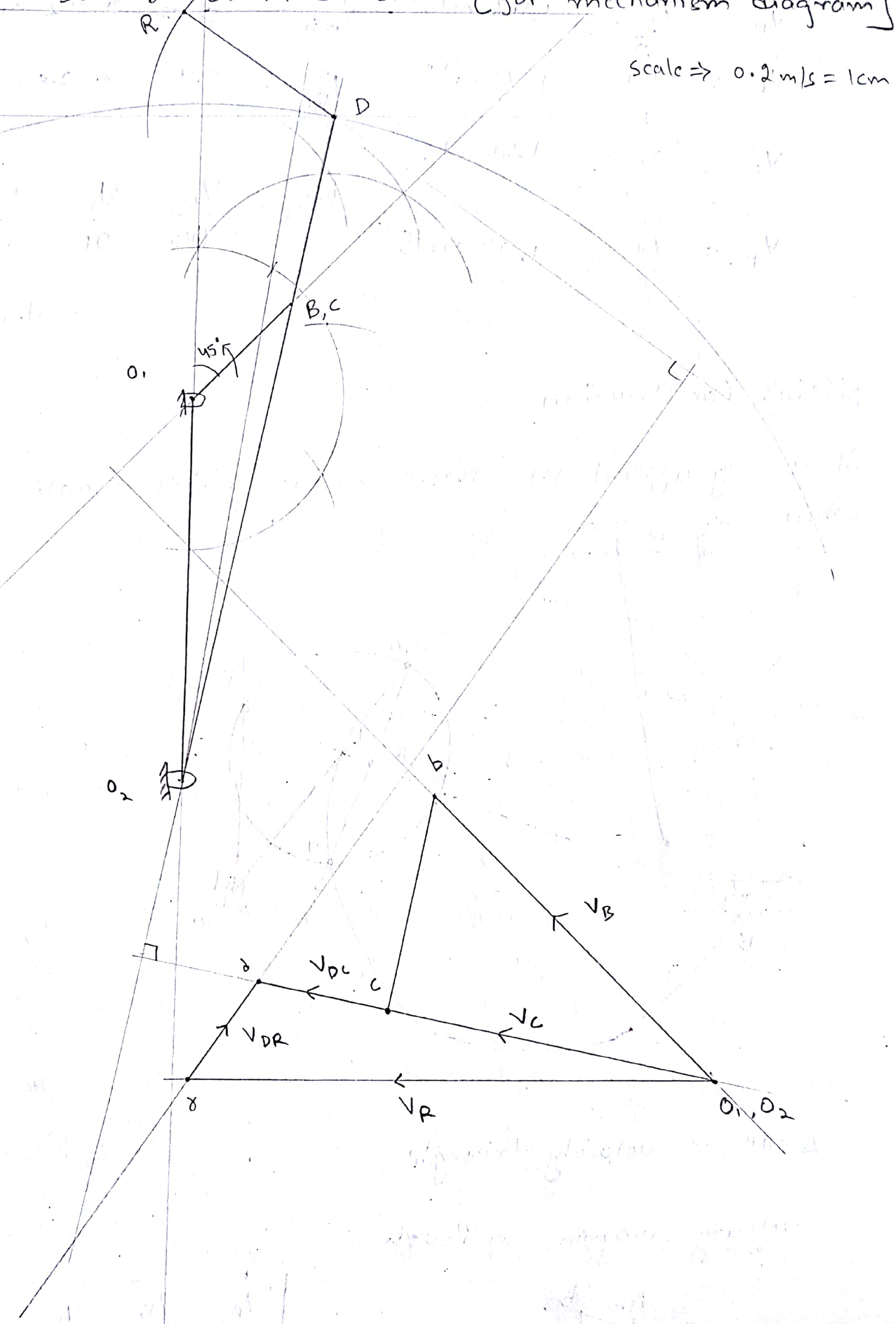
$\omega_{BC} = \frac{V_{BC}}{BC} = \frac{12.15/2.5}{0.6} = 8.1 \text{ rad/s}$

$\omega_{CD} = \frac{V_{CD}}{CD} = \frac{2.75/2.5}{.5} = 2.2 \text{ rad/s}$

WB
Ch-2
Q26

Scale :- 0.1 m ≡ 1cm [for mechanism diagram]

Scale ⇒ 0.2 m/s = 1cm



4/6/16

$V_B = \frac{7.33}{5} = 1.46$ m/s

$V_c = \frac{8.1}{5} = 1.62$ m/s

$V_D = \frac{8.56}{5} = 1.71$ m/s

$V_R = \frac{9.6}{5} = 1.92$ m/s

$V_{RD} = \frac{2.1}{5} = 0.42$ m/s

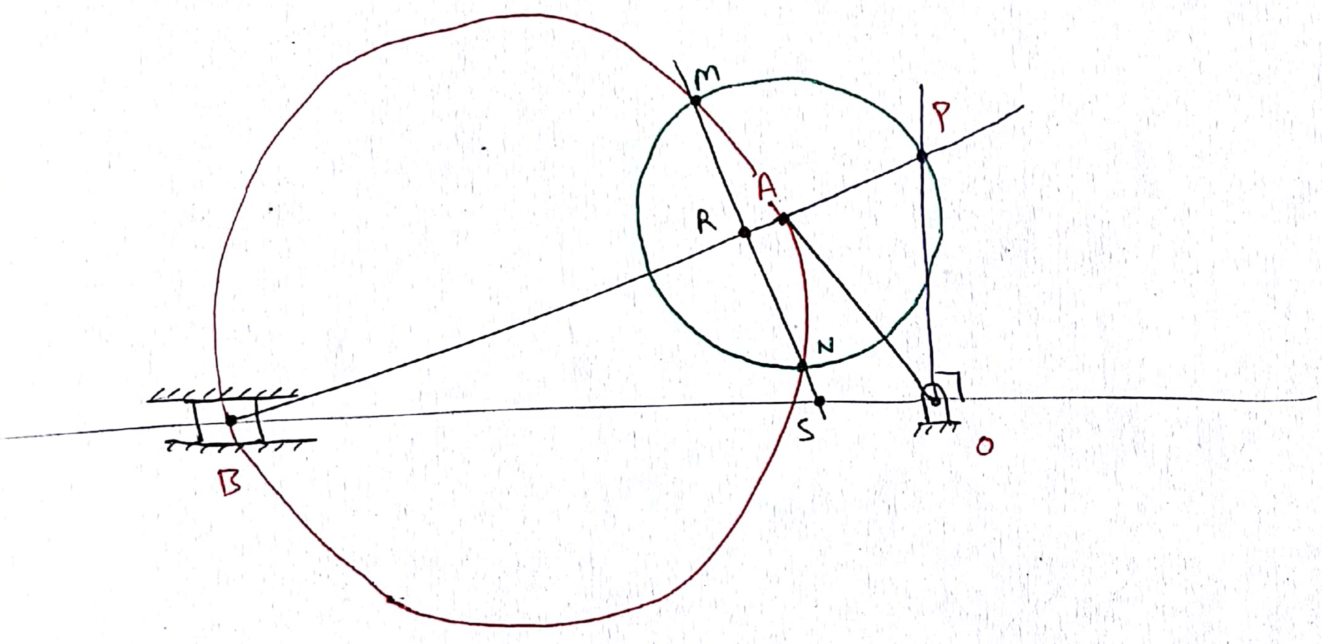
$V_{BC} = \frac{3.9}{5} = 0.78$ m/s

$\omega_{OD} = \frac{V_{OD}}{OD} = \frac{V_D}{OD} = \frac{1.71}{1.25}$

$= 1.36$ rad/s

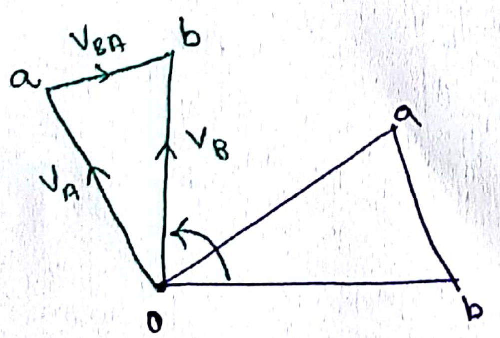
Klein's Construction

It is only applied in Basic Single slider crank mechanism when $\alpha_{crank} = 0$



Δ OAP $\Rightarrow$ velocity triangle

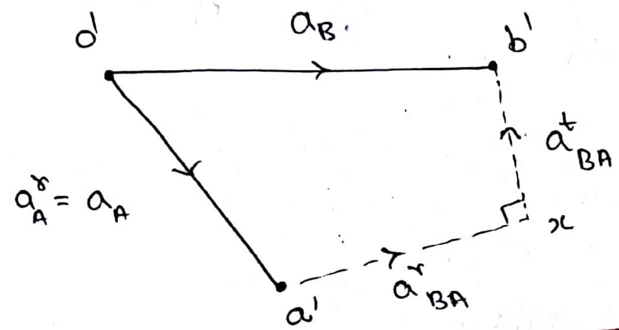
velocity diagram in Rough



$$\boxed{\frac{V_A}{OA} = \frac{V_B}{OP} = \frac{V_{BA}}{AP} = \omega_{crank}}$$

Quadrilateral OARS ⇒ Acceleration quadrilateral

Acceleration diagram in rough



$$\boxed{\frac{a_A}{OA} = \frac{a_B}{OS} = \frac{a^r_{BA}}{RA} = \frac{a^t_{BA}}{RS} = \omega^2_{crank}}$$

Note:

1) If O & P concides
↳ $OP = 0$ ⇒ $V_B = 0$ ⇒ piston is at its dead center.

2) If O and S concides
↳ $OS = 0$
⇒ $a_B = 0$ ⇒ piston is at mid position

3) If A and R concides
$AR = 0$
⇒ $a^r_{BA} = 0$ ⇒ BA is in pure translation

4) If R and S concides
$RS = 0$
⇒ $a^t = 0 = (BA) \times \cancel{\alpha_{BA}}^{0}$ ⇒ Connecting rod is at constant angular velocity.

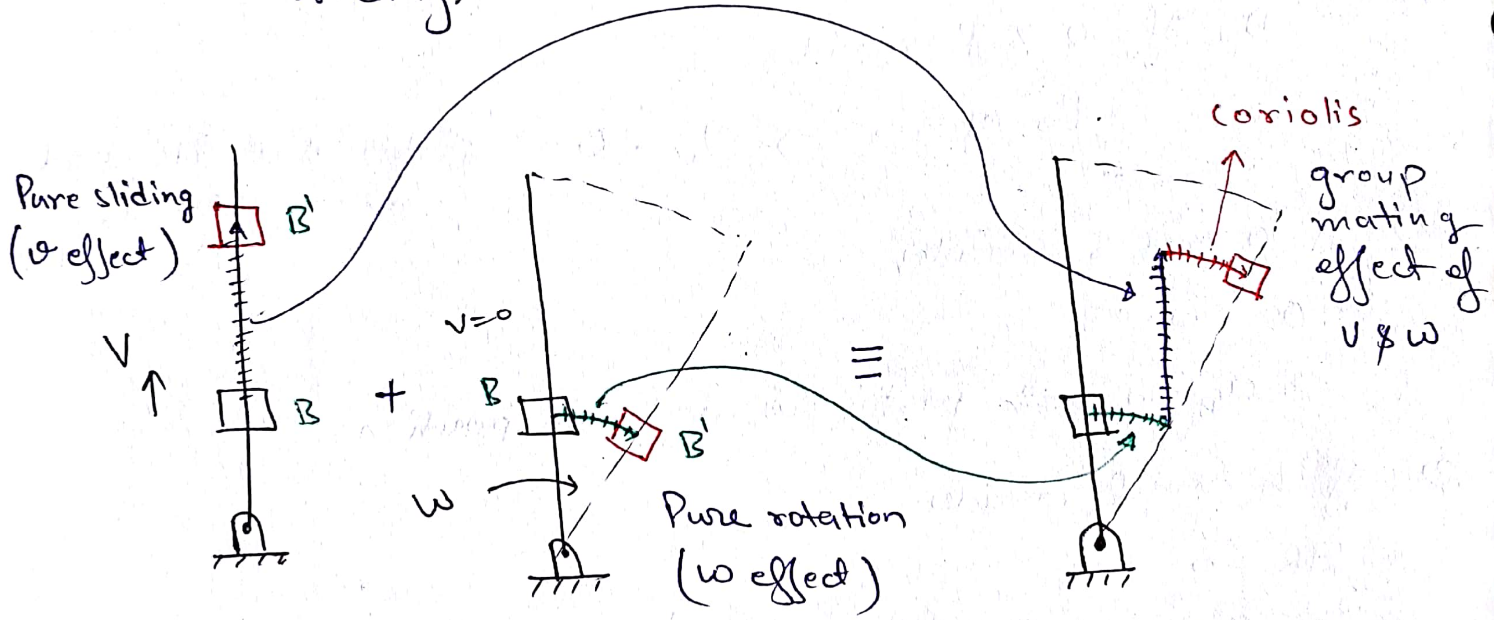
Corialis Acceleration (a^c)

This acceleration will always be associated with slider when the slider is sliding on rotating body. The magnitude of this acceleration is

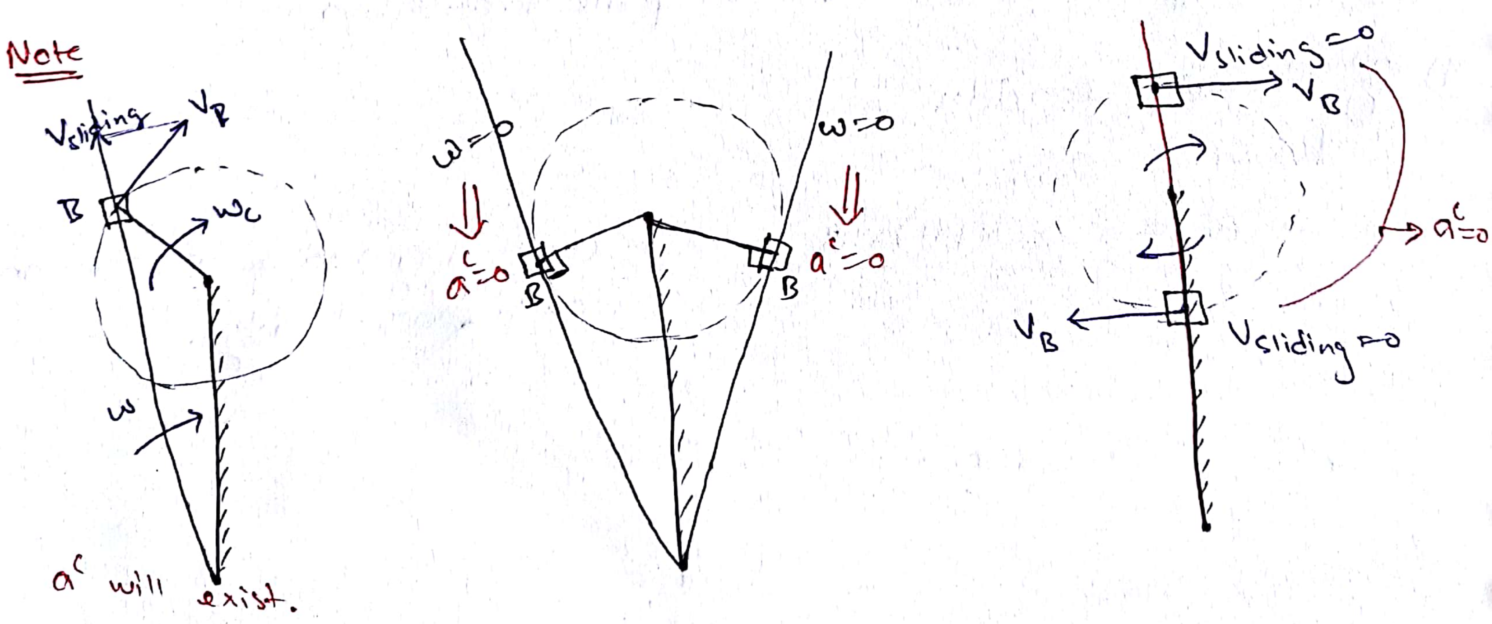
$$\boxed{a^c = 2v\omega}$$

v = sliding velocity of slider

ω = angular velocity of body on which slider is sliding.



Note

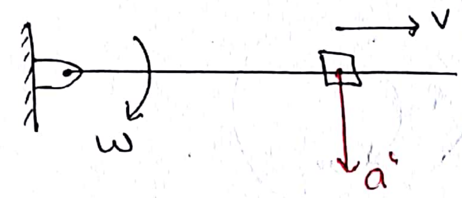


Direction of a^c

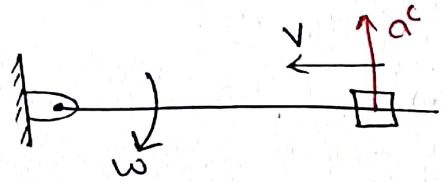
1. Take the sense of $\vec{\omega}$
2. Rotate the $\vec{v}$ in the same sense by 90°

eg:-

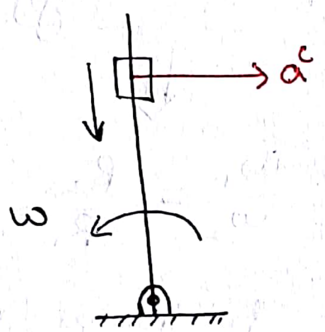
1)



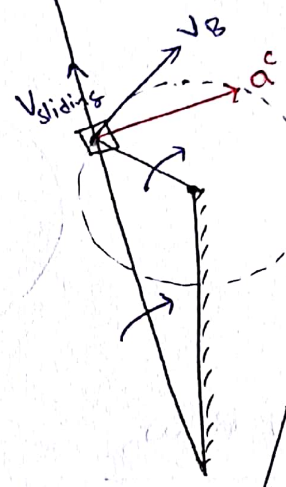
2)



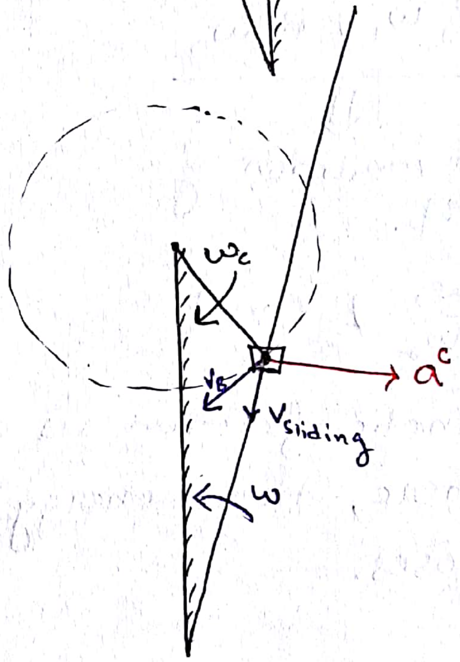
3)



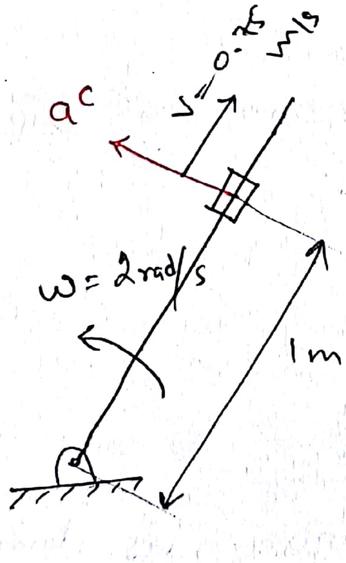
4.



5.



Ques:



find the Acceleration of slider.

Solu^n

$a^r = r\omega^2 = 1 \times (2)^2 = 4 \text{ m/s}^2$

$a^t = r\alpha = 0$

$a^c = 2v\omega = 2 \times 0.75 \times 2$

$= 3 \text{ m/s}^2$

$a = \sqrt{(3)^2 + (4)^2} = 5 \text{ m/s}^2$

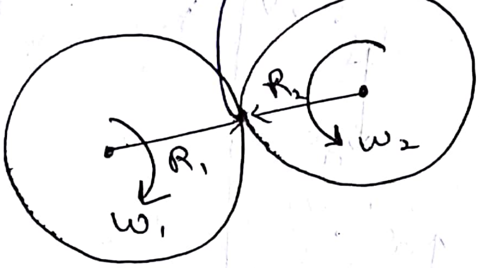
CHAPTER - 3

GEARS

⇓

used in Power transmission

θ = point of contact



$R_1\omega_1 \neq R_2\omega_2$

⇓

In condition of slip.

$\frac{\omega_1}{\omega_2} \neq$ const

- Kinetic friction is there, hence energy loses.

$R_1\omega_1 = R_2\omega_2$

⇓

condition for no sliping [Pure rolling]

⇓

- $\frac{\omega_1}{\omega_2} = \frac{R_2}{R_1} =$ const.

friction is static

$0 \leq f_s \leq \mu N$

★ All those drives in which phenomenon of slip is possible are called negative drives

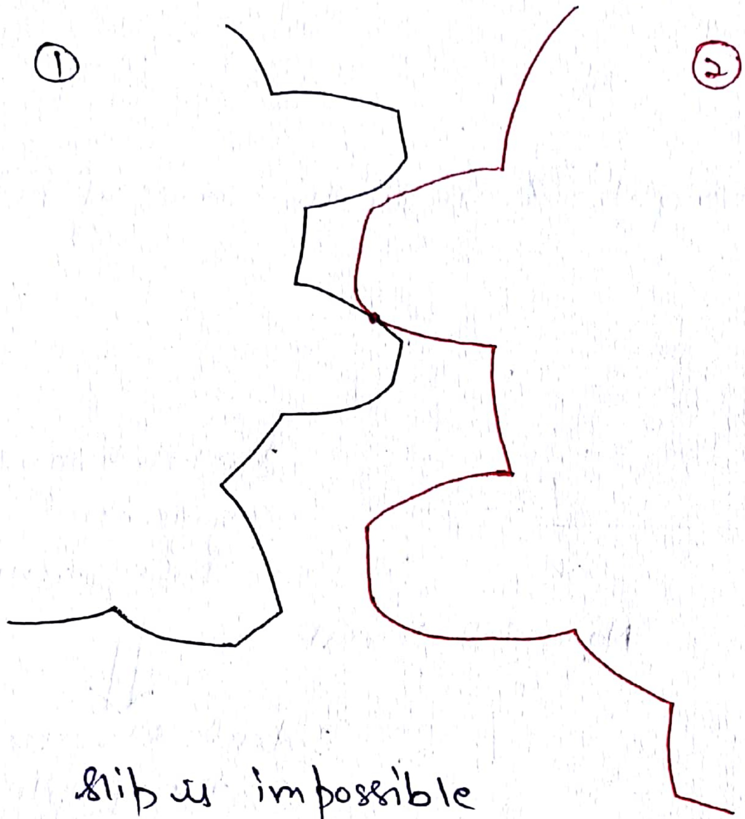
eg:- Belt drive
- Rope drive
- Chain drive

} 70% usage

★ In some system a very high accuracy is demanded for the velocity ratio to be constant i.e. $\frac{\omega_1}{\omega_2}$ = const.

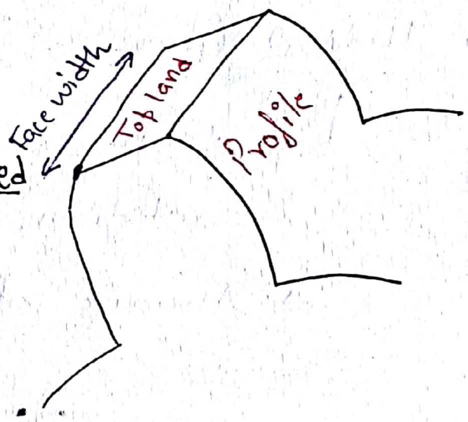
- when slip is impossible ⇒ Positive drive ⇒ Gear drive

monday => 6/6/16
Thermodynamics Test



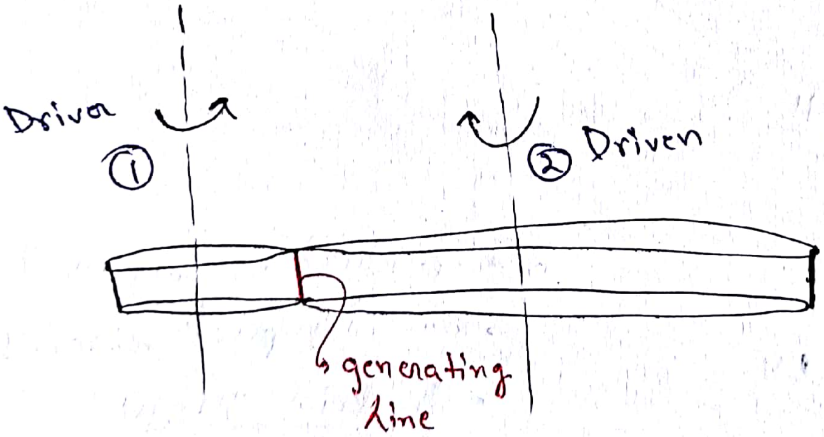
slip is impossible
⇓
Positive drive
⇓
gear drive → very costly

General Gear classification



A) According to Axes of the shaft connected

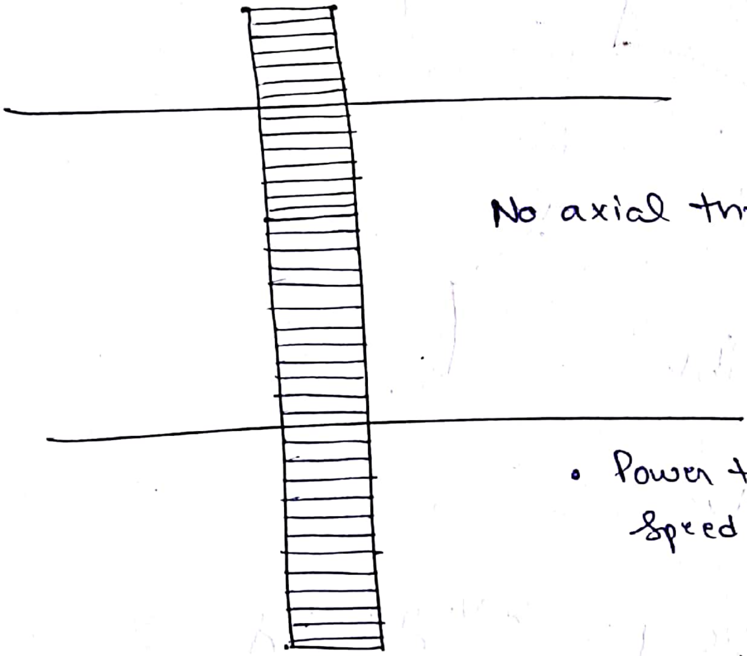
1. Both axes are parallel



Pure rolling motion can be transmitted b/w two cylindrical surface in contact.

- Spur Gear
  ↳ teeth are straight & ∥ to the axis of Rotation

99% fail
1% use



No axial thrust

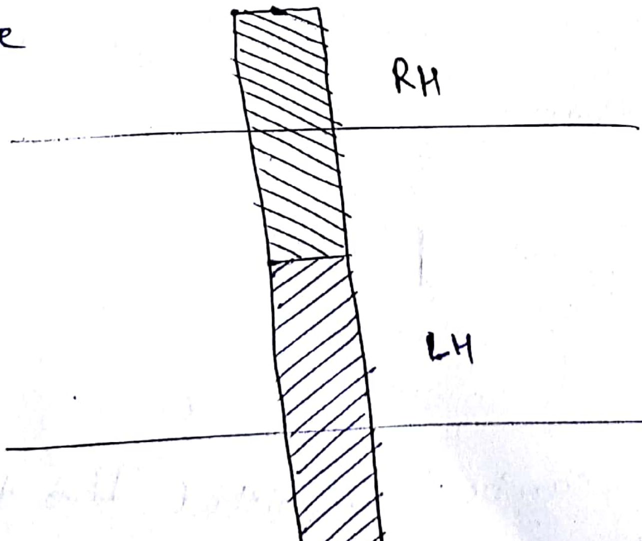
Instantenous engagement & Disengagement
⇓
Impact stresses on the profile

- Power transmission at low speed & low load.

- Helical Gears < Left hand / Right hand
  ↳ teeth are straight & inclined to axis of rotation.

→ Always opposite hand gears are engaged.

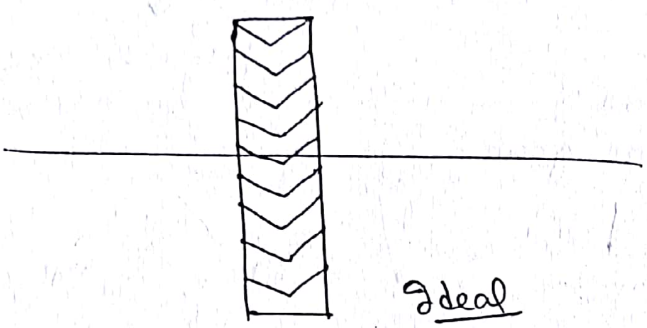
98% use



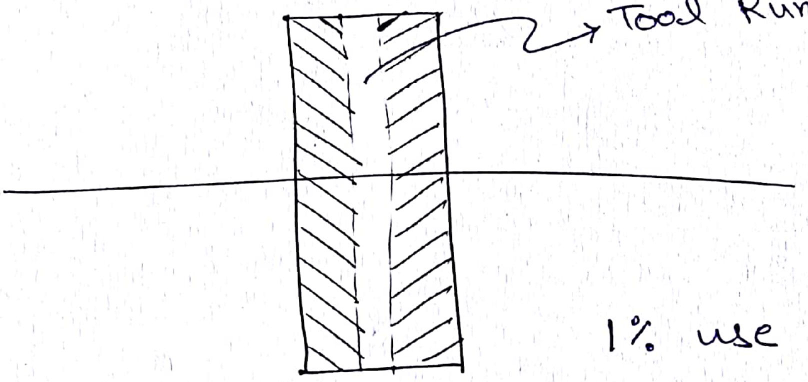
- Gradual engaggement
- no impact stresses
- Axial thrust

12/6/16

- Double helical Gears [Herringbone Gear]
  ↳ To minimise axial thrust.



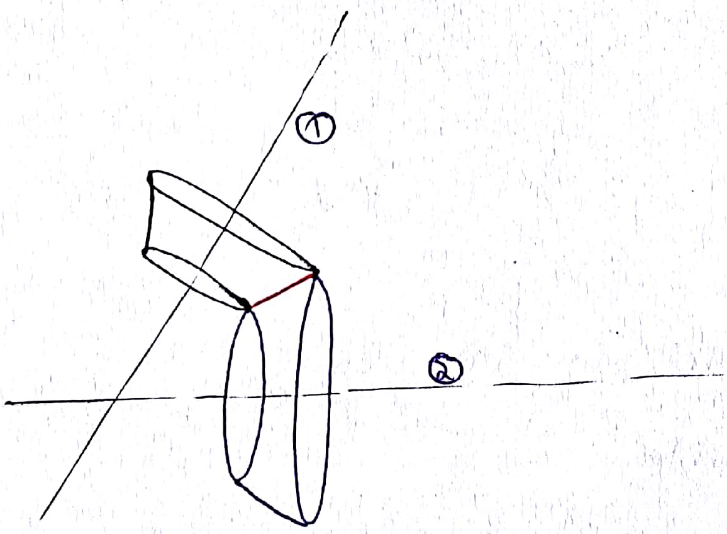
Ideal



Tool Run out

1% use [High cost]

2) When both axis are intersecting



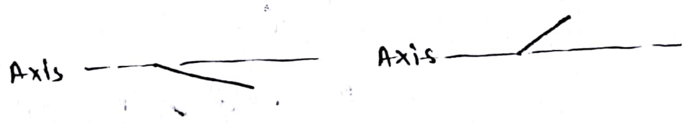
Pure rolling motion can be transmitted b/w two conical surface in contact.

Bevel Gear

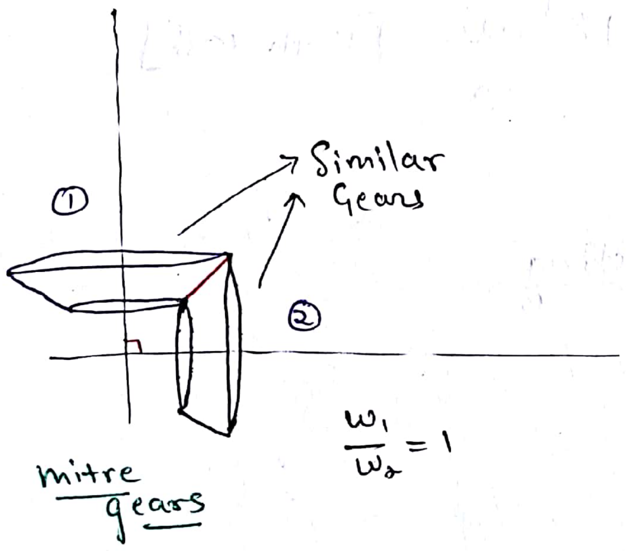
- Straight Bevel Gear
- Zerol Bevel gear
- Helical Bevel gear
  - RH
  - LH

Almost same

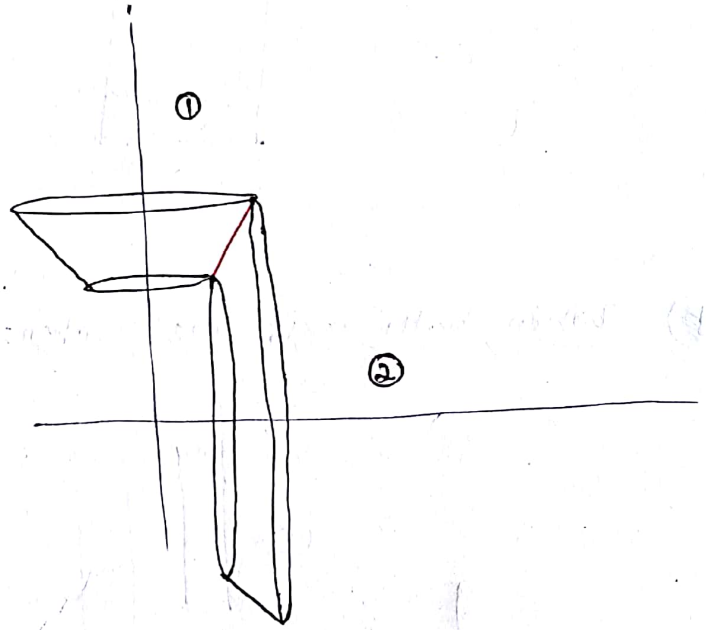
- Impact stresses
- 1 % use



Mitre Gears



$$\frac{\omega_1}{\omega_2} = 1$$

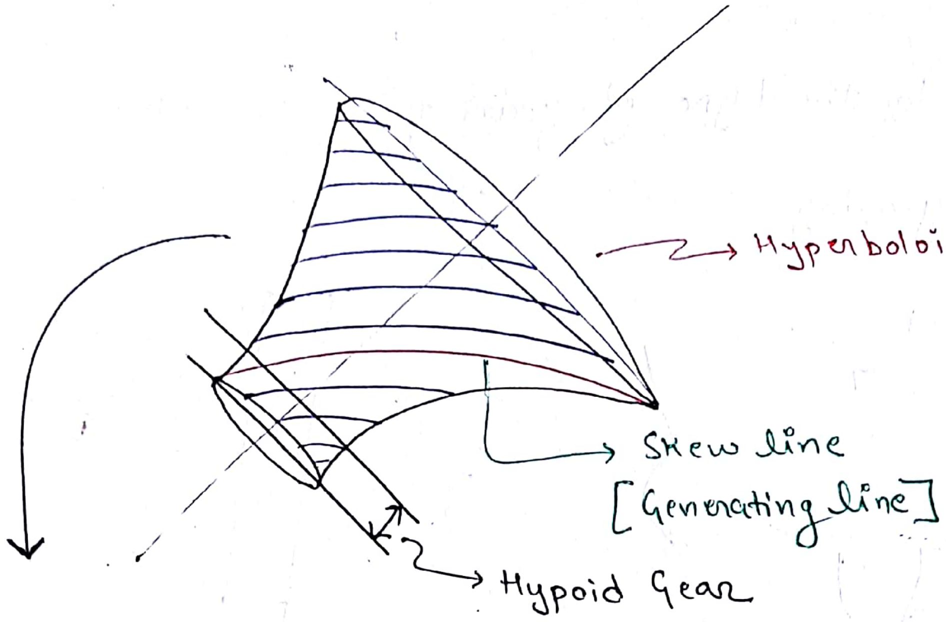


3) Axis are neither parallel nor intersecting

- Pure rolling is impossible
- Rolling is possible

⇓

Rotation + Partial sliding

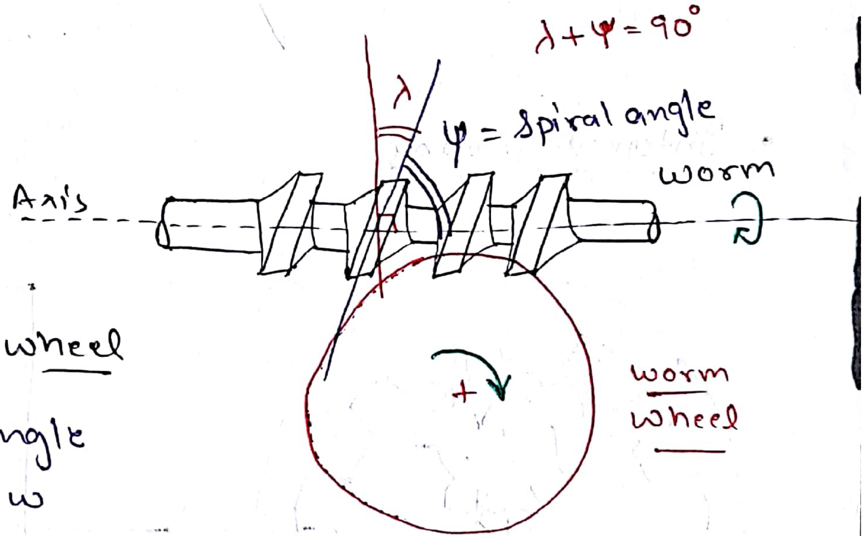


Spiral Gear or Skew Bevel gear

Sometimes when the space b/w the shafts is very less, then some portion of Hyperboloid is used to form spiral gears known as Hypoid Gears.

Worm & worm wheel

Spiral Gears



Worm	Worm wheel
• Spiral angle very high (ψ)	• Spiral angle very low
• very less dia.	• Very high Dia

This combination of Gear is famous for speed reduction Ratio.

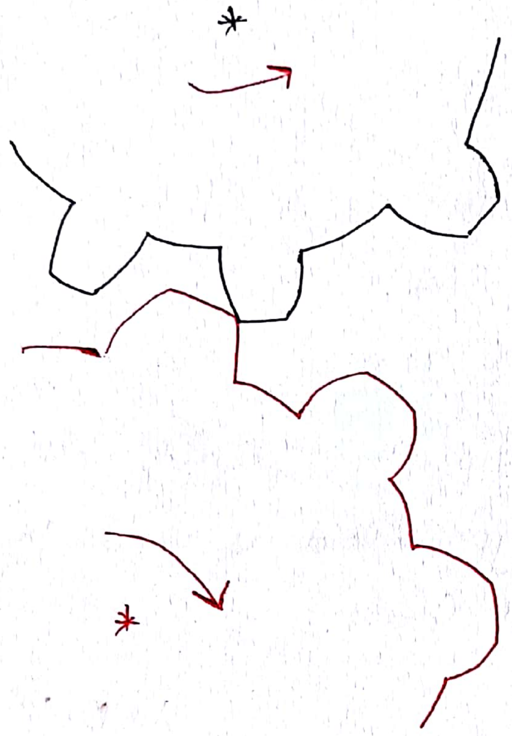
worm : worm wheel

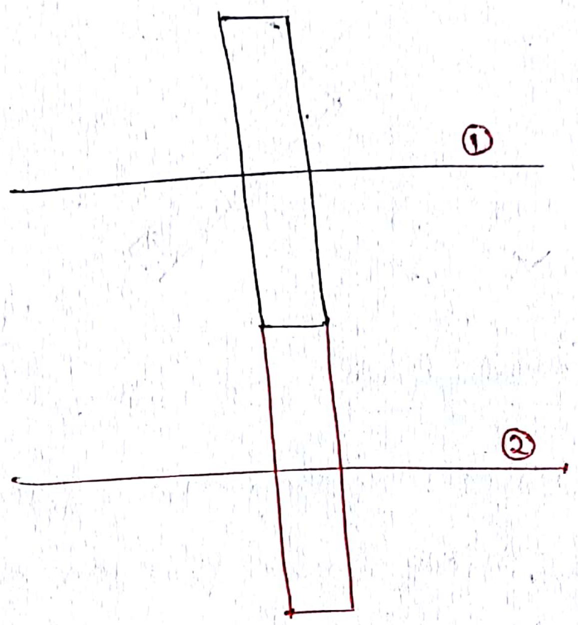
Driver

10 : 1
30 : 1
100 : 1
1000 : 1
1250 : 1

B) According to the type of gearing

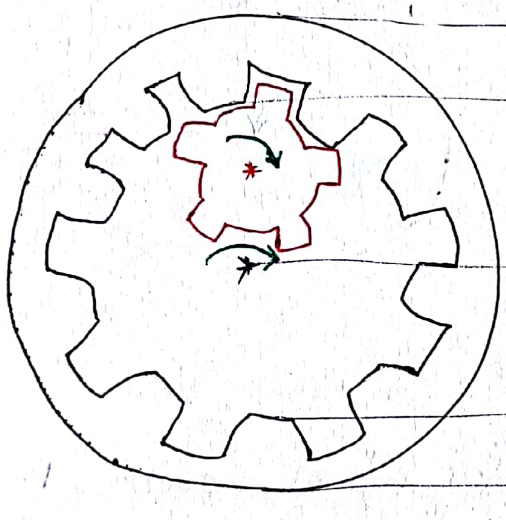
- External gearing

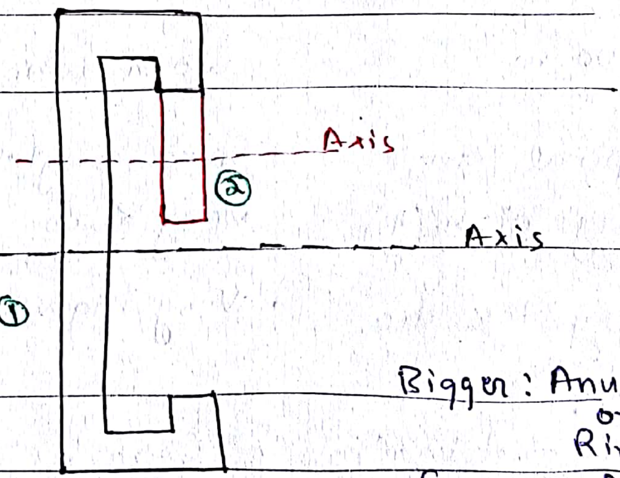




Bigger : Gear
Smaller : Pinion

- Internal Gearing





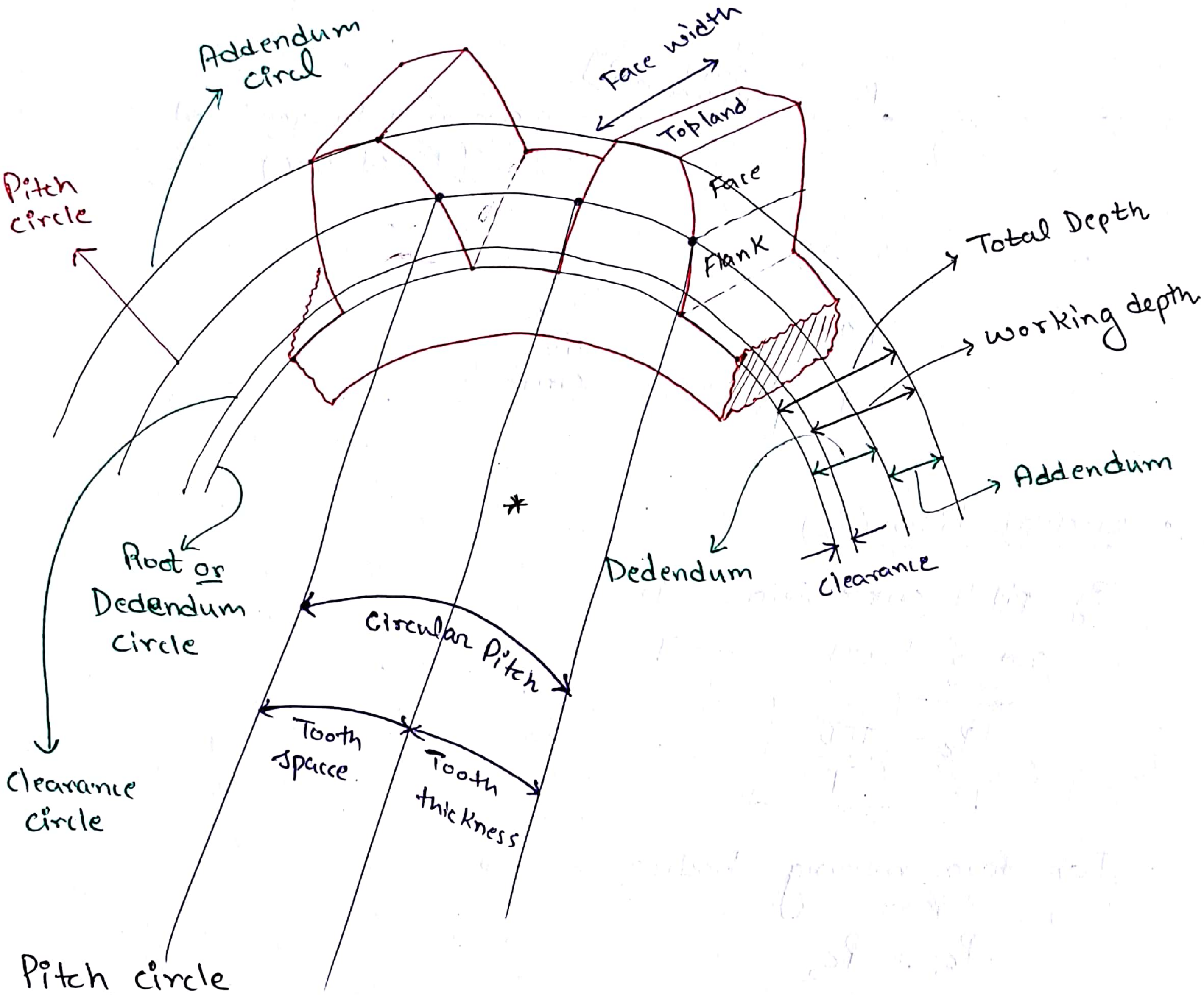
Bigger : Anular or Ring
Smaller : Pinion

Note :
- If more than one gear mounted on a shaft [same shaft] they are known as compound Gears & have same speed.
- Generally in Power transmission the smaller bodies are made as driver

$\underline{\underline{P}} = \underline{T}\,\underline{\underline{\omega}}$ → High for smaller bodies

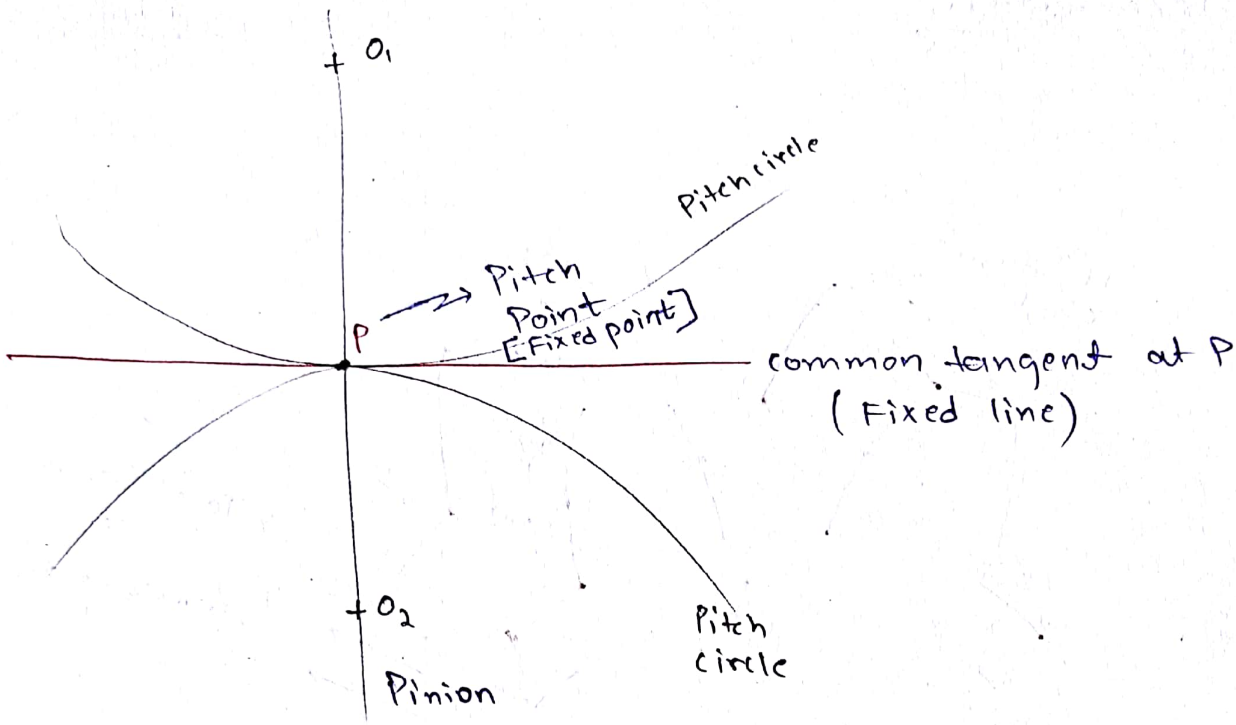
less torque is required ←

Gear Terminology



- **Pitch circle**

It is an Imaginary circle in the gears where pure rolling motion is observed when the mating gears are transmitting power. Being an imaginary circle it cannot be the physical characteristic of gear but being the most important circle. It is one of the biggest specification of the gear. The size of any gear is specified by diameter of pitch circle.



- Circular Pitch (P_c)

If pitch circle Dia = D
no. of teeth = T

$$\boxed{P_c = \frac{\pi D}{T}}$$

For two mating bodies

$$P_{c_1} = P_{c_2}$$

$$\frac{\cancel{\pi} D_1}{T_1} = \frac{\cancel{\pi} D_2}{T_2} \Rightarrow \boxed{\frac{D_1}{T_1} = \frac{D_2}{T_2}}$$

- Module (m)

$$\boxed{m = \frac{D}{T}}$$

D is always in 'mm'

$$P_c = \frac{\pi D}{T} = \pi m$$

for two mating gears $m_1 = m_2$

- <u>Diamterical Pitch</u> (P_d)

$$P_d = \frac{T}{D}$$

$$P_d \times P_c = \frac{\cancel{T}}{\cancel{D}} \times \frac{\pi \cancel{D}}{\cancel{T}} = \pi$$

- <u>Working Depth</u>

working depth = Addendum + Deddendum − clearance

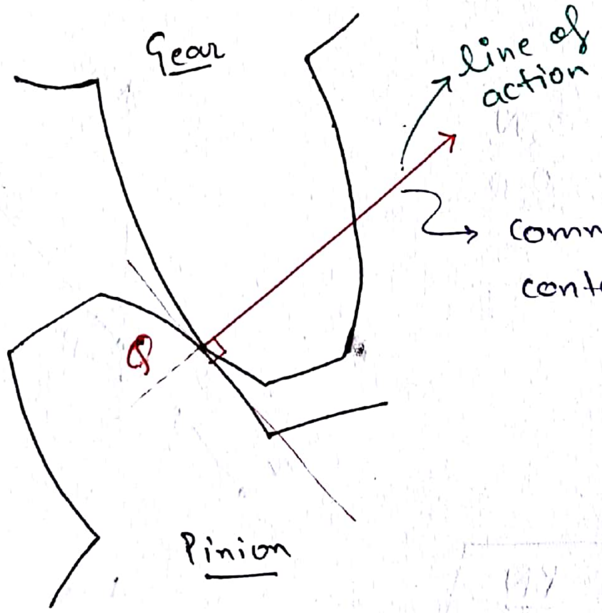
or

= Sum of addendum of Both mating Gears.

- Tooth Space − Tooth thickness of mating gear = Backlash

⇓

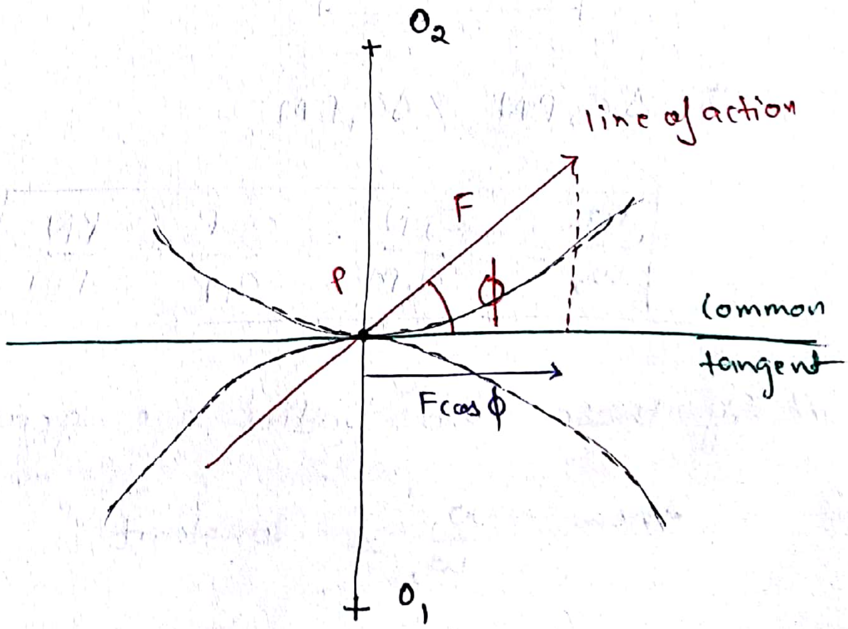
To avoid jamming of teeth due to Thermal Expansion.



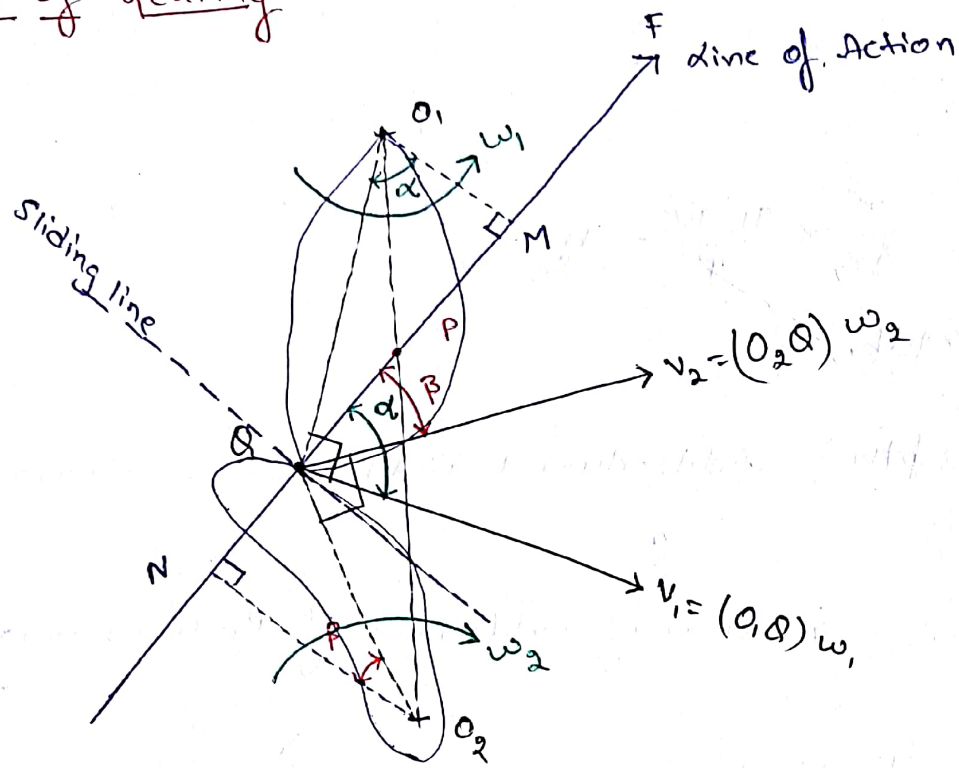
↳ common normal at the point of contact (P)

- <u>Pressure angle</u>(ϕ)

Angle b/w line of action and common tangent at P.



Law of Gearing

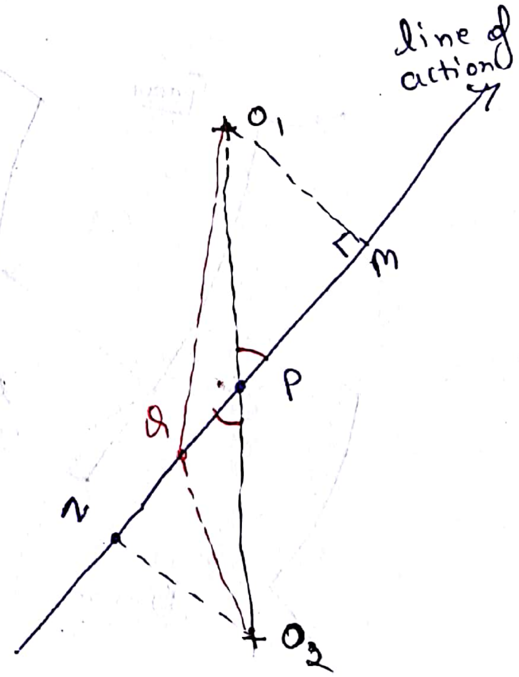


For proper contact

$$v_1 \cos\alpha = v_2 \cos\beta$$

$$\cancel{O_1Q}\,.\,\omega_1\,.\,\frac{O_1M}{\cancel{O_1Q}} = \cancel{O_2Q}\,.\,\omega_2\,.\,\frac{O_2N}{\cancel{O_2Q}}$$

$$\boxed{\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M}}$$



In ΔO_2PN & ΔO_1PM

$$\boxed{\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M} = \frac{O_2P}{O_1P} = \frac{PN}{PM}}$$

If these two bodies are gear

then $\frac{\omega_1}{\omega_2} =$ Constant

$$\Rightarrow \frac{O_2P}{O_1P} = \text{const.}$$

O_1 & O_2 already fixed $\Rightarrow$ P is a fixed Point

"line of action must always pass through the fixed point (Pitch Point) on the line joining the centers of rotation of gears."

For body to be gear

$\Downarrow$

line of action must pass through Pitch Point 'P'.

$\hookrightarrow$ Common normal at contact point 'Q'. $\downarrow$ on profile

"The mating profile should be made in such a way that law of gearing is always satisfied."

$\underbrace{\hspace{6cm}}$

Conjugate Profiles

- $\hookrightarrow$ Involute
- $\hookrightarrow$ Cycloidal

Velocity of sliding

$$V_{sliding} = |v_1 \sin\alpha - v_2 \sin\beta|$$

$$= \left|\cancel{O_1Q} \cdot \omega_1 \cdot \frac{QM}{\cancel{O_1Q}} - \cancel{O_2Q} \cdot \omega_2 \cdot \frac{QN}{\cancel{O_2Q}}\right|$$

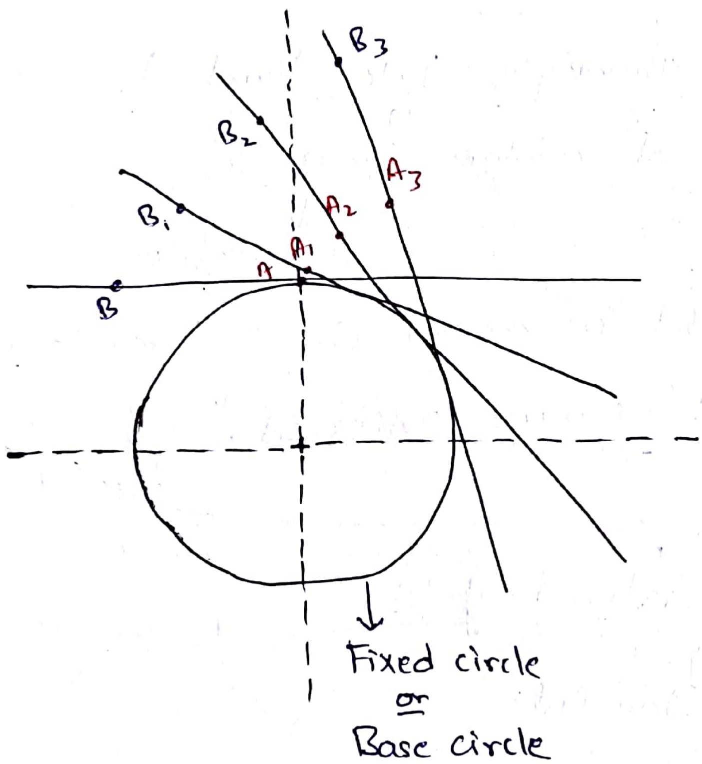
$$= |\omega_1(QP + PM) - \omega_2(PN - QP)|$$

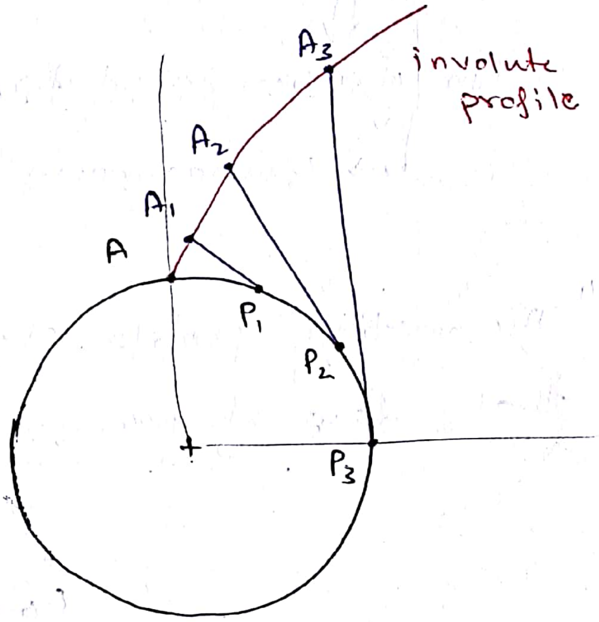
$$\boxed{V_{sliding} = (\omega_1 + \omega_2)\,QP}$$

Involute Profile

[By nature conjugate]

"It is defined as the locus of point on line which rolls without sliping on the fixed circle and this fixed circle which is the generator of involute profile is known as Base circle."





In reality

$AP_1 \longrightarrow 0$

$P_1P_2 \longrightarrow 0$

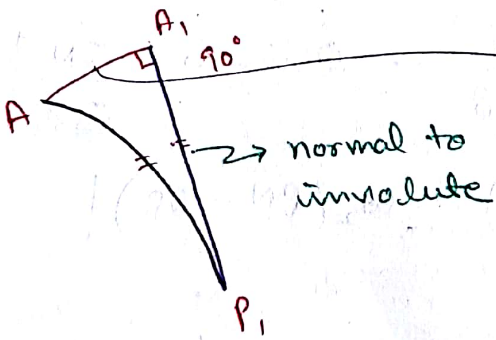
$P_2P_3 \longrightarrow 0$

} Differential length division

$\text{Arc}(AP_1) = P_1A_1$

$\text{Arc}(AP_2) = P_2A_2$

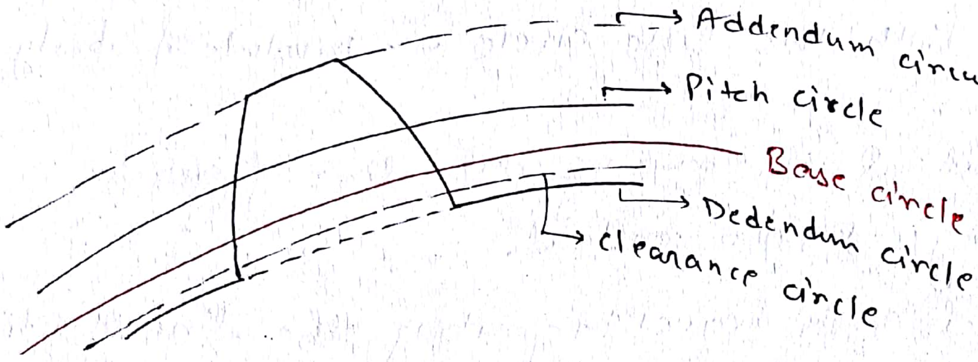
$\text{Arc}(AP_3) = P_3A_3$



Differential portion of circle having center P_1 & radius P_1A_1.

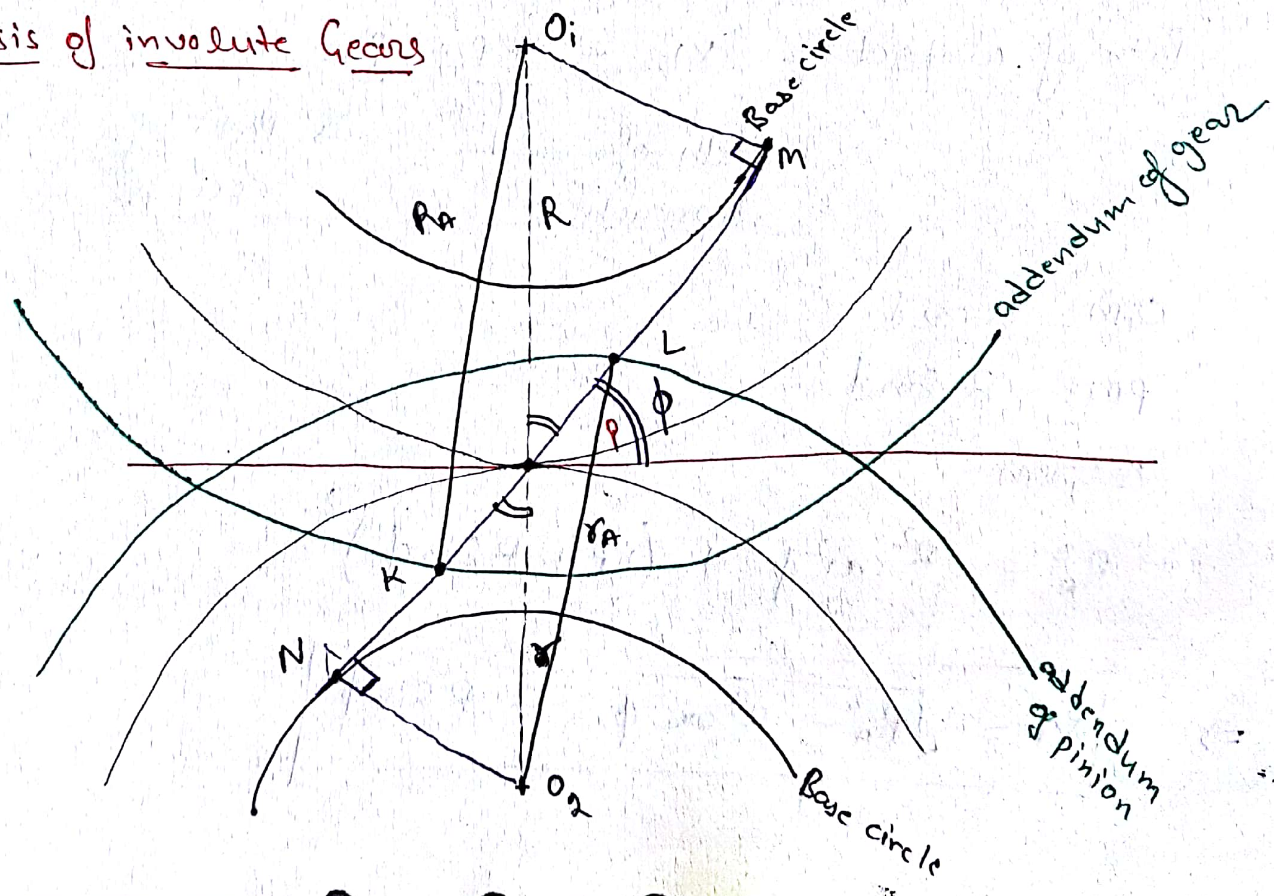
"Normal drawn at any point on involute curve will beome tangent to its base circle automatically".

Position of Base circle in involute gear



- In external gears if $r_{base} \downarrow$ then $\underset{\substack{\downarrow \\ \text{limitation}}}{\underline{\phi \uparrow}}$ $\underline{20 \text{ to } 25^\circ}$
- Base circle in external gear can never come on dedendum circle.
- Non-involute portion can never be eleminated in External gears.

Analysis of involute Gears



Start of the engagement = K
end of the engagement = L

Line of Action

- passes through pitch point ⟵ Law of gearing
- Tangent to Both the Base circle ⟵ Involute property

Point of contact Q is changing but line of action is not changing ⇒ ϕ = constant.

Point of contact Q is travelling along the line of action
↳ locus of Q = Straight line.

⇒ The time interval in which Q is travelling from start to end of engagement is called one engagement period.

⇒ Distance travelled by Q in one engagement period is called Path of contact.

$$\text{Path of contact} = KL = \underbrace{KP}_{\text{Path of approach}} + \underbrace{PL}_{\text{Path of reccess.}}$$

$O_1M = R\cos\phi$

$PM = R\sin\phi$

ΔO_1KM

$$R_A^2 = R^2\cos^2\phi + (KP + R\sin\phi)^2$$

$$\Rightarrow \boxed{KP = \sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi}$$

Similarly,

$$\boxed{PL = \sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi}$$

$KP + PL = KL =$ Path of contact.

Arc of contact

" When the point of contact is travelling from start of engagement to end of engagement, the Distance travelled by Pinion or gear along there pitch circle in this period is known as arc of contact "

Arc of contact : Travel of gear/pinion along their pitch circle in one engagement period.

Arc of Approach : $\dfrac{\text{Path of Approach}}{\cos\phi}$

Arc of recess : $\dfrac{\text{Path of recess}}{\cos\phi}$

$$\boxed{\text{Arc of contact} = \frac{\text{Path of contact}}{\cos\phi}}$$

In one engagement period

$$(\text{Angle turned})_{\text{By pinion}} = \frac{\text{Arc of contact}}{r} \ \text{radian}$$

$$(\text{Angle turned})_{\text{By gear}} = \frac{\text{Arc of contact}}{R} \ \text{radian}$$

$$\boxed{\text{Contact ratio} = \frac{\text{Arc of contact}}{\text{Circular Pitch } (P_c)}}$$

→ no. of Pairs engaged in one engagement period.

→ Contact ratio ≥ 1 for proper messing.

Generally C.R. $\in$ [1.2 to 1.8]

- Spur Gears $\in$ [2 to 3]

eg:- Contact ratio 1.28

one pair is engaged in full engagement period. But in 28% time of engagement period along with this pair one more pair is engaged. Therefore no. of average pairs in mess comes out to be 1.28.

Velocity Ratio

$$\frac{\omega_P}{\omega_G} = \frac{T}{t} \quad > 1$$

$$\frac{\omega_G}{\omega_P} = \frac{t}{T} \quad < 1$$

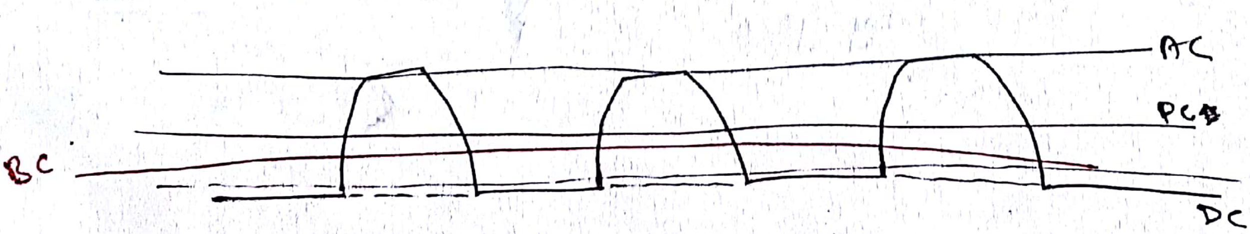
Gear Ratio

$$\boxed{G = \frac{T}{t}}$$

always ≥ 1

Rack

⇓ Gear of ∞ Pitch circle dia [Biggest Gear]



WB
Ch-3
Q51

given ⇒ $t = 24$

$T = 36$

$m = 8\text{ mm}$

$\phi = 20°$

Addendum (each) = 7.5 mm

$N_p = 450$ rpm

$$\frac{N_g}{N_p} = \frac{t}{T} = \frac{450 \times 24}{36} = 300 \text{ rpm}$$

<u>Gear</u> : $R = \frac{mT}{2} = \frac{8 \times 36}{2} = 144$ mm

$R_A = 144 + 7.5 = 151.5$ mm

<u>Pinion</u> : $r = \frac{mt}{2} = \frac{8 \times 24}{2} = 96$ mm

$r_A = 96 + 7.5 = 103.5$ mm

Path of contact = KP + PL

$$= \sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi$$

$$+$$

$$\sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi$$

$= \underline{18.88}$ mm $+ \underline{17.9}$ mm $= \underline{36.78}$ mm

Arc of contact $= \frac{KL}{\cos\phi} = \underline{39.144}$ mm.

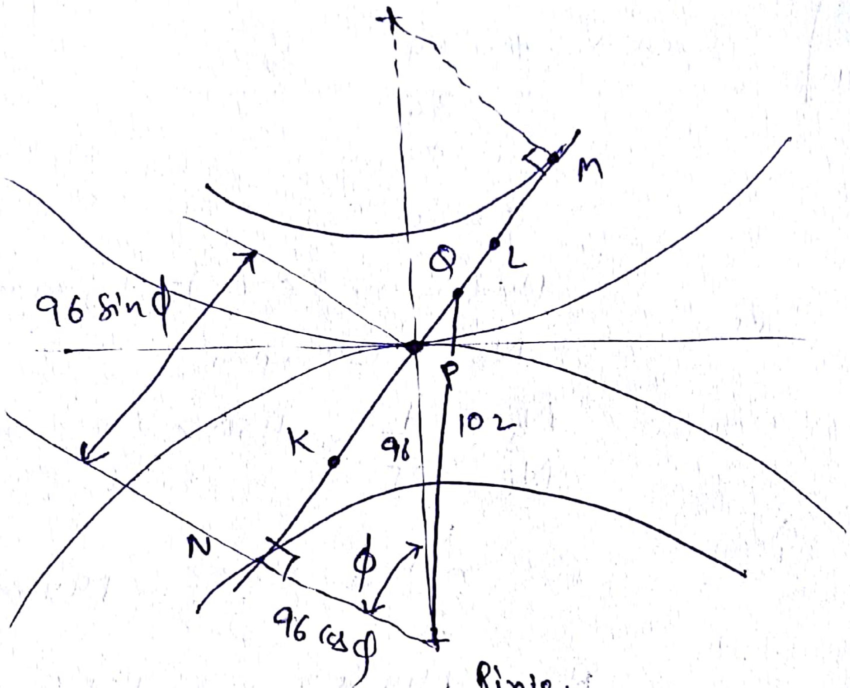
(i) $(\text{Angle})_{\text{pinion}} = \frac{\text{Arc of contact}}{r} \times \frac{180}{\pi} = \underline{23.36°}$

(ii) $V_{\text{sliding}} = (\omega_p + \omega_g)\, QP$

$= \frac{2\pi}{60}(300 + 450) \times QP = 1.16$ m/s

13/6/16

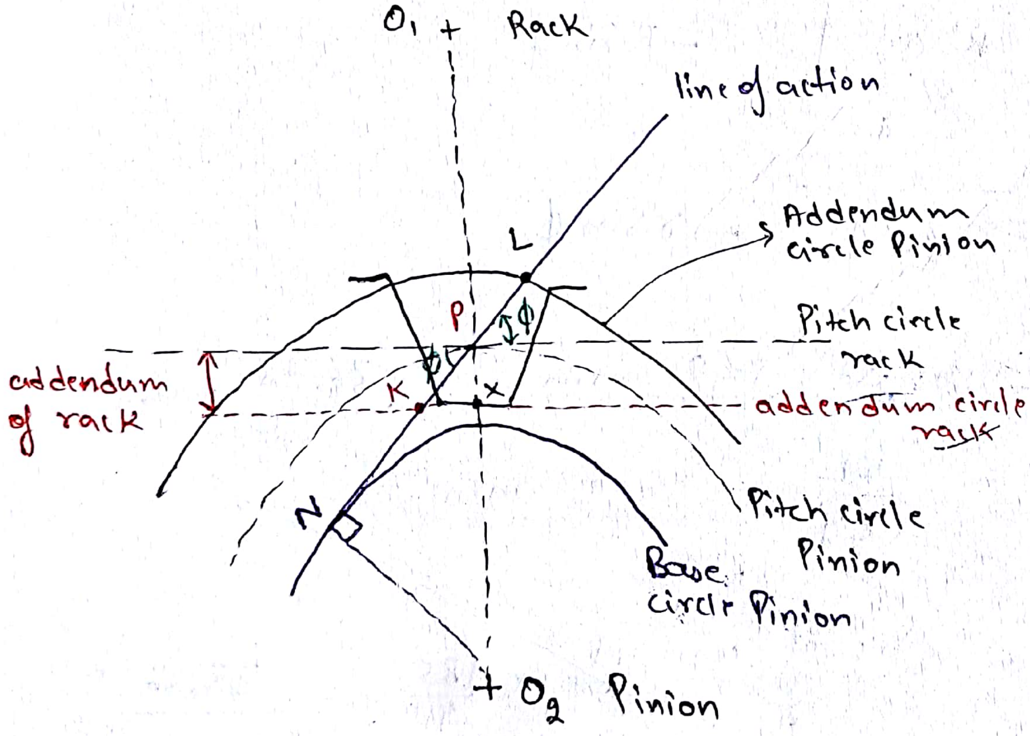
Calculation of QP



$$(102)^2 = (96\cos\phi)^2 + (96\sin\phi + QP)^2$$

$\Rightarrow$ QP = 14.77 mm

Path of contact in Rack & Pinion arrangement



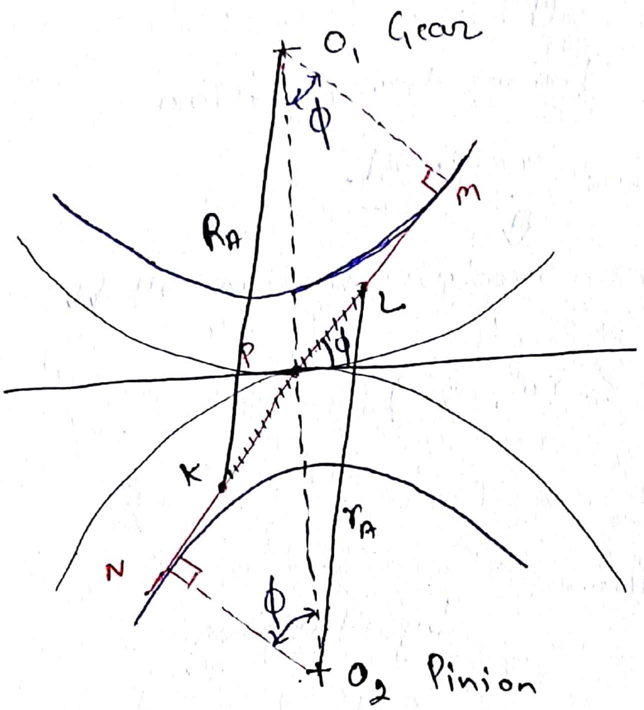
$$\sin\phi = \frac{Px}{KP}$$

$$KP = \frac{Px}{\sin\phi} = \frac{\text{addendum of rack}}{\sin\phi}$$

Path of contact = KL = KP + PL

$$\boxed{KL = \frac{A_{rack}}{\sin\phi} + \sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi}$$

Interference



If $r_A > O_2M$

- Involute tip of pinion will touch non involute portion of flank of gear
- Involute & non-involute contact, Law of gearing is not satisfied.
- Involute tip of pinion will remove some material from non-involute flank portion of gear.
- This removal of material is a process called "under cutting".

⇒ Similar phenomenon will happen if $R_A > O_1N$

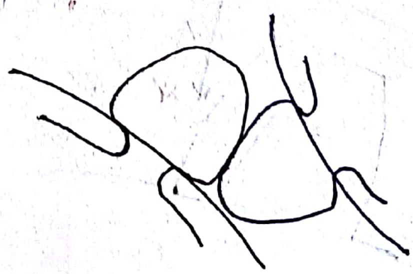
$$\underbrace{\hspace{6cm}}_{\text{Interference}}$$

Last safety points of K & L are N & M
Critical points
Interference points

Methods to prevent Interference

1. Under-cut Gear

- Under cutting is done by cutting tool at the time of manufacturing.
- Strength of tooth is less at root
- Limitation
  ⇓
  used upto low power transmission



2. Increase the Pressure Angle (ϕ)

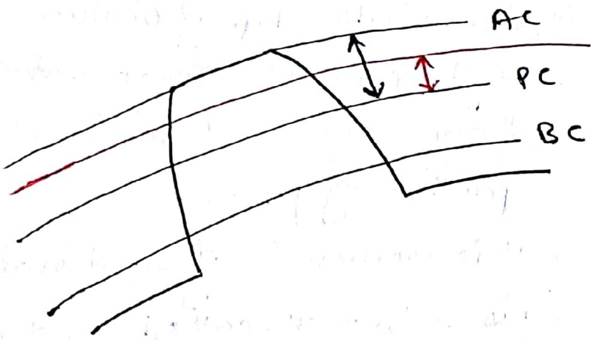
- Used in medium to high Power transmission
- If ϕ increase when r_{Base} reduced.

⇓

non involute Portion will ↓↓

Limitation [$\phi \leq 20^\circ, 25^\circ$]

3. By stubbing the teeth



By stubbing

- Addendum ↓
- Addendum circle radius ↓
- Interference ↓↓

By stubbing

- No change in ϕ
- But Addendum circle radius ↓

↳ Path of contact ↓

⇓

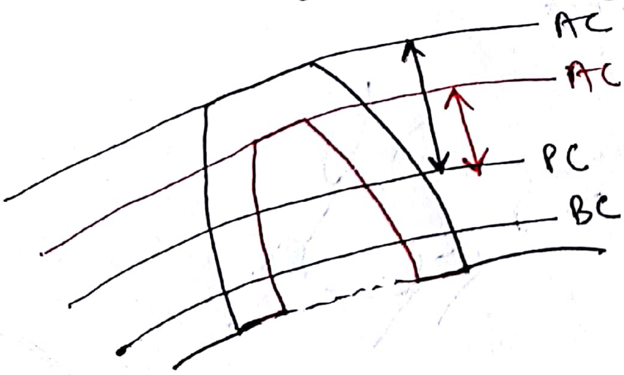
Arc of contact ↓

⇓

Contact ratio ↓ [∴ $P_c = \frac{\pi D}{T}$ ⇓ const.]

Limitation mini. >1

4. Increasing number of teeth



If no. of teeth ↑

- Addendum circle radius ↓
- Interference ↓↓
- No change in ϕ

If no. of teeth increased

- Arc of contact decrease

  but $P_c = \frac{\pi D}{T}$ ↓↓

  $\Rightarrow$ contact ratio ↑↑

- No Limitation, Best method

Fractional Addendum (A)

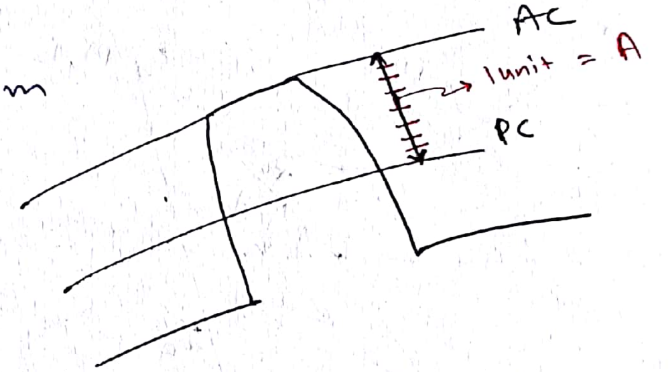
A → fractional addendum for 1mm module in order to avoid interference.

Therefore

Addendum required in order to avoid interference = mA [m = module]

for eg:- Addendum = 7.5 mm

module = 8 mm



$mA = 7.5$

$A = \frac{7.5}{m} = \frac{7.8}{8}$

Involute Gear System

1. Full depth involute.

full depth Involute ⇒ $(14\frac{1}{2}°, 20°)$

Addendum = Standard Addendum

= one module value

$\cancel{m}A = 1\cancel{m}$

$\boxed{A = 1}$

2. Stubbed involute $(20°, 25°)$

Addendum < Standard addendum

$\cancel{m}A < 1\cancel{m}$

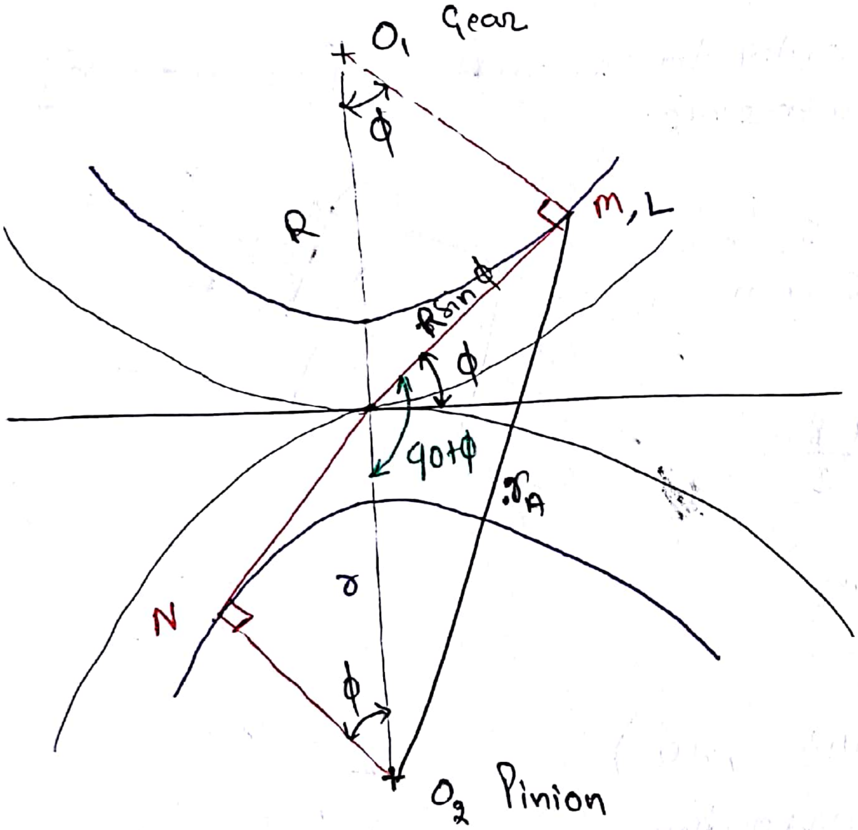
$\boxed{A < 1}$

Best Gear in market

↳ 20° stubbed involute

- Lesser interference
- minimum no. of teeth requirement is less
- cost is less
- Stronger tooth

Minimum no. of teeth requirement on Pinion/Gear In order to avoid interference:-



$\Delta\, O_2PL$

$$r_A^2 = r^2 + R^2\sin^2\phi - 2rR\sin\phi\,(\cos(90+\phi))$$

$$r_A^2 = r^2 + (R^2 + 2rR)\sin^2\phi$$

$$= r^2\left[1 + \left(\frac{R}{r}\right)\left(\frac{R}{r} + 2\right)\sin^2\phi\right]$$

$$r_A = r\sqrt{1 + G(G+2)\sin^2\phi} - 1$$

Addendum of Pinion $= r_A - r$

$$mA_p = r \times \sqrt{1 + G(G+2)\sin^2\phi} - 1$$

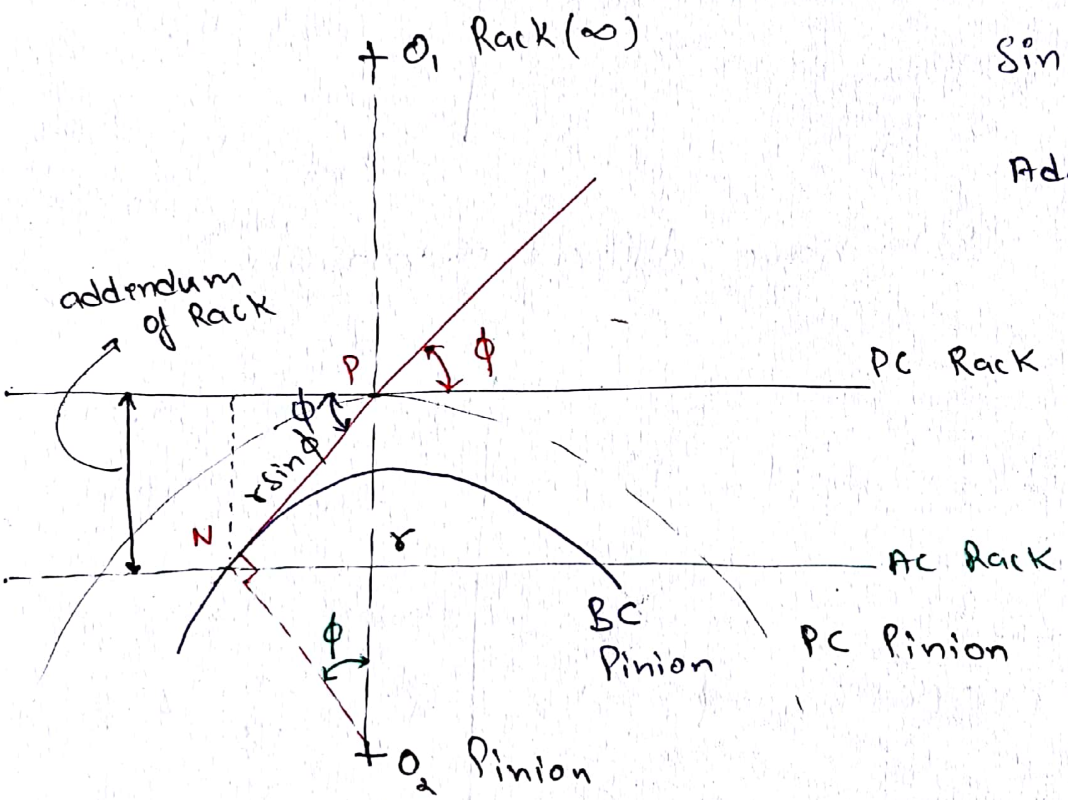
$$\cancel{m}A_p = \frac{\cancel{m}t_{min}}{2} \times \sqrt{1 + G(G+2)\sin^2\phi} - 1$$

$$\boxed{t_{min} = \frac{2A_p}{\sqrt{1 + G(G+2)\sin^2\phi} - 1}}$$

Similarly,

$$\boxed{T_{min} = \frac{2A_g}{\sqrt{1+\frac{1}{q}\left(\frac{1}{q}+2\right)\sin^2\phi} - 1}}$$

minimum no. of teeth requirement on Pinion in order to avoid interference in involute Rack & pinion arrangement



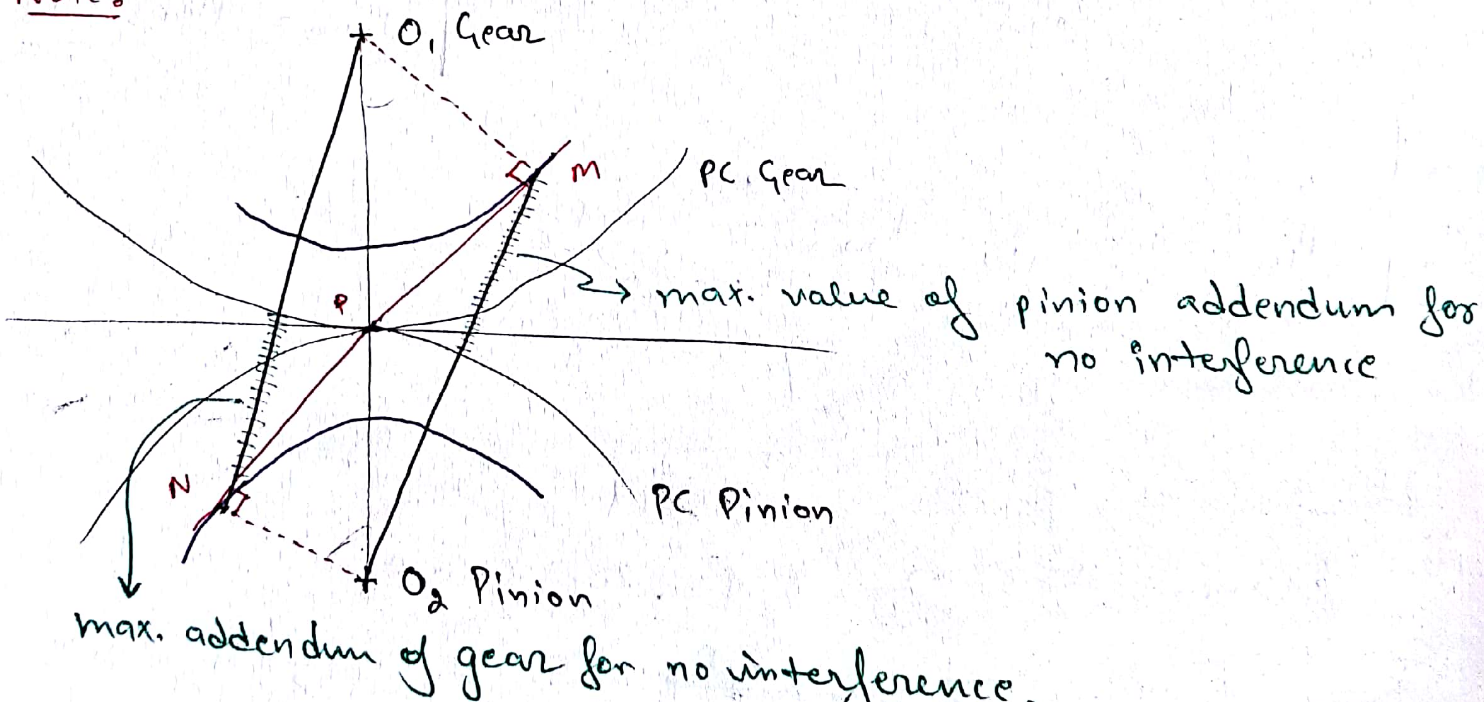
$$\sin\phi = \frac{\text{Add. of Rack}}{r\sin\phi}$$

$$\text{Add. rack} = r\sin^2\phi$$

$$\cancel{m}A_R = \frac{\cancel{m}\,t_{min}}{2}\sin^2\phi$$

$$\boxed{t_{min} = \frac{2A_R}{\sin^2\phi}}$$

Note:



If Gear and Pinion ⇒ Same addendum

- First Gear must be safe.

$$T_{min} = \frac{2A_g}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi}-1}$$

- Pinion will be automatically safe

$$t_{min} = \frac{T_{min}}{G}$$

eg:- $T_{min} = 28$

$G = 3$

$t_{min} = \frac{28}{3}$

$= 9.33$

⇓

$t_{min} = \underline{10}$

So $T_{min} = 10 \times 3 = \underline{\underline{30}}$

If Gear | Pinion Both have different addendum:

- $$T_{min} = \frac{2A_g}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi}-1}$$

- $$t_{min} = \frac{2A_p}{\sqrt{1+G(G+2)\sin^2\phi}-1}$$

- $$G = \frac{T_{min}}{t_{min}}$$

$G = 3$

eg:- $T_{min} = 39.01$

$T_{min} = 40$

$t_{min} = 13$

$G = \frac{40}{13} \neq 3$

$t_{min} = \underline{14}$

$T_{min} = 14 \times 3 = \underline{\underline{42}}$

WB
Q52
ESE
2007

$G = 3$

$A_p = A_g = 1$

$\phi = 20°$

find $t_{min} = ?$

(i) $$T_{min} = \frac{2A_g}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi}-1} = 45$$

$t_{min} = \frac{45}{3} = 15$ $\underline{\text{Ans.}}$

(ii) If $t_{min} = 15-3 = 12$ & $\phi = 20°$ find A_g

$$T_{min} = 12 \times 3 = 36 = \frac{2A_g}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi}-1}$$

$\Rightarrow A_g = 0.8$

20% stubbing

Ques: all data same but $A_g = 1$ find increase in ϕ

$$36 = \frac{2A_g^{\to 1}}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1} \Rightarrow \phi = \underline{\cancel{\quad} \; 22.53^\circ}$$

$\downarrow$

3   1.77

WB
Q50
ESE
1998

$\phi = 20^\circ$

$m = 10\ mm$

$A_p = A_g = 1$

$\left.\begin{matrix} T = 50 \\ t = 13 \end{matrix}\right\} G = \frac{50}{13}$

(i) $T_{min} = \dfrac{2A_g}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1}$

$= 60$

yes Interference is there

(ii) $T_{min} = 50 = \dfrac{2A_g}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1}$

$\Rightarrow \phi = \underline{\cancel{\quad} \; 21.87^\circ}$

Ques

$G = 4$

$A_p = A_g = 1$

$\phi = 20^\circ$

$T_{min} = ?$

$T_{min} = \dfrac{2A_g}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1}$

$= 62$

$t_{min} = \frac{62}{4} = 15.5 \Rightarrow 16$

$T_{min} = 16 \times 4 = \underline{64}$ Ans

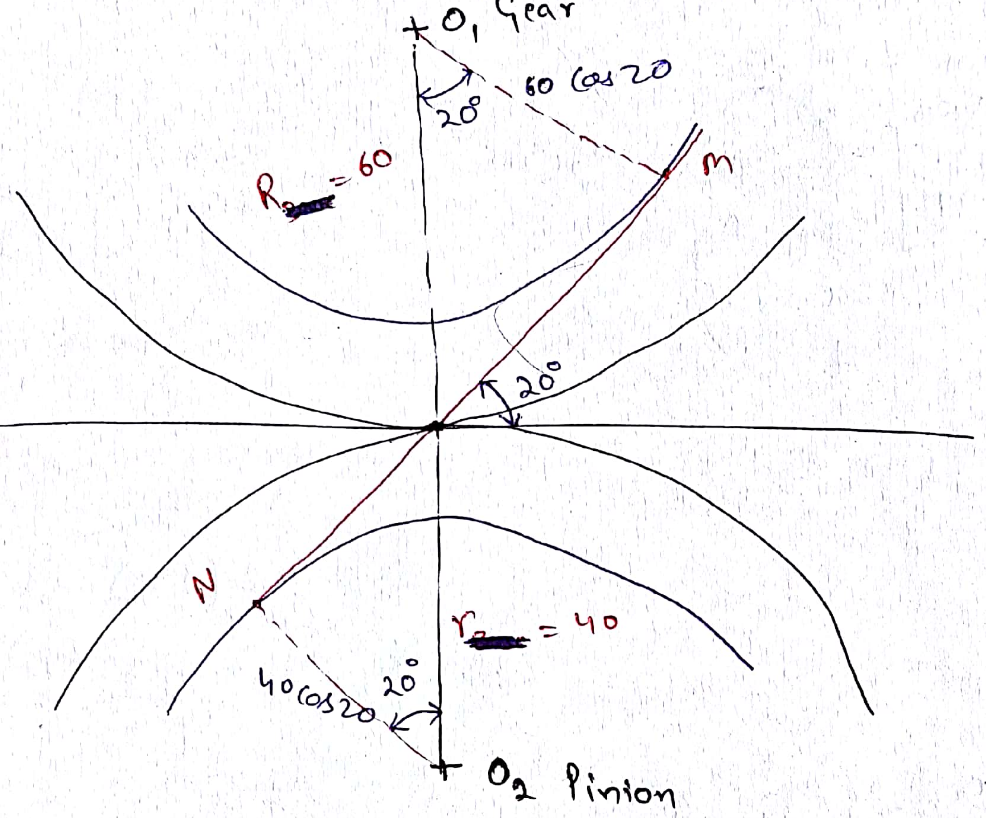
Effect of variation of center distance due to vibration on the performance of involute Gears:

eg:- Center distance = 100 mm

$R = 60$ mm

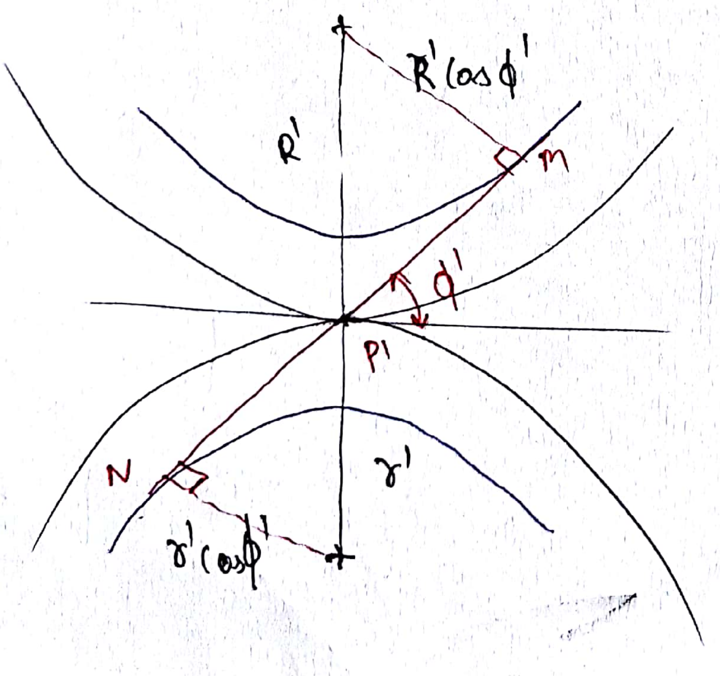
$r = 40$ mm

$\phi = 20^\circ$



At a moment when center of gears are shifted and center distance increased by 2%

New $O_1'O_2' = 102$ mm



Base radius not changed bcz it is physical parameter

$R' \cos\phi' = 60 \cos 20$

$r' \cos\phi' = 40 \cos 20$

$(r' + R')\cos\phi' = (40 + 60)\cos 20$

$\cos\phi' = \dfrac{100 \cos 20}{102}$

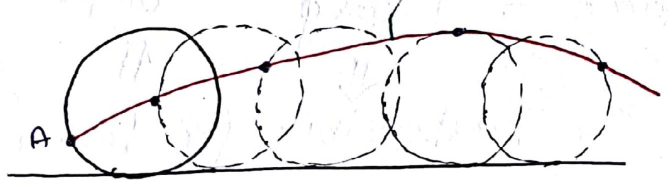
$\phi' = ?$

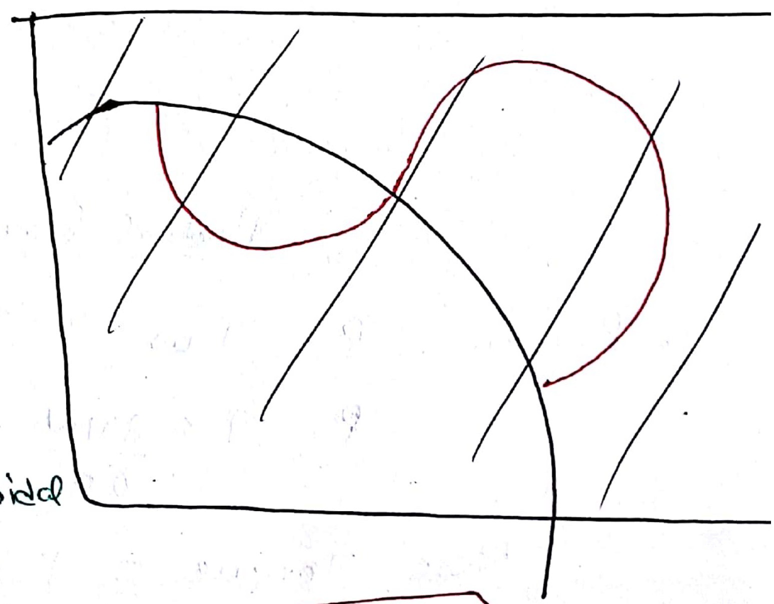
Because of vibration [$\neq 0$, close to zero]

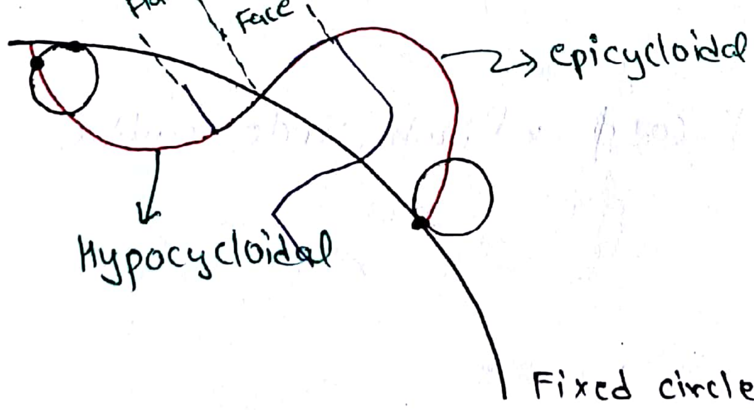
- Center distance is changing
- Pitch circle is changing
- Pitch Point is changing
- ϕ is changing
- $r_{Base} \longrightarrow$ no change
- $\frac{\omega_1}{\omega_2} = \frac{O_2 N}{O_1 M}$ = constant.

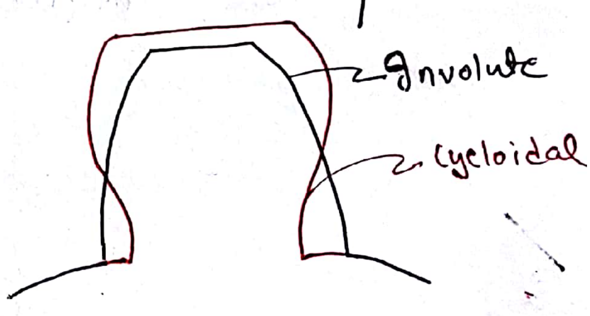
CYCLOIDAL PROFILE: [By nature Conjugate]

" It is defined as the locus of the point on the circumference of circle which roles without slipping on fixed straight line."









Advantages of cycloidal over involute

- Per tooth cost is more but overall cost of gear is less.
- Interference absent.
- Flank wider ⇒ stronger tooth.
- Convex - concave connection ⇒ less wear ⇒ life is more.

Variation of Pressure Angle:

$\phi_{max} \longrightarrow$ Start of engagement

$\phi = 0 \longrightarrow$ At Pitch point

$\phi_{max} \longrightarrow$ At end of engagement
[In reverse dir^n]

Note: In mating Gears

- Normal thrust b/w the teeth $= F$

Tangential force $= F\cos\phi$ ⇓ responsible for Power Transmissions

Power component ⇑

Radial force $= F\sin\phi$

- Power $= P = T\omega$

$$P = T \times \frac{2\pi N}{60}$$

Here, Torque $= F\cos\phi \times$ Pitch circle radius.

CHAPTER - 4

GEAR TRAIN

⇓

Combination of Gears

⇓

why

- large center Distance
- Velocity ratio required is very high or very low.
- multiple velocity ratios are required.

Any Gear train is combination of

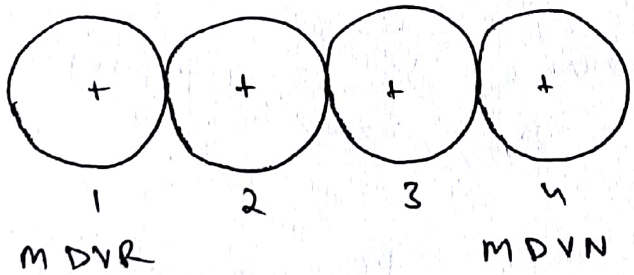
- Main Driver
- Main Driven
- Intermediate Gears
- Arm

$$\text{Speed ratio (S.R.) of Gear train} = \frac{(\omega)_{\text{main driver}}}{(\omega)_{\text{main driven}}}$$

$$\text{Train value} = \frac{1}{S.R.} = \frac{\omega_{\text{main driven}}}{\omega_{\text{main driver}}}$$

Simple Gear Train

Every shaft is having only one Gear in use.



(1,2) :- $\frac{\omega_1}{\omega_2} = \frac{T_2}{T_1}$ ⟶ ①

$m_1 = m_2 = m_3 = m_4$

2,3 :- $\frac{\omega_2}{\omega_3} = \frac{T_3}{T_2}$ ⟶ ②

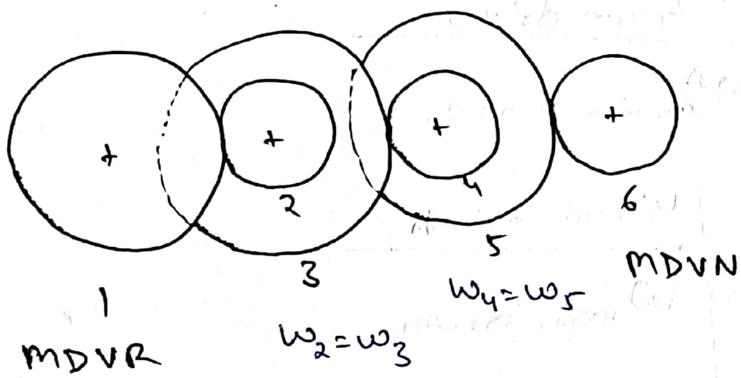
3,4 :- $\frac{\omega_3}{\omega_4} = \frac{T_4}{T_3}$ ⟶ ③

① × ② × ③ ⟹ $\boxed{\frac{\omega_1}{\omega_4} = \frac{T_4}{T_1} = S.R.}$

- If no. of idlers are → even → Dir^n opposite
  - ↳ odd → Dir^n same

Compound Gear Train

Atleast one of the intermediate shaft must have more than one Gear in use.



DVR : 1, 3, 5

DVN : 2, 4, 6

$m_1 = m_2$

$m_3 = m_4$

$m_5 = m_6$

1,2 :- $\frac{\omega_1}{\omega_2} = \frac{T_2}{T_1}$ —— (i)

3,4 : $\frac{\omega_3}{\omega_4} = \frac{T_4}{T_3}$ —— (ii)

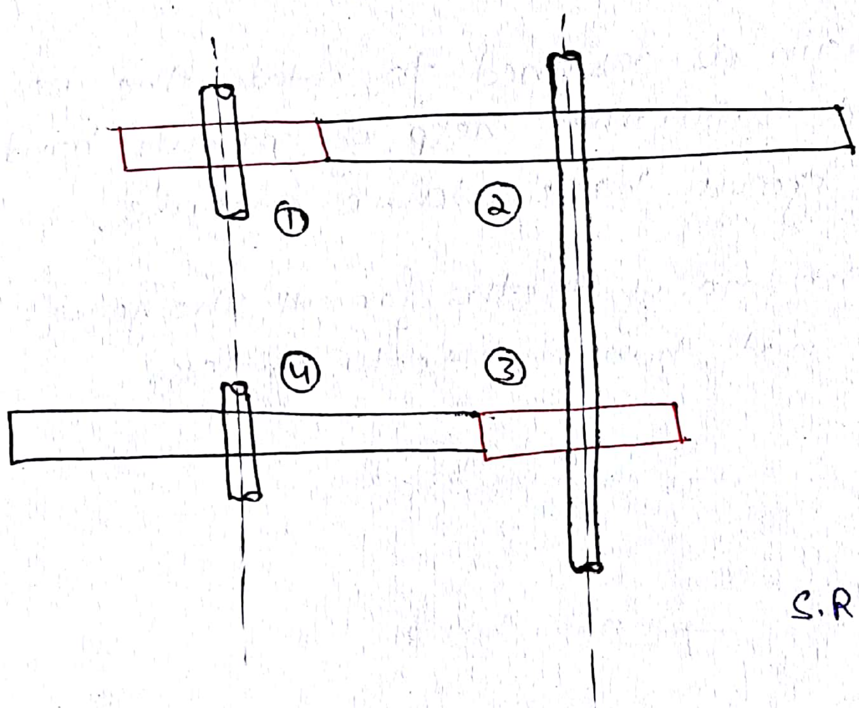
5,6 : $\frac{\omega_5}{\omega_6} = \frac{T_6}{T_5}$ —— (iii)

(i) × (ii) × (iii) ⇒ $\boxed{\dfrac{\omega_1}{\omega_6} = \dfrac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5}} = S.R.$

In General, $S.R. = \dfrac{\omega_1}{\omega_6} = \dfrac{\text{Product of no. of teeth on DVN}}{\text{Product of no. of teeth on DVR}}$

- Reverted Gear Train

That compound Gear Train which is used to connect co-axil shafts.



$m_1 = m_2 = m$

$m_3 = m_4 = m'$

DVR : 1, 3

DVN : 2, 4

$S.R. = \dfrac{\omega_1}{\omega_4} = \dfrac{T_2 \times T_4}{T_1 \times T_3}$

Note: If In the problem of reverted gear train, speed reduction is given same then,

$$\boxed{\frac{T_2}{T_1} = \frac{T_4}{T_3}}$$

A General concept

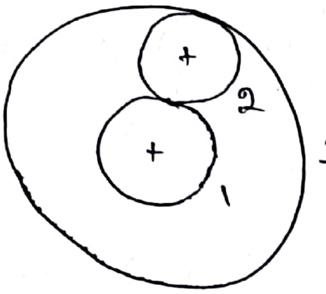
$r_1 + r_2 = r_3 + r_4$

$$\frac{mt_1}{\cancel{2}} + \frac{mt_2}{\cancel{2}} = \frac{m'T_3}{\cancel{2}} + \frac{m'T_4}{\cancel{2}}$$

$$m(T_1 + T_2) = m'(T_3 + T_4)$$

If all gears have same module then, $T_1 + T_2 = T_3 + T_4$

eg:



$r_1 + 2r_2 = r_3$

$T_1 + 2T_2 = T_3$

Ques: A riveted Gear train is designed to rotate the hour hand of the clock with the help of minute hand. Assuming all the gears have same module.

Determine :- 1. The no. of teeths for all the gears.

Any wheel must not have 11 teeth or less.

Soln

$\omega_1 = \frac{2\pi}{60 \times 60}$ rad/s $\rightarrow$ minute hand

$\omega_2 = \frac{2\pi}{12 \times 60 \times 60}$ rad/s $\rightarrow$ hour hand

- $\frac{\omega_1}{\omega_2} = 12$
- $\left(\frac{T_2}{T_1}\right) \times \left(\frac{T_4}{T_3}\right) = 12 \quad \text{—(i)}$
- $T_1 + T_2 = T_3 + T_4 \quad \text{—(ii)}$
- $T_{min} > 11$

Assume

$T_1 = 12$

&

$\frac{T_2}{T_1} = 4$ & $\frac{T_4}{T_3} = 3$

$T_2 = 48$

$T_1 + T_2 = 60 = T_3 + 3T_3$

$T_3 = 15$

$T_4 = 45$

Ans $\Rightarrow$ $T_1 = 12$
$T_2 = 48$
$T_3 = 15$
$T_4 = 45$

Another Assumption

Let $T_1 = 12$

$\frac{T_2}{T_1} = 3$ & $\frac{T_4}{T_3} = 4$

$T_2 = 36$

$T_1 + T_2 = 48 = T_3 + 4T_3$

$\cancel{T_3 = \frac{48}{5}}$

Assume

Let $T_1 = 12$

& $\frac{T_2}{T_1} = 6$ & $\frac{T_4}{T_3} = 2$

$T_2 = 72$

$T_1 + T_2 = 84 = T_3 + 2T_3$

$T_3 = \frac{84}{3} = 28$

$T_4 = 56$

Ans $\Rightarrow$ $T_1 = 12$
$T_2 = 72$
$T_3 = 28$
$T_4 = 56$

WB Q48

$m_1 = m_2 = 2$

$m_3 = m_4 = 3$

$\omega_4 < \frac{\omega_1}{12} \Rightarrow \frac{\omega_1}{\omega_2} > 12$

$T_1 = T_3 = 24$ (given)

$\frac{\omega_1}{\omega_4} > 12$

$\frac{T_2 \times T_4}{T_1 \times T_3} > 12$

$T_2 \cdot T_4 > 6912$ — (1)

$m(T_1 + T_2) = m'(T_3 + T_4)$

$2(24 + T_2) = 3(T_4 + 24)$

$48 + 2T_2 = 72 + 3T_4$

$3T_4 = 2T_2 - 24$

$T_4 = \frac{2}{3}(T_2 - 12)$ — (2)

By (1) & (2)

$T_2 \cdot \frac{2}{3}(T_2 - 12) > 6912$

$T_2^2 - 12T_2 - 10368 > 0$

$\Rightarrow T_2 > 108$

Assume

$T_2 = 109 \Rightarrow T_4 = \frac{2}{3}(109 - 12)$

$T_4 = 64.66$

Assume

$T_4 = 65$

$T_3 = 24 - 1 = 23$

$2(24 + T_2) = 3(23 + 65)$

$T_2 = 108$

$T_1 = 24$
$T_2 = 108$
$T_3 = 23$
$T_4 = 65$

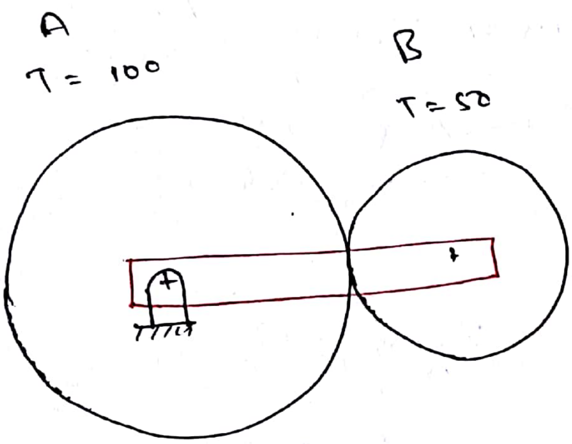
$\frac{\omega_1}{\omega_4} = \frac{108 \times 65}{24 \times 23} = 12.71$

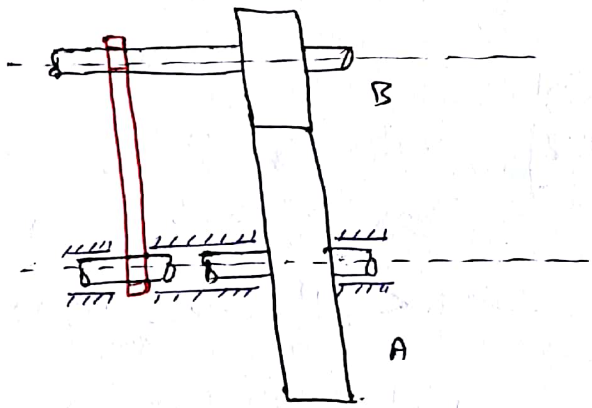
$C.D. = r_1 + r_2 = 132$ mm

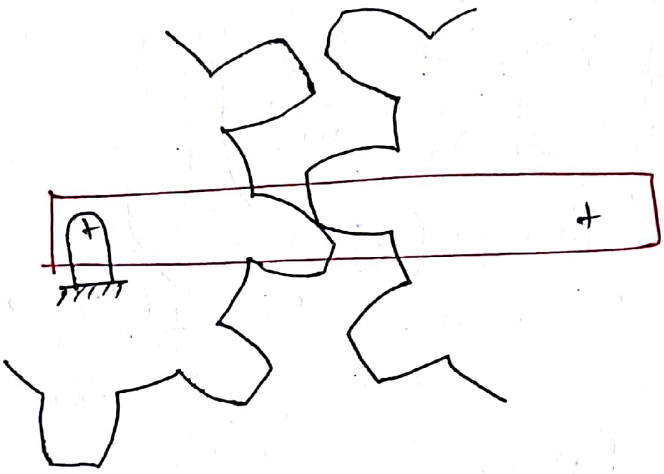
Epicyclic Gear Train

" Apart from the rotation of gears if any gear axis is also rotating w.r.t. som other axis, then the train will be known as epicyclic gear train, it may be Simple epi-cyclic, compound epicyclic, reverted epicyclic and so on."

To rotate the axis of the gear, A link is used which is known as arm or carrier.







$l = 4$

$j = 3$

$h = 1$

$F = 3(4-1) - 2\times3 - 1$

$= 9 - 6 - 1 = 2$

Let Gear A ⇓ 100 rpm clock

Gear B

→ By Gear A, 200 rpm AC

→ Arm → 200 rpm (AC)

Arm → 200 rpm (clock)

Arm → 300 rpm (clock)

400 rpm AC

0 rpm

100 rpm clock

Ques All gears have same module

$T_A = 20$

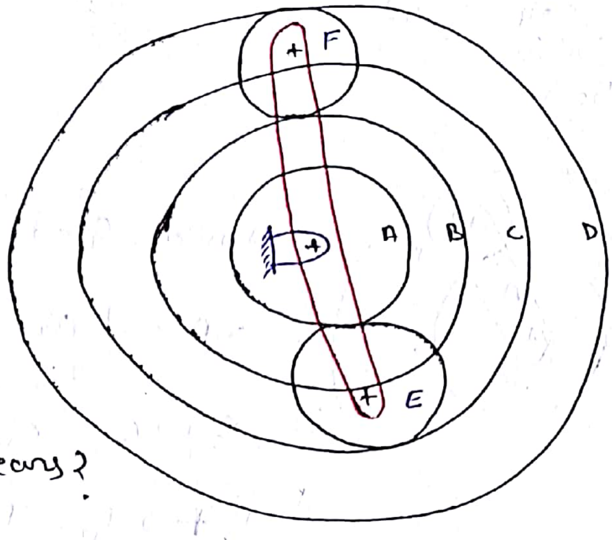
$T_B = 30$

$T_E = T_F = 10$

If Gear D = Fixed

Arm → 100 rpm (AC)

Find rpm of rest 5 gears?



Solu^n

$$T_A + 2T_E = T_C = 40$$

T_A: 20, T_E: 10

$$T_B + 2T_F = T_D = 50$$

T_B: 30, T_F: 10

motion	Arm	A/B (20, 30)	E_{10}	C_{40}	F_{10}	D_{50}
Arm fixed Assume & A rotates by +x Clock	0	x	$-x\times\frac{2\cancel{0}}{1\cancel{0}}$	$-x\times\frac{2\cancel{0}}{1\cancel{0}}\times\frac{1\cancel{0}}{\cancel{40}_{2}}$	$-x\times\frac{3\cancel{0}}{1\cancel{0}}$	$-x\times\frac{3\cancel{0}}{1\cancel{0}}\times\frac{1\cancel{0}}{5\cancel{0}}$
Arm Free	y	$x+y$	$y-2x$	$y-\frac{x}{2}$	$y-3x$	$y-\frac{3x}{5}$
	$= -100$					$= 0$

Solu^n ⇒

$$y = -100 \quad ; \quad y - \frac{3x}{5} = 0 \Rightarrow -\frac{3x}{5} = 100 \Rightarrow -x = \frac{500}{3}$$

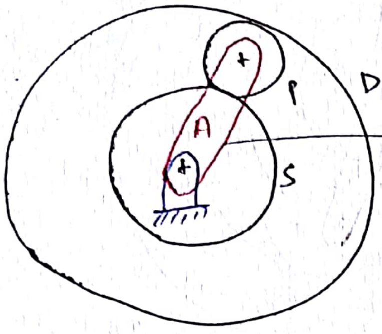
$$E = y - 2x = -100 - \left(-\frac{500}{3}\right) = \frac{-300+500}{3} = -\frac{200}{3}$$

$$A/B\ \cancel{E} = y + \cancel{2}x = -100 - \frac{500}{3} = \frac{-800}{3}$$

$$C = y - \frac{x}{2} = -100 + \frac{500}{2\times 3} = \frac{-100}{6} = -\frac{50}{3}$$

$$F = y - 3x = -100 + \cancel{3}\left(+\frac{500}{\cancel{3}}\right) = 400$$

Planetary Gear Train
(Epi- cyclic)



D = Anular Gear

Arm / Carrier

$$r_s + 2r_p = r_D$$

$$\boxed{T_s + 2T_p = T_D}$$

I st Input

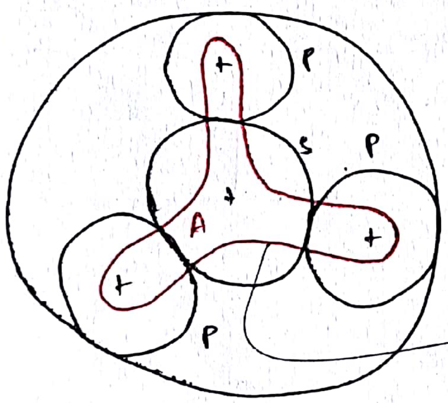
Sun	Ring
Fixed	input
input	Fixed

II Input

⇓

Arm

- Generally in planetary Gear trains, no. of planets are more than 1 because → Balancing gear train
  → For load distribution on the no. of planets in high power transmission.



Arm / carrier (Spider

WB Q49

$$\frac{252}{3.5} = T_D$$

$$T_D = 72$$

$$T_s + 2T_p = T_D$$

$$N_D = 0$$

$$N_s = 5N_A$$

$$T_P = \frac{72-18}{2} = \frac{54}{2} = 27$$

Arm (A)	S	P	D
0	$+x$	$-x\frac{T_s}{T_p}$	$-x\frac{T_s}{T_p} \times \cancel{\frac{T_p}{T_D}}$
y	$y+x$	$y - \frac{T_s}{T_p} x$	$y - \frac{T_s}{72}$

$$y + x = 5y$$

$$\boxed{x = 4y}$$

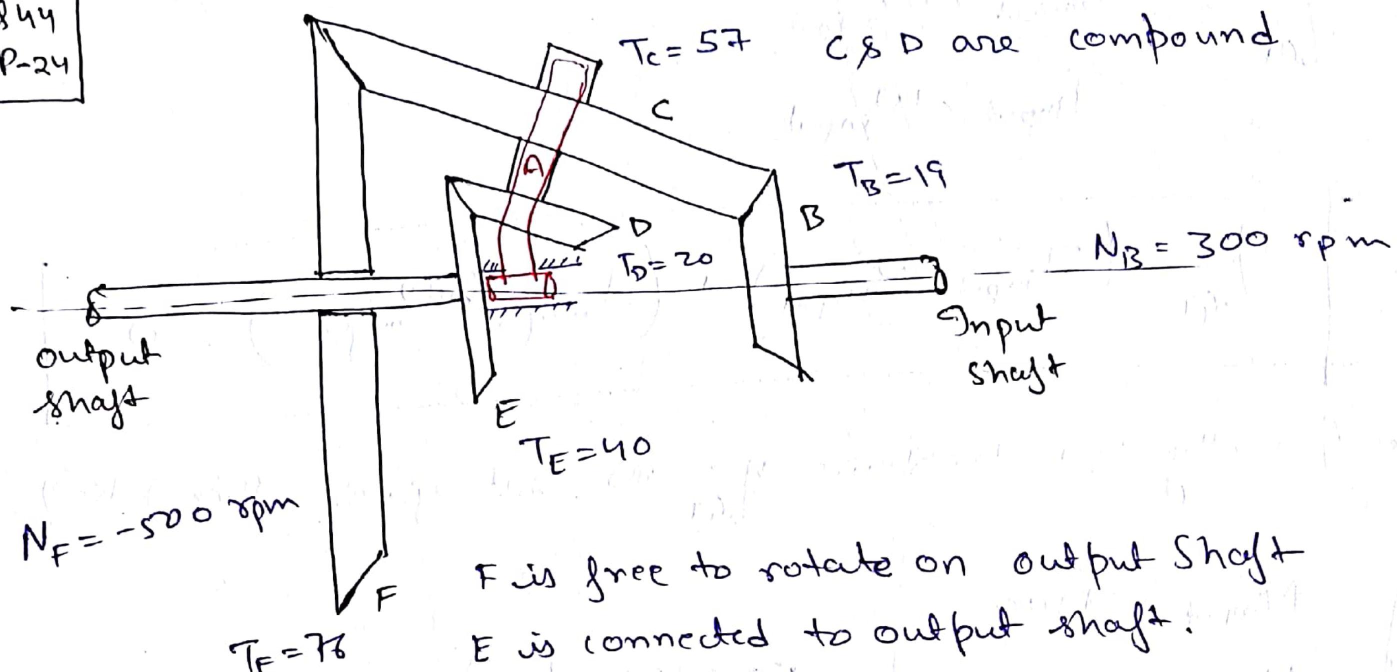
$$y - x\frac{T_s}{T_D} = 0$$

$$y - 4y\frac{T_s}{72} = 0$$

$$18 = T_s \Leftarrow y\left(1 - \frac{T_s}{18}\right) = 0$$

Direction Considerations in Bevel Epicyclic Gear trains :

WB
Q44
P-24



F is free to rotate on output Shaft
E is connected to output shaft.

Arm	B 19	C/D 57 20	E 40	F 76
0	x	$\pm x \frac{T_B}{T_C}$	$-x \frac{T_B}{T_C} \times \frac{T_D}{T_E}$	$-x \frac{T_B}{\cancel{T_C}} \times \frac{\cancel{T_C}}{T_F}$
y	$y+x$	$y \pm x \frac{19}{57}$	$y - x\frac{19}{57} \times \frac{20}{40}$	$y - \frac{19}{76} x$

$$\left.\begin{aligned} y+x &= 300 \\ y-\frac{x}{4} &= -500 \end{aligned}\right\} \quad x + \frac{x}{4} = 800 \Rightarrow x = 640$$

$$y = 300 - 640 = -340$$

$$N_E = y - \frac{x}{6} = -340 - \frac{640}{6} = -446.66 \text{ rpm}$$

Fixing Torque / Holding Torque in Epi-cyclic Gear Train

Total external torque in Epi-cyclic Gear train

$$\Sigma T = 0$$

$$\boxed{T_{input} + T_{output} + T_{fixing} = 0} \quad \text{--- (1)}$$

Power Balance

$$\eta_{GT} = \frac{T_{output} \times \omega_{output}}{T_{input} \times \omega_{input}}$$

$$\boxed{\eta_{GT} \times (T_{input} \cdot \omega_{input}) + (T_{output} \cdot \omega_{output}) = 0}$$

- Generally in Problems η_{GT} is not given then take $\eta_{GT} = 1$

WB P-22 Q34

$N_{input} = +100$

$N_{out} = +250$

$T_{inp} = +50$

$T_{Fixing} = ?$

$50 \times 100 + T_{out} \times 250 = 0$

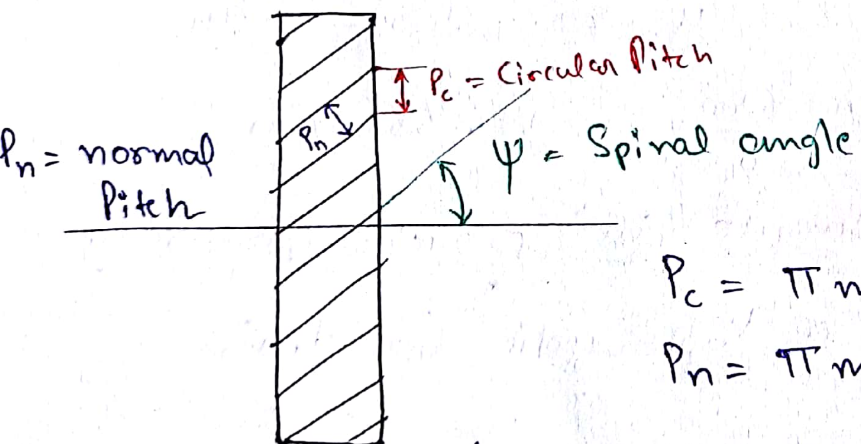
$T_{out} = -20$

$T_{out} + T_{inp.} + T_{Fixin} = 0$

$-20 + 50 + T_{Fixing} = 0$

$\Rightarrow T_{Fixing} = -30$

Terminology of Helical & Spiral Gear



$P_c = \pi m$

$P_n = \pi m_n$

m_n = normal module

- for two mating gears [Helical or Spiral] normal module is same.

$m_{n_1} = m_{n_2}$	
$P_{n_1} = P_{n_2}$	

$P_c \cos\psi = P_n$	
$m \cos\psi = m_n$	

θ = Angle b/w two shafts

1 → Driver

2 → Driven

$$\boxed{\theta = \psi_1 + \psi_2}$$ for same hand helical gear

$$\boxed{\theta = \psi_1 - \psi_2}$$ for opposite hand helical gear

Note :- In problems, if it is not mentioned about the hands of mating Gears, Then take same hand.

i.e. $\boxed{\theta = \psi_1 + \psi_2}$

Velocity Ratio (VR)

$$VR = \frac{\omega_2}{\omega_1}$$

$$V_{normal} = V_1 \cos\psi_1 = V_2 \cos\psi_2$$

$$VR = \frac{\omega_2}{\omega_1} = \frac{V_2/r_2}{V_1/r_1} = \frac{V_2 r_1}{V_1 r_2} = \frac{\cos\psi_1}{\cos\psi_2} \times \frac{\frac{m_1 T_1}{\cancel{2}}}{\frac{m_2 T_2}{\cancel{2}}}$$

$$VR = \frac{\cancel{\cos\psi_1} \times T_1 \times \cancel{m_n} \times \cancel{\cos\psi_2}}{\cancel{\cos\psi_2} \times T_2 \times \cancel{\cos\psi_1} \times \cancel{m_n}}$$

$$\boxed{VR = \frac{T_1}{T_2} = \frac{\omega_2}{\omega_1}}$$

Center Distance

$$\text{Center Distance} = r_1 + r_2$$

$$= \frac{m_1 T_1}{2} + \frac{m_2 T_2}{2}$$

$$= \frac{m_n T_1}{\cos\psi_1 \, 2} + \frac{m_n}{\cos\psi_2} \times \frac{T_2}{2}$$

$$\boxed{\text{Center Distance} = \frac{m_n}{2}\left[\frac{T_1}{\cos\psi_1} + \frac{T_2}{\cos\psi_2}\right]}$$

Efficiency (η)

$$\eta = \frac{P_{output}}{P_{input}} = \frac{F_2 V_2}{F_1 V_1}$$

$$\boxed{\eta = \frac{\cos(\psi_2 + \phi)}{\cos(\psi_1 - \phi)} \cdot \frac{\cos\psi_1}{\cos\psi_2}}$$

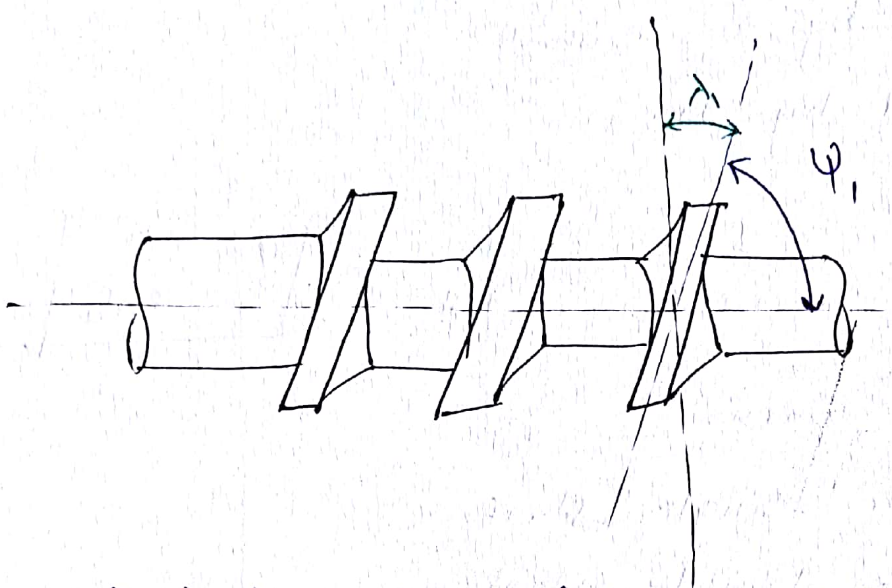
max. Efficiency

$$\boxed{\eta_{max} = \frac{1 + \cos(\theta + \phi)}{1 - \cos(\theta - \phi)}}$$

Here, $\theta = \psi_1 + \psi_2$

at $\psi_1 = \frac{\theta + \phi}{2}$; $\psi_2 = \frac{\theta - \phi}{2}$

Worm & worm wheel terminology



λ_1 = lead angle

$$\boxed{\lambda_1 + \psi_1 = 90^\circ}$$

$$\psi_1 = 90 - \lambda_1$$

- Angle b/w worm & worm wheel $= 90^\circ$ i.e. $\boxed{\theta = 90^\circ}$

$$\theta = \psi_1 + \psi_2$$

$$\psi_1 + \psi_2 = 90^\circ \Rightarrow \psi_1 = 90 - \psi_2$$

$$\psi_1 + \lambda_1 = 90^\circ \Rightarrow \boxed{\psi_2 = \lambda_1} \quad \boxed{\psi_1 = 90^\circ - \lambda_1}$$

- If coefficient of friction (μ) is given

$$\mu = \tan\phi \Rightarrow \boxed{\phi = \tan^{-1}\mu}$$

- Center Distance

$$= \frac{m_n}{2}\left[\frac{T_1}{\cos\psi_1} + \frac{T_2}{\cos\psi_2}\right] = \frac{m_2 \cancel{\cos\psi_2}}{2} \times \frac{1}{\cancel{\cos\psi_2}}\left[\frac{T_1\cos\psi_2}{\cos\psi_1} + T_2\right]$$

$$= \frac{m_2}{2}\left[\frac{T_1\cos\lambda_1}{\cos(90-\lambda_1)} + T_2\right]$$

$$\boxed{\text{Center Dist.} = \frac{m_2}{2} \times \left[T_1\cot\lambda_1 + T_2\right]}$$

- **Velocity Ratio (V.R.)**

$$VR = \frac{\text{Angle turned by worm wheel}}{\text{Angle turned by worm}}$$

$$VR = \frac{l/r_2}{2\pi} = \frac{l}{2\pi r_2} = \frac{l}{\pi d_2} \Rightarrow \boxed{VR = \frac{l}{\pi d_2}}$$

Here, l = lead of worm

$l = P$ for single start

$l = 2P$ for Double start

$l = 3P$ for tripple start.

- **Efficiency ($\theta = 90°$)**

$$\eta = \frac{\cos(\lambda_1 + \phi)}{\cos(90° - (\lambda_1 + \phi))} \times \frac{\cos(90 - \lambda_1)}{\cos \lambda_1}$$

$$\boxed{\eta = \frac{\tan(\lambda_1)}{\tan(\lambda_1 + \phi)}}$$

- **max. efficiency**

$$\boxed{\eta_{max.} = \frac{1 - \sin\phi}{1 + \sin\phi}}$$

CHAPTER-8

MECHANICAL VIBRATIONS

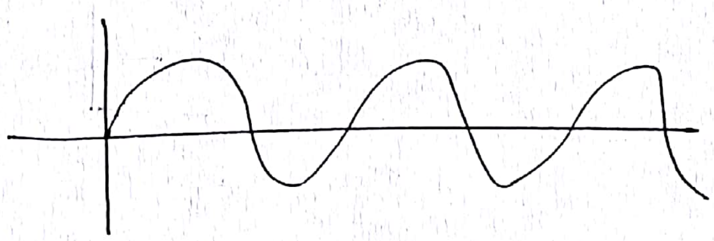
⇓

- To & fro Periodic motion about their equilibrium position.
- Harmonic motion
- Oscillations

- equilibrium Position
- mean Position
- Zero Position

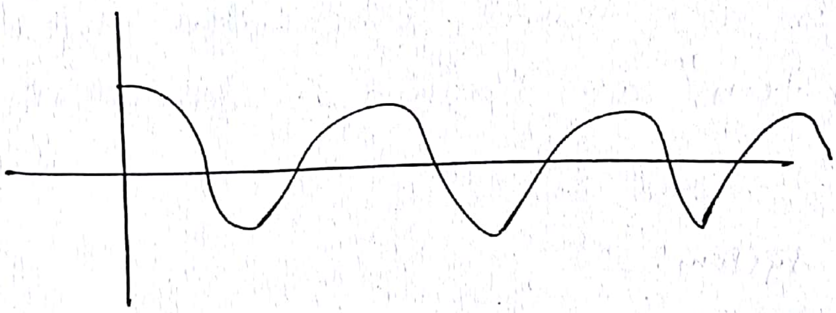
$\underbrace{\qquad\qquad\qquad}_{\text{All are same words}}$

Sine wave



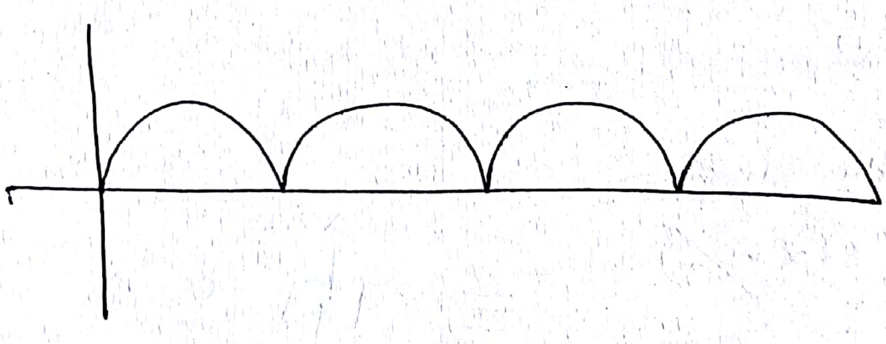
Vibrations

Cosine wave



Vibrations

$\sin^2$ wave



No Vibrations

Any vibrating system is a combination of

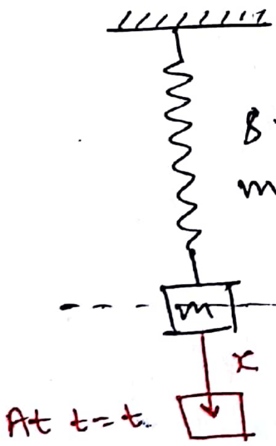
1. mass (m) ⇒ Kinetic Energy storing device
2. Stiffness (s)
   - Spring constant
   - P.E. storing device
3. Some energy loss (continously happening)
   - Kinetic friction $\neq 0$
   - Damping.
4. $F_{unbalanced} \neq 0$

1, 2, 3 — Vib in non-running m/c, here Vib. are introduced due to external disturbance given initially.

1, 2, 3, 4 — Vibration in Running machine

Natural Vibration

"The vibrations in which there is no kinetic friction at all as well as there is no external force after the initial release of the system." are known as natural vibrations.



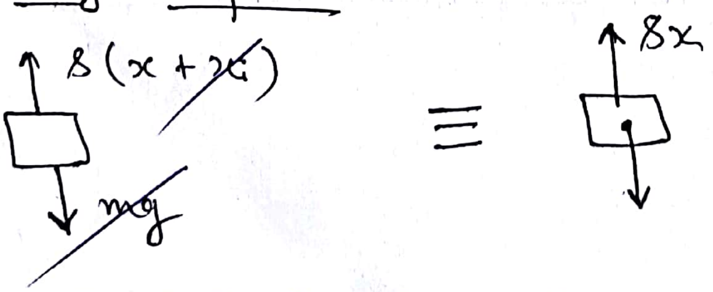
s = Stiffness of system
m = mass of system

mean (equil. position)

$x = f(t) = ??$

System free body diagram

At t = t

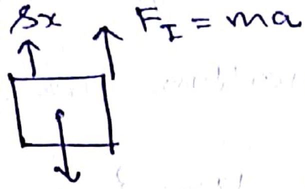


$0 - sx = ma$

$\boxed{sx + ma = 0}$

D'Alembert's Principle

System free body diagram



$$\boxed{sx + ma = 0}$$

$$ma + sx = 0$$

$$m\ddot{x} + sx = 0$$

$$\boxed{\ddot{x} + \left(\frac{s}{m}\right)x = 0}$$ ⇒ eqn of natural system

- The solution will be

$$\boxed{x = R\sin\left(\sqrt{\frac{s}{m}}\,t + \phi\right)}$$

R & ϕ are constant.

⇓

- Vibrations with frequency ⇒ $\boxed{\omega_n = \sqrt{\frac{s}{m}}}$ rad/s

- Amplitude = Const. = R

- Time period = $\boxed{T_n = \frac{2\pi}{\omega_n}}$ sec

- frequency = $\boxed{f_n = \frac{\omega_n}{2\pi}}$ Hz

R & ϕ are found by initial conditions:

- At $t=0$: $x = x_0$, $\dot{x} = 0$

- At $t=0$: $x = x_0$, $\dot{x} = V_0$

- At $t=0$: $x = 0$, $\dot{x} = V_0$

Finally eq^n of natural vibration is

$$\boxed{\ddot{x} + (\omega_n)^2 s = 0}$$

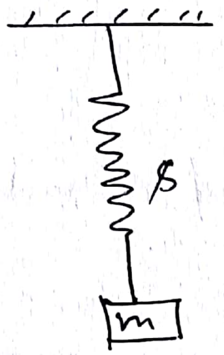
Ques: The natural vibration equation ~~are~~ is

$5\ddot{x} + 2s = 0$ find $\omega_n = ?$

Solu^n $\ddot{x} + \frac{2}{5}s = 0$

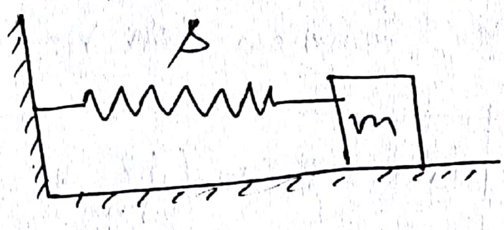
$\omega_n^2 = \frac{2}{5} \Rightarrow \omega_n = \sqrt{\frac{s}{m}} = \sqrt{\frac{2}{5}}$

Note : 1.



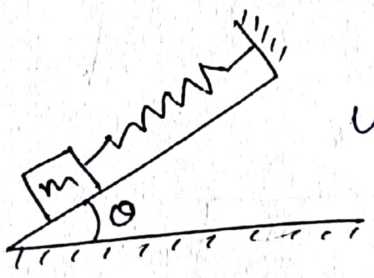
$\omega_n = \sqrt{\frac{s}{m}}$

2.



$\omega_n = \sqrt{\frac{s}{m}}$

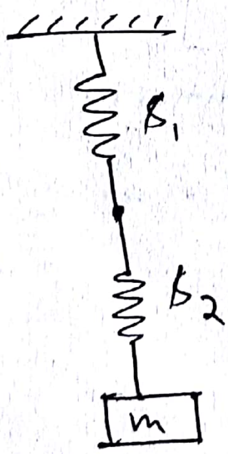
3.



$\omega_n = \sqrt{\frac{s}{m}}$

Combination of springs

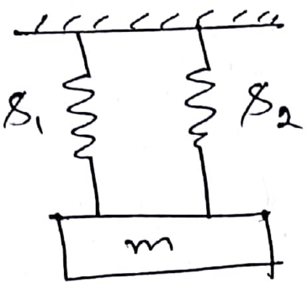
1. Series



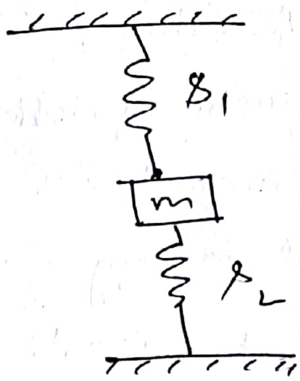
$\frac{1}{s} = \frac{1}{s_1} + \frac{1}{s_2}$

$\omega_n = \sqrt{\frac{s}{m}}$

2. Parallel



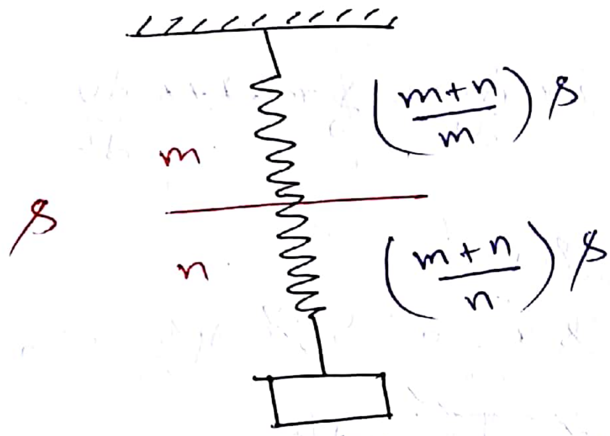
or



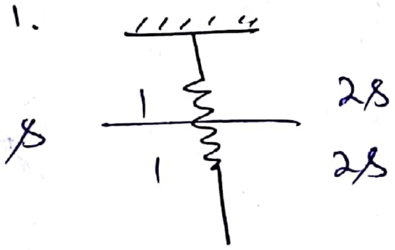
$$s = s_1 + s_2$$

$$\omega_n = \sqrt{\frac{s}{m}}$$

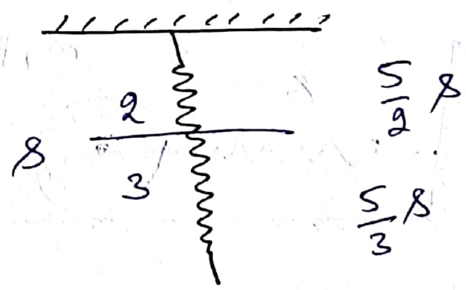
Cutting of springs



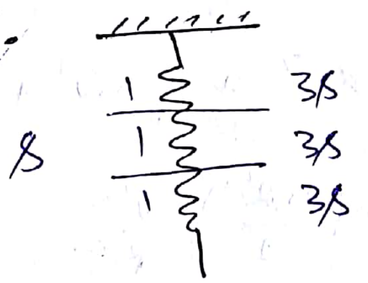
Ex :- 1.



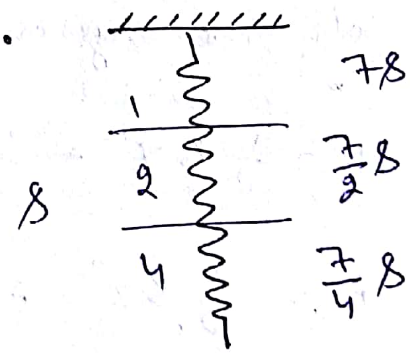
3.



2.



4.



Energy Method

In natural vibrations

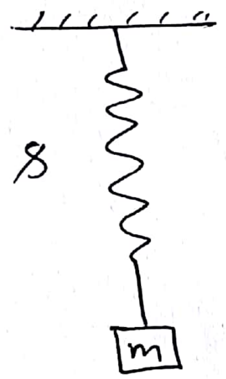
Kinetic friction $= 0$

Energy losses $= 0$

$\Rightarrow$ Total energy = Constant.

$E = \text{const.}$

$$\frac{dE}{dt} = 0$$



At $t = t$

System energy $= E = \frac{1}{2} s x^2 + \frac{1}{2} m v^2$

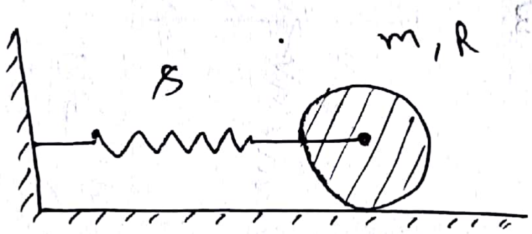
$$\frac{dE}{dt} = \cancel{\frac{1}{2}} s \times \cancel{2} x \cancel{\frac{dx}{dt}} + \cancel{\frac{1}{2}} m \times \cancel{2} \cancel{v} \times \frac{dv}{dt} = 0$$

$$ma + sx = 0$$

$$m\ddot{x} + sx = 0 \Rightarrow \ddot{x} + \left(\frac{s}{m}\right)x = 0$$

$$\omega_n = \sqrt{\frac{s}{m}} \text{ rad/s}$$

Ques:



At $t = t$

energy of the system $= E = \frac{1}{2} s x^2 + \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$

$$E = \frac{1}{2} s x^2 + \frac{1}{2} m v^2 + \frac{1}{2} \times \left(\frac{m\cancel{R^2}}{2}\right) \frac{v^2}{\cancel{R^2}}$$

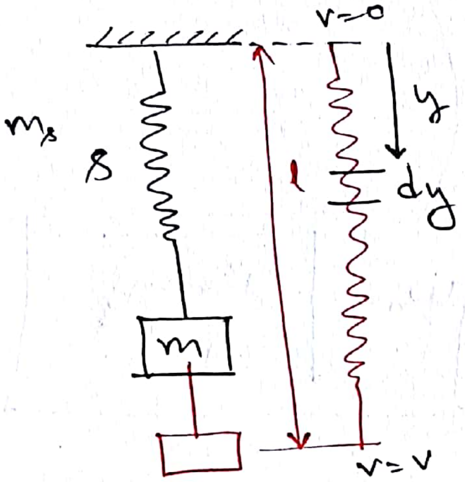
$$E = \frac{1}{2} s x^2 + \frac{1}{2}\left(\frac{3m}{2}\right) v^2$$

Shape	Value of I
Ring or hollow Cylinder	mR^2
Disc or Solid Cylinder	$\frac{mR^2}{2}$
hollow Sphere	$\frac{2}{3} mR^2$
Solid sphere	$\frac{2}{5} mR^2$

On comparing

$$\boxed{\omega_n = \sqrt{\frac{s}{3m/2}}} \text{ rad/s}$$

Spring mass System [Spring is also having mass]



$$KE_{Spring} = \int_0^l \frac{1}{2}\left[\left(\frac{m_s}{l}\right) dy\right] \times \left[\frac{y}{l} \cdot v\right]^2$$

$$= \frac{1}{6} m_s v^2$$

At $t = t$

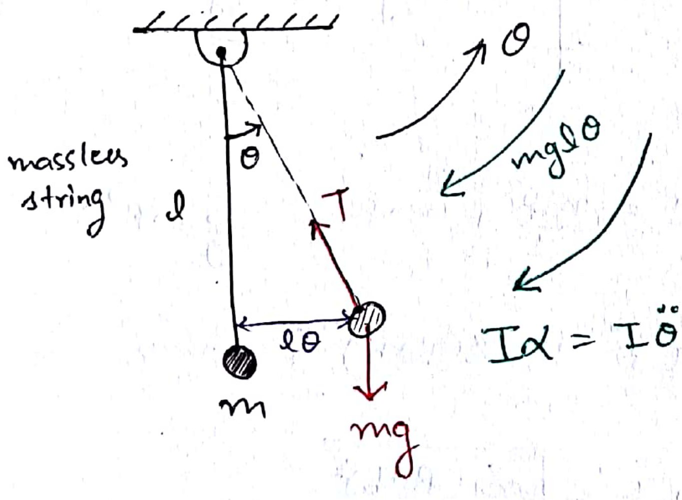
$$E = \frac{1}{2} s x^2 + \frac{1}{2} m v^2 + \frac{1}{6} m_s v^2$$

$$E = \frac{1}{2} s x^2 + \frac{1}{2}\left(m + \frac{m_s}{3}\right) v^2$$

$$\Rightarrow \boxed{\omega_n = \sqrt{\frac{s}{m + \frac{m_s}{3}}}} \text{ rad/s}$$

Torque method

[For small oscillations]



$$I\alpha = I\ddot{\theta}$$

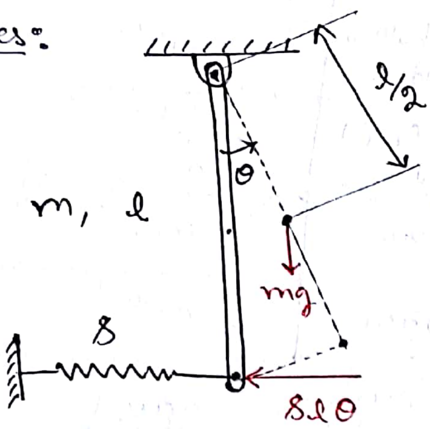
$$I\alpha + mgl\theta = 0$$

$$\cancel{m}l^{\cancel{2}}\ddot{\theta} + \cancel{m}g\cancel{l}\theta = 0$$

$$\ddot{\theta} + \left(\frac{g}{l}\right)\theta = 0$$

$$\boxed{\omega_n = \sqrt{\frac{g}{l}}} \text{ rad/s}$$

Ques:



θ

$mg\frac{l}{2}\theta$

$sl^2\theta$

$I\ddot{\theta}$

$$I = \frac{ml^2}{12} + m\left(\frac{l}{2}\right)^2$$

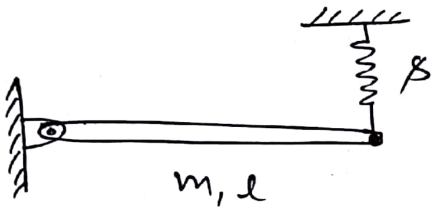
$$I = \frac{ml^2}{3}$$

$$I\ddot{\theta} + sl^2\theta + mg\frac{l}{2}\theta = 0$$

$$\ddot{\theta} + \left(\frac{sl^2 + mg\frac{l}{2}}{I}\right)\theta = 0$$

$$\boxed{\omega_n = \sqrt{\frac{sl^2 + mg\frac{l}{2}}{I}}}$$

Note:- Horizontal System

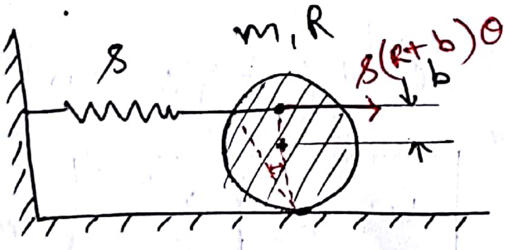


mg torque is cancelled by (sx_i) torque

Therefore mg will not be considered

$$\omega_n = \sqrt{\frac{s\cancel{l^2}}{m\cancel{l^2}/3}} \Rightarrow \boxed{\omega_n = \sqrt{\frac{3s}{m}}}$$

Ques:



θ

$s(R+b)^2\theta$

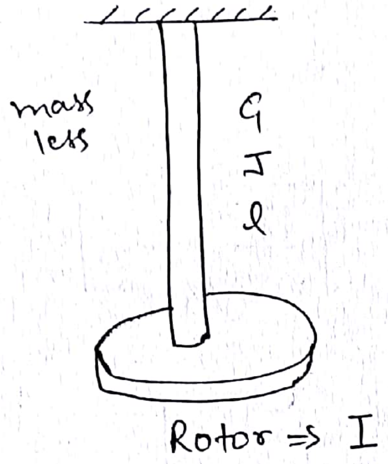
$I\ddot{\theta}$

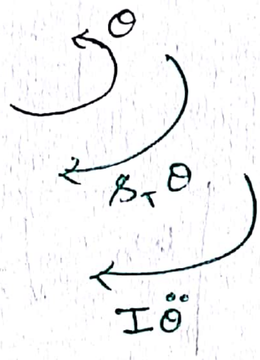
$$I = \frac{mR^2}{2} + mR^2 = \frac{3}{2}mR^2$$

$$I\ddot{\theta} + s(R+b)^2\theta = 0$$

$$\omega_n = \sqrt{\frac{s(R+b)^2}{\frac{3}{2}mR^2}}$$

Torsional vibrations:





Torsional stiffness = $s_T = \frac{GJ}{l}$

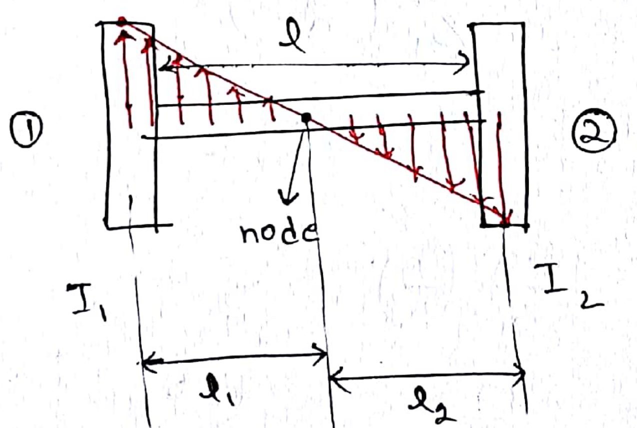
$$I\ddot{\theta} + s_T\theta = 0$$

$$\boxed{\omega_n = \sqrt{\frac{s_T}{I}}}$$

Note: If rod is also having mass

$$\omega_n = \sqrt{\frac{s_T}{I + \frac{I_r}{3}}}$$

Two rotor system



$$l_1 + l_2 = l$$

$$\sqrt{\frac{s_{T_1}}{I_1}} = \sqrt{\frac{s_{T_2}}{I_2}}$$

$$\frac{GJ}{l_1 I_1} = \frac{GJ}{l_2 I_2}$$

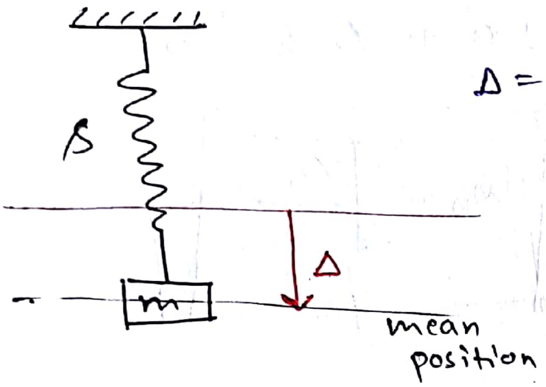
$$\Rightarrow \boxed{l_1 I_1 = l_2 I_2}$$

\# If no. of rotor = n
then,
no. of node point = n-1

Method of static deflection of mass (Δ)

* Rayleigh's method

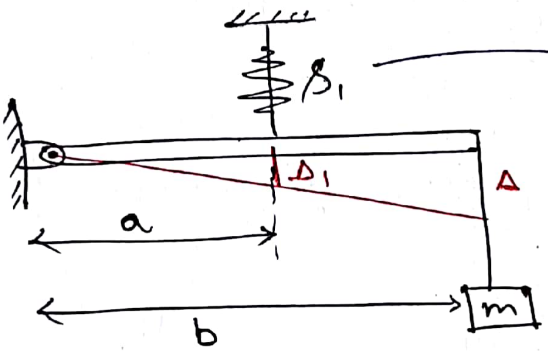
Basic spring mass system



$$\Delta = \frac{mg}{s} \quad ; \quad \sqrt{\frac{g}{\Delta}} = \sqrt{\frac{g}{mg/s}} = \sqrt{\frac{s}{m}}$$

$$\boxed{\omega_n = \sqrt{\frac{g}{\Delta}} = \sqrt{\frac{s}{m}}}$$

Ques: 1.



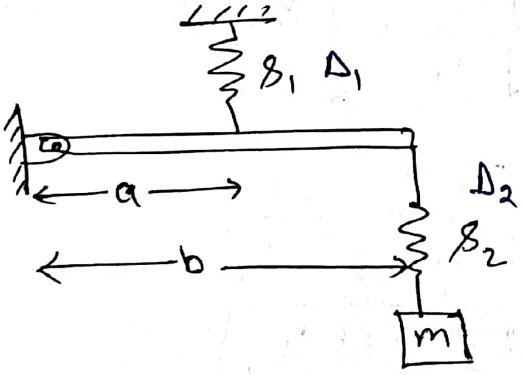
$$\Delta_1 = \frac{mg}{s_1}\left(\frac{b}{a}\right)$$

$$\left[\because mgb = F \times a\right]$$

$$\Delta = \frac{mg}{s_1}\left(\frac{b^2}{a^2}\right)$$

$$\omega_n = \sqrt{\frac{g}{\Delta}}$$

Q2



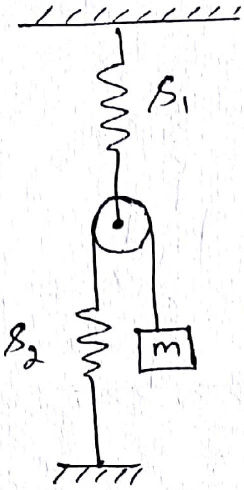
$$\left.\begin{aligned} \Delta_1 &= \frac{mg}{s_1}\left(\frac{b}{a}\right) \\ \Delta_2 &= \frac{mg}{s_2} \\ \Delta &= \Delta_1 \times \frac{b}{a} + \Delta_2 \end{aligned}\right\} \omega_n = \sqrt{\frac{g}{\Delta}}$$

Q3

$$\Delta_1 = \frac{2mg}{s_1} \quad ; \quad \Delta = 2\Delta_1$$

$$\omega_n = \sqrt{\frac{g}{\Delta}}$$

Q4



$$\Delta_1 = \frac{2mg}{s_1}$$

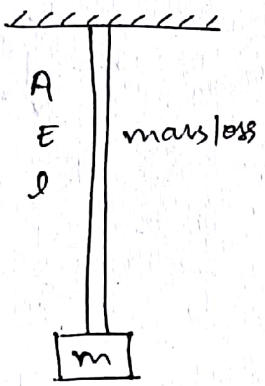
$$\omega_n = \sqrt{\frac{g}{\Delta}}$$

$$\Delta = 2\Delta_1 + \Delta_2$$

$$\frac{mg}{s_2} = \Delta_2$$

Longitudinal vibration of Beams : [vib. parallel to length of Beam]

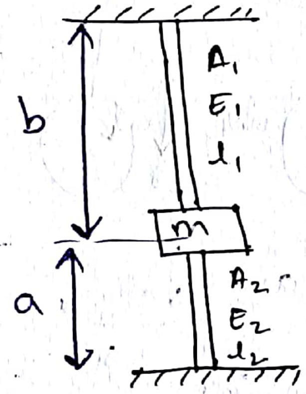
1.



$$\text{longitudinal stiffness} = s = \frac{AE}{l}$$

$$\omega_n = \sqrt{\frac{s}{m}}$$

2.



$$s_1 = \frac{A_1 E_1}{l_1}$$

$$s_2 = \frac{A_2 E_2}{l_2}$$

$$s = s_1 + s_2$$

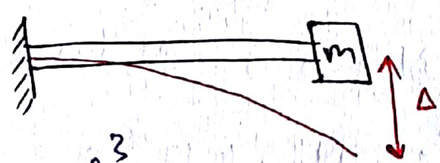
$$\omega_n = \sqrt{\frac{s}{m}}$$

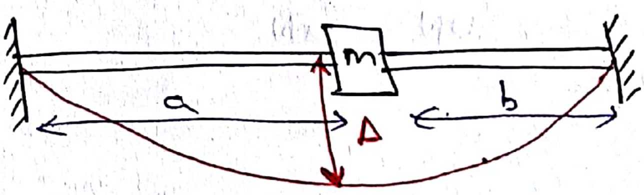
For lateral

$$\Delta = \frac{w a^3 b^3}{3EI\, l^3}$$

↳ At the point of loading

Transverse Vibration of Beams [vib. ⊥ar to length of Beam]





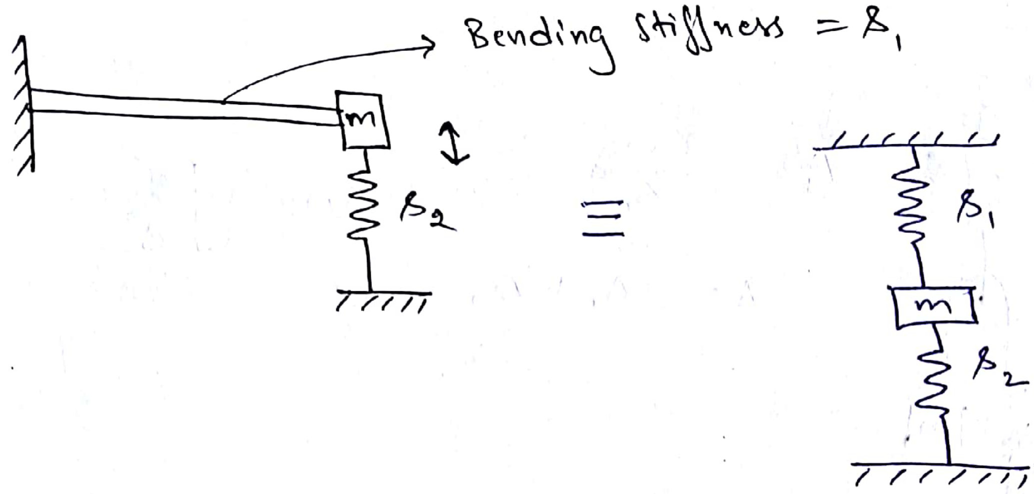
$$\Delta = \frac{wl^3}{3EI}$$

Δ = will be calculated by formulas in SOM

$$\Delta = \frac{w a^3 b^3}{3EI\, l^3}$$

$$\omega_n = \sqrt{\frac{g}{\Delta}}$$

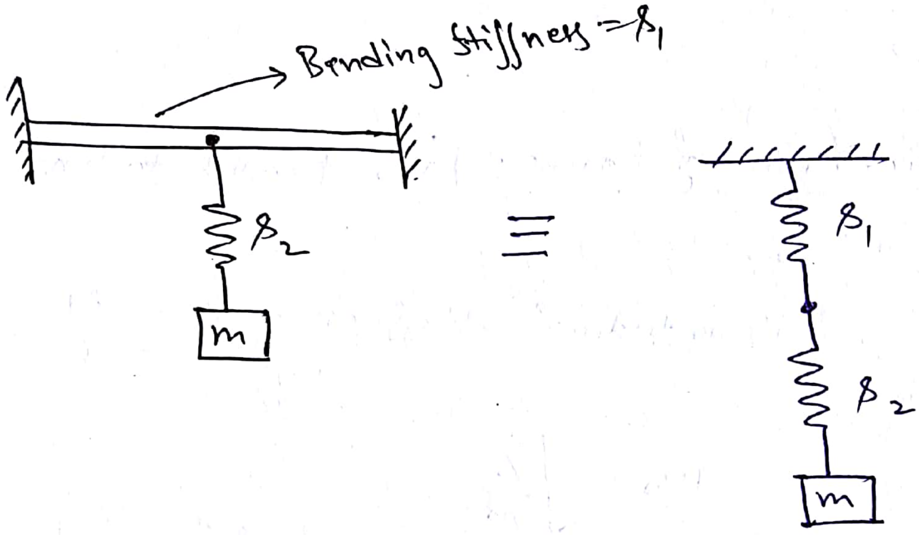
Q1:



$s = s_1 + s_2$

$\omega_n = \sqrt{\frac{s}{m}}$

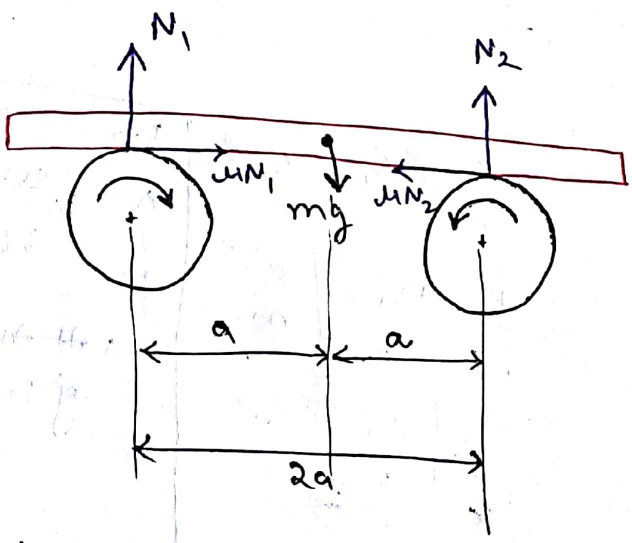
Q2:

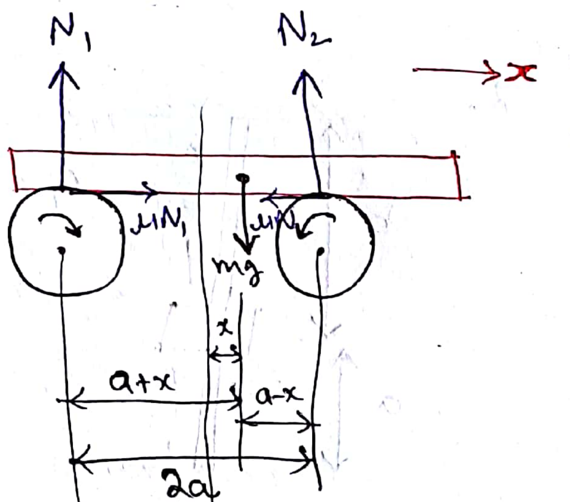


$\frac{1}{s} = \frac{1}{s_1} + \frac{1}{s_2}$

$\omega_n = \sqrt{\frac{s}{m}}$

Q3:
IES 2016





$N_1 = N_2 = \frac{mg}{2}$

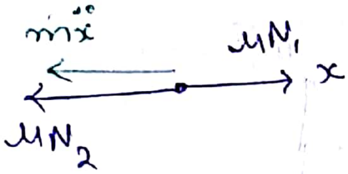
$\mu N_1 = \mu N_2$

$N_1 + N_2 = mg$

$N_1(a+x) = N_2(a-x)$

$\Rightarrow N_1 = mg\frac{(a-x)}{2a}$

$N_2 = mg\left(\frac{a+x}{2a}\right)$



$$m\ddot{x} + \mu N_2 - \mu N_1 = 0$$

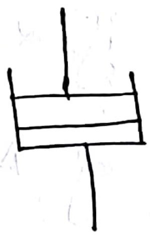
$$m\ddot{x} + \mu\left(\frac{mg}{2a}\right)\times(\cancel{a}+x-\cancel{a}+x) = 0$$

$$\ddot{x} + \left(\frac{\mu g}{a}\right)x = 0 \Leftarrow \cancel{m}\ddot{x} + \frac{\mu \cancel{m} g}{a}x = 0$$

$$\omega_n = \sqrt{\frac{\mu g}{a}}$$

Damping System
[Kinetic friction $\neq 0$]

- Technical name of Kinetic friction in any Vibrating System ⇒ Damping
- Damping is represented by the below symbol



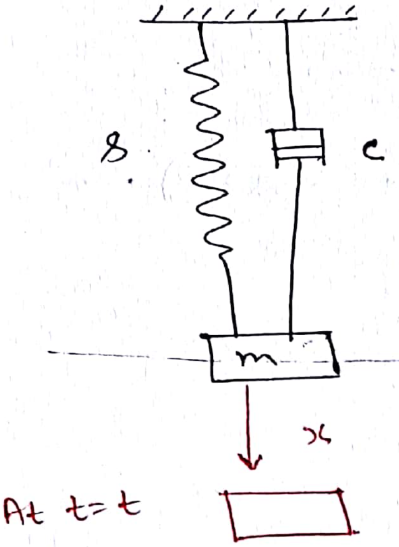
Damping in any System

- (Coulamb damping) Friction b/w Dry Surfaces [very high]
- Viscous Damping [very less]

- Damping force $\propto \dot{x}$

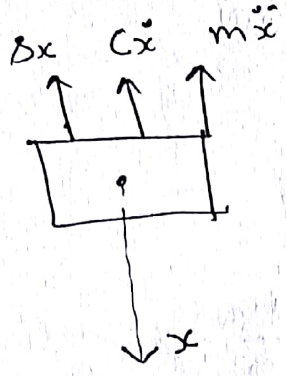
$$= \underline{\underline{c}}\,\dot{x}$$

c = coefficient of Damping [const. for a system]



mean
(eqᵃⁿ position)

$x = f(t) = ??$



$$m\ddot{x} + c\dot{x} + sx = 0$$

$$\ddot{x} + \left(\frac{c}{m}\right)\dot{x} + \left(\frac{s}{m}\right)x = 0$$

$$\boxed{\ddot{x} + \left(\frac{c}{m}\right)\dot{x} + (\omega_n)^2 x = 0}$$

equation of damped system

The solun of above equation is

$x = Ae^{\alpha_1 t} + Be^{\alpha_2 t}$ if $\alpha_1 \neq \alpha_2$

$x = (A + Bt)e^{\alpha t}$ if $\alpha_1 = \alpha_2 = \alpha$

where α_1 & α_2 are roots of Auxillary equation

$$\alpha^2 + \left(\frac{c}{m}\right)\alpha + (\omega_n)^2 = 0$$

$$\alpha_{1,2} = \frac{-\left(\frac{c}{m}\right) \pm \sqrt{\left(\frac{c}{m}\right)^2 - 4\omega_n^2}}{2}$$

$$\boxed{\alpha_{1,2} = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \omega_n^2}}$$

$$\frac{\left(\frac{c}{2m}\right)^2}{\omega_n^2} = \text{Degree of Dampness}$$

$$\sqrt{\frac{(c/2m)^2}{(\omega_n)^2}} = \zeta \longrightarrow \text{Damping factor / Damping ratio}$$

$$\zeta = \sqrt{\frac{c^2/2m^2 \times 2}{s/m}} = \frac{c}{2\sqrt{sm}}$$

$$2\zeta\omega_n = \cancel{2} \times \frac{c}{\cancel{2}\sqrt{\cancel{s}m}} \times \sqrt{\frac{\cancel{s}}{m}} = \frac{c}{m}$$

$$\boxed{2\zeta\omega_n = \frac{c}{m}}$$

<u>Finally</u> equation of Damped System

$$\boxed{\ddot{x} + (2\zeta\omega_n)\dot{x} + (\omega_n)^2 x = 0}$$

The solu^n is $x = Ae^{\alpha_1 t} + Be^{\alpha_2 t}$ if $\alpha_1 \neq \alpha_2$

$x = (A + Bt)e^{\alpha t}$ if $\alpha_1 = \alpha_2 = \alpha$

where $\boxed{\alpha_{1,2} = \left(-\zeta \pm \sqrt{\zeta^2 - 1}\right)\omega_n}$

- If $\zeta > 1$ $\Rightarrow$ over damped System
  [over Damping]
  [coulomb's damping]

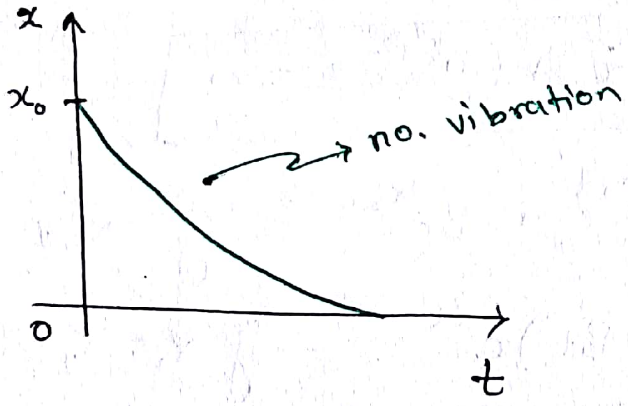
- If $\zeta = 1$ $\Rightarrow$ Critically Damped System
  [Critical Damping]

- If $\zeta < 1$ $\Rightarrow$ Under Damped Systems
  [under Damping]
  [Viscous Damping]

1. Over Damped Systems ($\zeta > 1$)

$$x = Ae^{\alpha_1 t} + Be^{\alpha_2 t}$$

$$x = Ae^{(-\zeta + \sqrt{\zeta^2-1})\omega_n t} + Be^{(-\zeta - \sqrt{\zeta^2-1})\omega_n t}$$

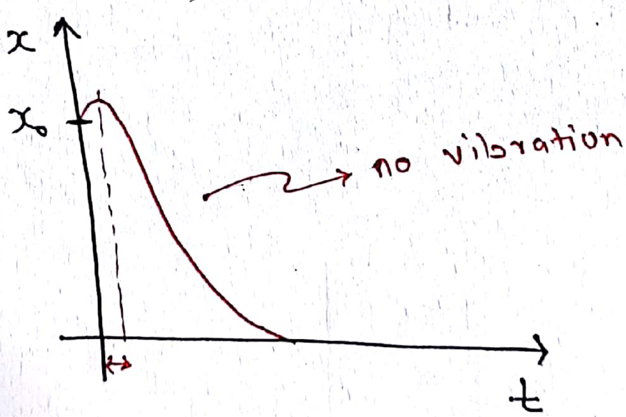


2. Critically Damped system ($\zeta = 1$)

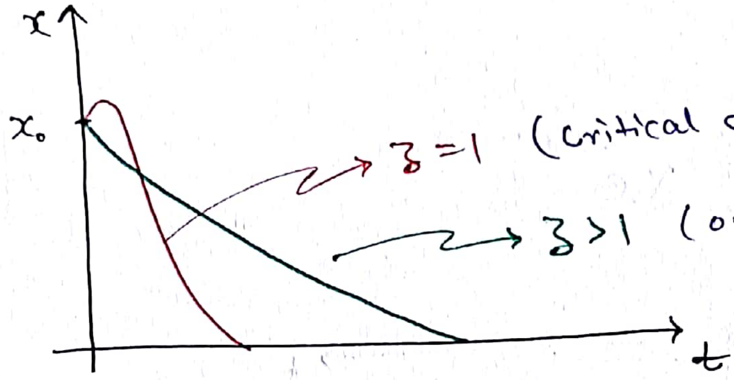
$\alpha_1 = \alpha_2 = \alpha = -\omega_n$

The solu^n will be

$$x = (A + Bt)e^{-\omega_n t}$$



Note :- Critical Damping Response is much fast than over Damping.



eg :- Door closer $\Rightarrow$ over damping

- A.K. 47 $\Rightarrow$ critically damped

3. Under damping ($\zeta < 1$)

$$\alpha_{1,2} = -\zeta\omega_n \pm i \underbrace{\sqrt{1-\zeta^2}\,.\,\omega_n}_{\omega_d}$$

$$\alpha_{1,2} = -\zeta\omega_n \pm i\,\omega_d$$

The soln is

$$x = A e^{\alpha_1 t} + B e^{\alpha_2 t}$$

$$= A e^{(-\zeta\omega_n + i\omega_d)t} + B e^{(-\zeta\omega_n - i\omega_d)t}$$

$$= e^{-\zeta\omega_n t}\left[\underbrace{(A+B)}_{X\sin\phi}\cos(\omega_d t) + i\underbrace{(A-B)}_{X\cos\phi}\sin(\omega_d t)\right]$$

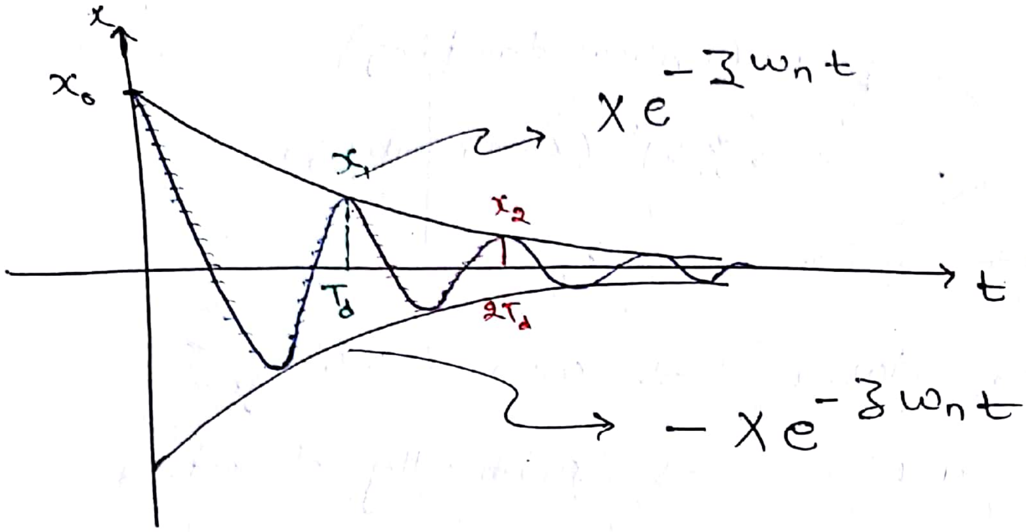
$$x = \underbrace{X e^{-\zeta\omega_n t}}_{\Downarrow}\left[\underbrace{\sin}_{\Downarrow}[(\omega_d t) + \phi]\right]$$

Decreasing Amplitude with time

Vibration with frequency ω_d

$$\omega_d = \sqrt{1-\zeta^2}\,.\,\omega_n$$

$$T_d = \frac{2\pi}{\omega_d}$$



At $t=0$

$$x_0 = Xe^{-\zeta\omega_n t} \times \sin(\omega_d t + \phi)$$

$$x_0 = X\sin\phi$$

At $t = T_d$

$$x_1 = Xe^{-\zeta\omega_n T_d}\,.\,\sin\phi$$

At $t = 2T_d$

$$x_2 = Xe^{-\zeta\omega_n 2T_d}\,.\,\sin\phi$$

Decrement ratio

$$\frac{x_0}{x_1} = \frac{x_1}{x_2} = \frac{x_3}{x_4} \text{ ------ } = e^{\zeta\omega_n T_d} = \text{const.}$$

Logarithmic decrement (δ)

$$\delta = \ln e^{\zeta\omega_n T_d} = \frac{\zeta\omega_n\,2\pi}{\sqrt{1-\zeta^2}\,.\,\omega_n} \Rightarrow \boxed{\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}}$$

Critical damping coefficient (C_c)

$$\frac{\cancel{2}\,\zeta\,\cancel{\omega_n} = c}{\cancel{2}\times 1\times \cancel{\omega_n} = C_c} \Rightarrow \boxed{\zeta = \frac{c}{C_c}} = \frac{\text{Actual Damping coefficient}}{\text{Critical Damping coefficient}}$$

WB P-45 Q55

$m = 7.5$ Kg

$T_d = \frac{35}{60}$ s

$\omega_d = \cancel{\frac{T_d}{2\pi}} \cdot \frac{2\pi}{T_d} = 10.77$

$\frac{x_1}{x_7} = 2.5$

$$\frac{x_1}{x_2}\cdot\frac{x_2}{x_3}\cdot\frac{x_3}{x_4}\cdot\frac{x_4}{x_5}\cdot\frac{x_5}{x_6}\cdot\frac{x_6}{x_7} = 2.5$$

$e^{6\delta} = 2.5$

$\Rightarrow \delta = \frac{\ln 2.5}{6} \Rightarrow \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} = \frac{\ln 2.5}{6}$

$(41.14)^2\zeta^2 = 1-\zeta^2$

$\zeta = \cancel{\ldots}\ 0.02429$

$2\zeta\omega_n = \frac{c}{m}$

$\Rightarrow c = \cancel{0.015}\ 3.925$

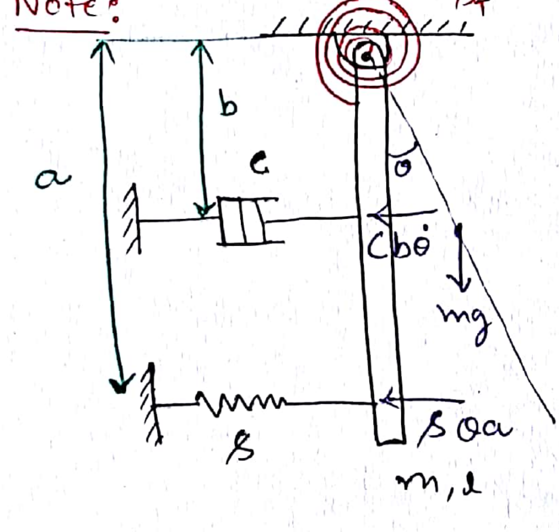
$\omega_d = \sqrt{1-\zeta^2}\cdot\omega_n \Rightarrow \omega_n = 10.77$

$\omega_n = \sqrt{\frac{\delta}{m}} \Rightarrow \delta = 869.94$

$\zeta = \frac{c}{C_c} \Rightarrow C_c = 161.0169$

$$\boxed{\begin{gathered}\delta = 0.1527 = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}\\ 1-\zeta^2 = \left(\frac{2\pi}{0.1527}\zeta\right)^2\\ \zeta = 0.02429\end{gathered}}$$

Note:



θ

$mg\frac{l}{2}\theta$

$Cb^2\dot{\theta}$

$\delta a^2\theta$

$\delta_T\theta$

$I\ddot{\theta}$

$I = \frac{ml^2}{3}$

$$I\ddot{\theta} + Cb^2\dot{\theta} + \left(\frac{mgl}{2} + \delta a^2 + \delta_T\right)\theta = $$

eqn of Damped

$m\ddot{x} + c\dot{x} + \delta x = 0$

Q1 The Damping coefficient in Vib. eqn will be

$$= Cb^2$$

Q2 $\omega_n = ?$

$$\omega_n = \sqrt{\frac{\frac{mgl}{2} + k_T + k a^2}{I}}$$

Q3 $\zeta = ?$

$$2\zeta\omega_n = \frac{Cb^2}{I} \Rightarrow \zeta = \frac{Cb^2}{2\omega_n I}$$

Q4 $C_c = ?$

$$C_c b^2 = 2\omega_n I$$

Q5 $\omega_d = ?$

$$\omega_d = \sqrt{1-\zeta^2} \cdot \omega_n$$

Note: when system becomes horizontal mg torque will not be considered.

Vibrations caused by unbalanced forces in running mechanical system

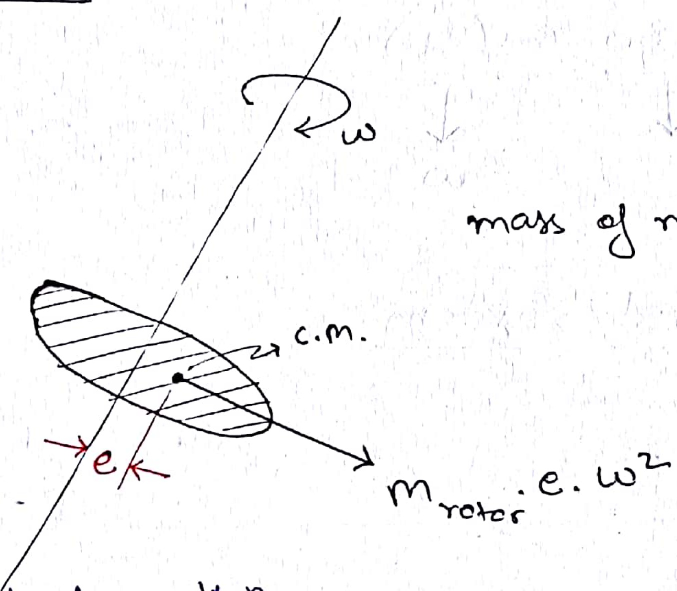
m = mass which is under vibration

or

machine mass

Analysis will be done on complete m/c mass

1. Rotating unbalance



mass of rotor = m_{rotor}

At $t = t$

F_{un} in a particular dir^n

$$F_{un} = m_{rotor}.e.\omega^2 \sin\theta$$

$$F_{un} = (m_{rotor}.e.\omega^2)\sin(\omega t) = F_0 \sin(\omega t)$$

F_0 = max. value

ω = forced frequency.

2. Reciprocating unbalance:

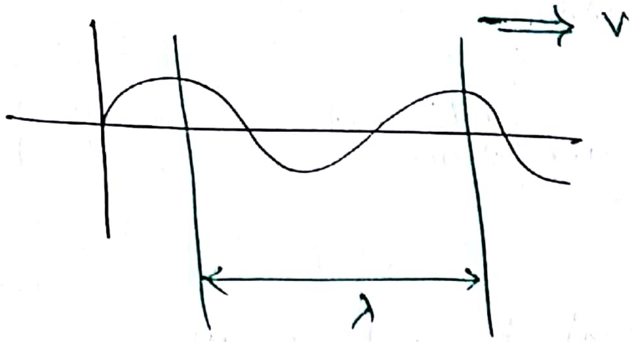
$$F_{un} = m_{reci.} \times r \times \omega^2 \sin \omega t = F_0 \sin \omega t$$

r = crank radius

F_0 = max. value

ω = forced frequency (crank speed)

3. wave form



$F_0 = 200\,N$ [given]

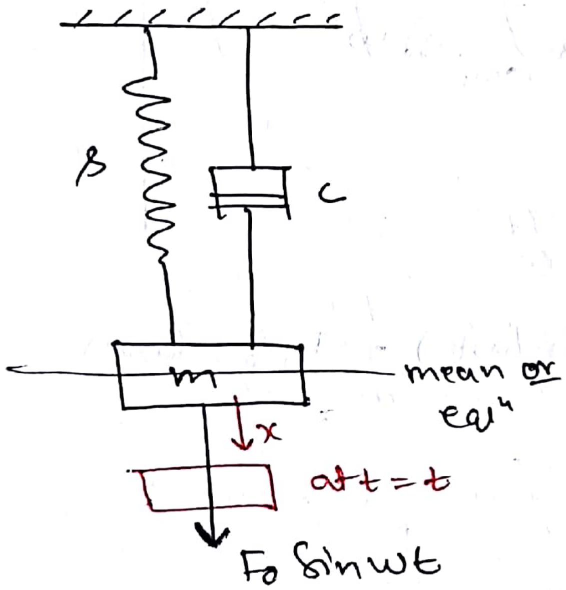
$f = \frac{V}{\lambda}$

$\omega = 2\pi f$

4. Direct form

$$F_{un} = \underbrace{200}_{F_0} \cos(\underbrace{3}_{\omega}t)$$

Forced damped System

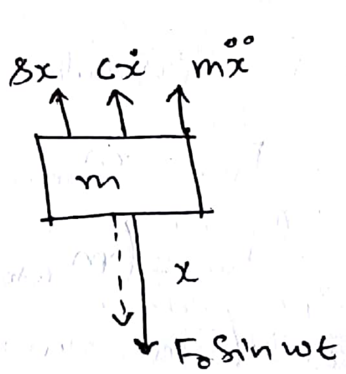


F_0 = max. value

ω = forced frequency

At t = t

F.B.D.



$$m\ddot{x} + c\dot{x} + sx - F_0 \sin(\omega t) = 0$$

$$\boxed{\ddot{x} + (2\zeta\omega_n)\dot{x} + (\omega_n)^2 x = \frac{F_0}{m}\sin(\omega t)}$$

equation of forced damped system.

The solu^n of above equ^n

$x = CF + PI$

$$CF \begin{cases} \zeta > 1 \\ \zeta = 1 \end{cases} \text{no vibrations}$$

$$\zeta < 1 \;] \text{ vibrations}$$

After some time $CF = 0$

then, $x = PI$

$$PI = \frac{\frac{F_0}{m}\sin(\omega t)}{D^2 + (2\zeta\omega_n)D + \omega_n^2}$$

$$= \frac{\frac{F_0}{m}\sin(\omega t)}{(\omega_n^2 - \omega^2) + (2\zeta\omega_n)D} \times \left[\frac{(\omega_n^2 - \omega) - (2\zeta\omega_n)D}{(\omega_n^2 - \omega) - (2\zeta\omega_n)D}\right]$$

$$= \frac{\frac{F_0}{m}\left[\overbrace{(\omega_n^2 - \omega^2)}^{R\cos\phi}.\sin(\omega t) - \overbrace{(2\zeta\omega\omega_n)}^{R\sin\phi}\cos(\omega t)\right]}{(\omega_n^2 - \omega^2) + (2\zeta\omega\omega_n)^2}$$

$$= \frac{F_0/m \, \cancel{R} \sin(\omega t - \phi)}{R^{\cancel{2}}}$$

$$\boxed{PI = \frac{F_0/s}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2}} \times \underline{\sin}(\omega t - \phi)}$$

⇓

Amplitude

⇓

Independent of time

Vibration with forced frequency (ω) (speed)

After some time CF → 0

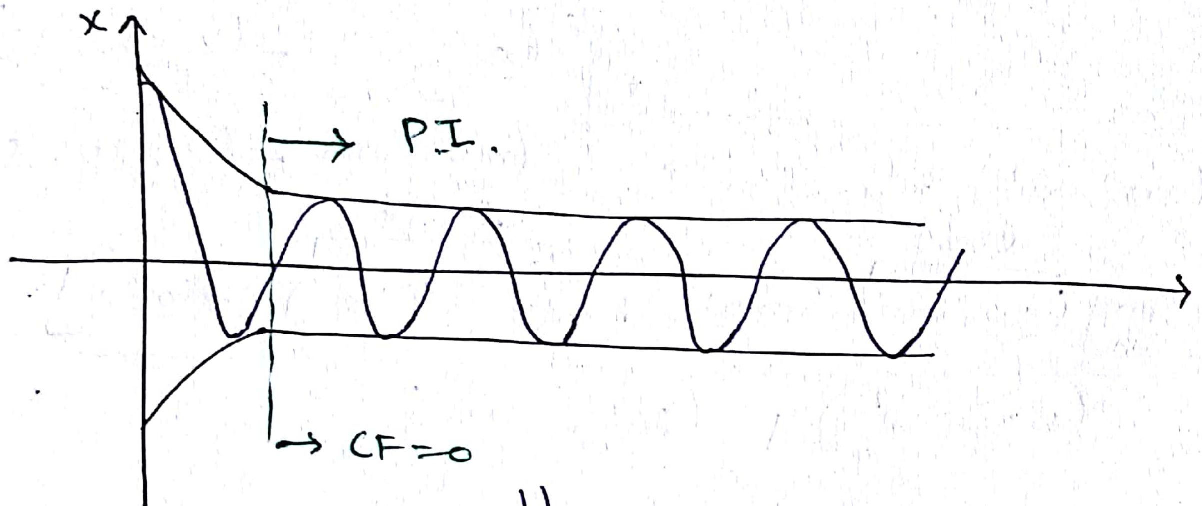
$x = PI$

$$\boxed{x = A \sin(\omega t - \phi)}$$

$$A = \frac{F_0/s}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2}}$$

→ Amplitude of steady-state Vibration (forced vib.)

- Vibration in running system will never stop.



⇓

To stop fatigue, Every mechanical running system must have one running life.

Magnification factor (M.F)

$$M.F = \frac{A}{F_0/s}$$

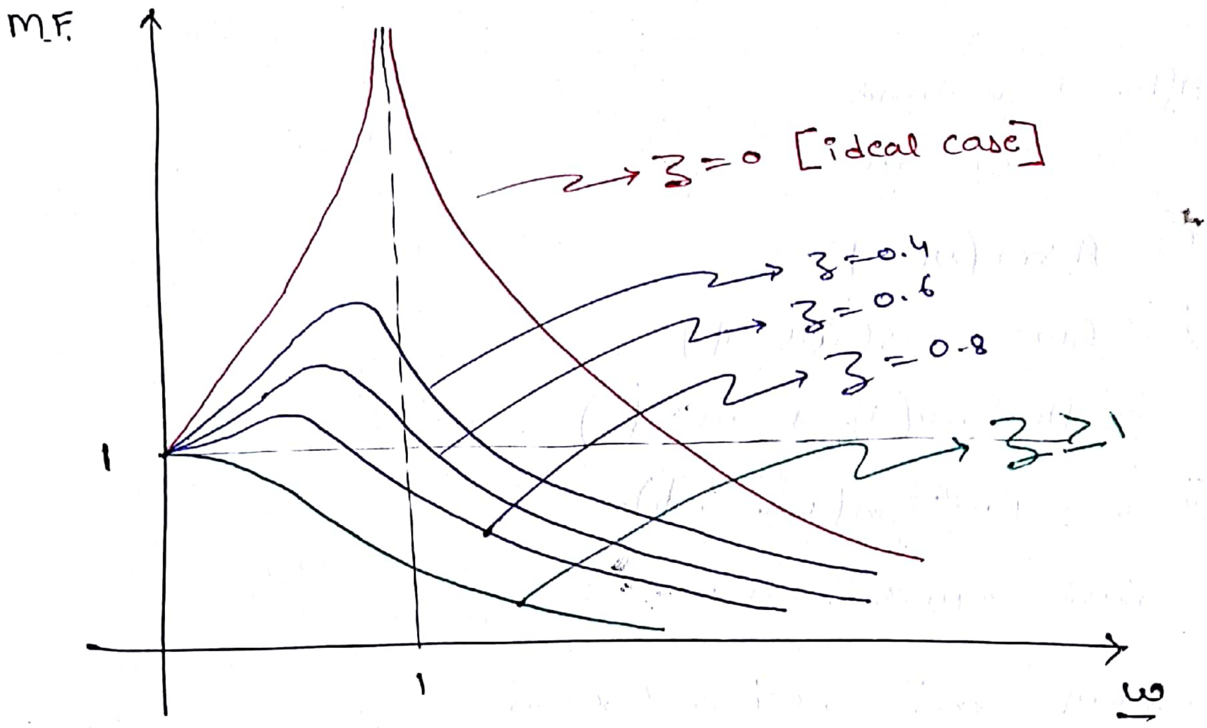
⇓

Strength of A

$$\boxed{MF = \frac{1}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2}}}$$

⇒ Depends on

1. $\frac{\omega}{\omega_n}$
2. ζ



$\frac{\omega}{\omega_n} = 1$

[Resonance]

1. underdamping ↑

$\Rightarrow \zeta \downarrow \Rightarrow MF \uparrow \Longrightarrow A \uparrow \Longrightarrow$ Running life ↓

2. $A_{max.}$ at [MF_{max} at]

at $\frac{\omega}{\omega_n} < 1$ underdamping [viscous damping]

at $\frac{\omega}{\omega_n} = 1$ no damping

at $\frac{\omega}{\omega_n} = 0$ over/critical damping [colombs damping]

3. $A_{Resonance}$

at resonance $\frac{\omega}{\omega_n} = 1$

so, $\boxed{A_{Resonance} = \frac{F_0/s}{2\zeta}} \propto \frac{1}{\zeta}$

Note:

After some time

$CF \to 0$

$$x = A \sin(\omega t - \phi)$$

$$\dot{x} = A\omega \cos(\omega t - \phi)$$

$$= A\omega \sin(\pi/2 + \omega t - \phi)$$

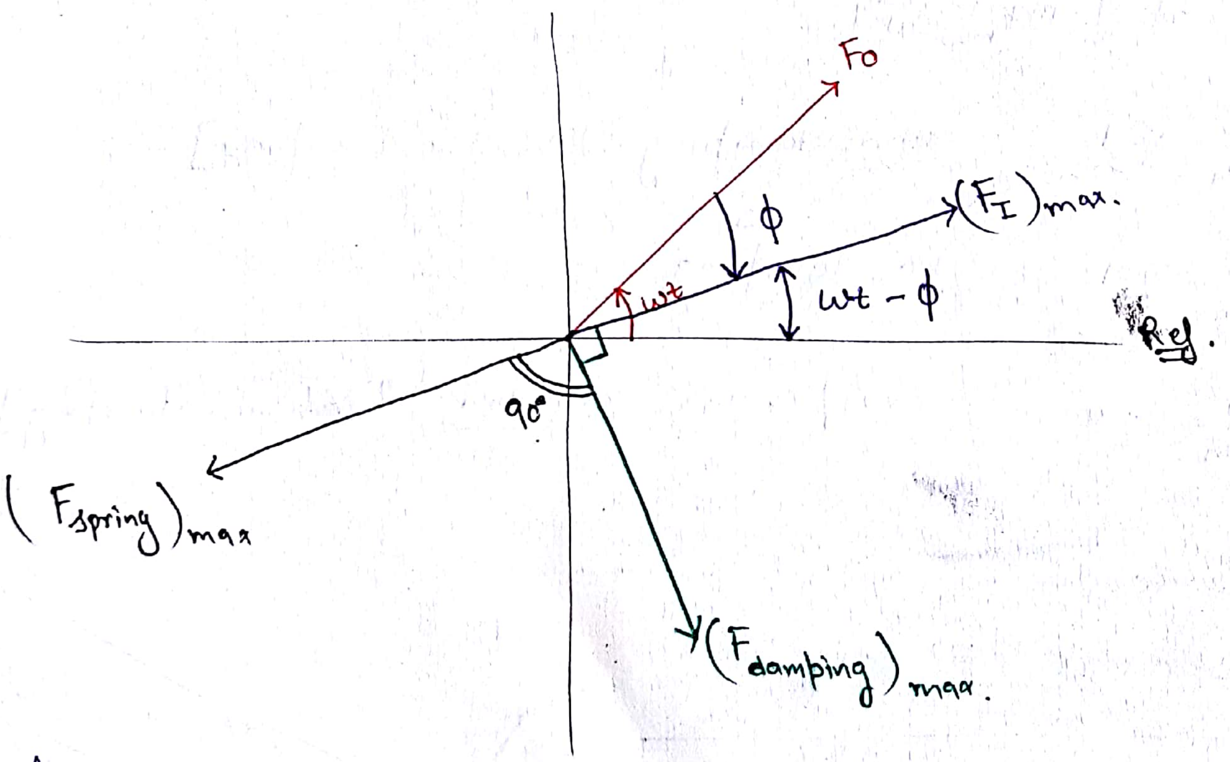
$$\ddot{x} = -A\omega^2 \sin(\omega t - \phi)$$

The Basic equation was

$$F_0 \sin\omega t - m\ddot{x} - c\dot{x} - s x = 0$$

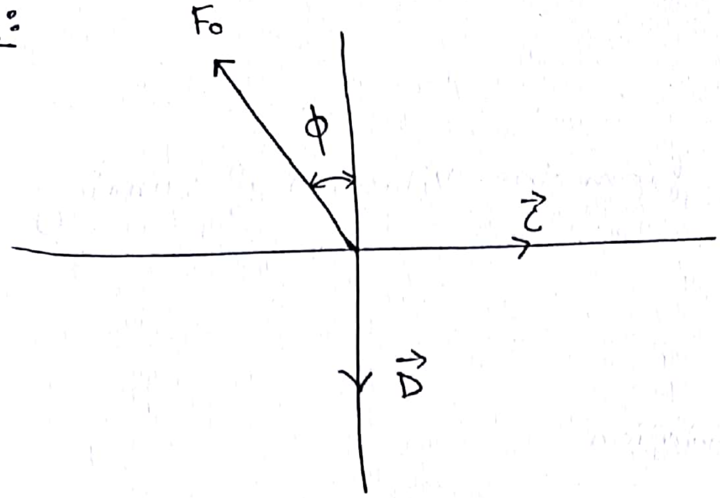
$$\underbrace{F_0}_{(F_{un})_{max}} \sin\omega t + \underbrace{mA\omega^2}_{(F_I)_{max}} \sin(\omega t - \phi) \underbrace{-cA\omega}_{(F_{damping})_{max}} \sin\left(\frac{\pi}{2} + \omega t - \phi\right) \underbrace{-sA}_{(F_{spring/elastic})_{max}} \sin(\omega t - \phi) = 0$$

At $t' = t$ phasor diagram



- Any moment, the maximum responses of spring & damping forces are mutually perpendicular.

Ques:

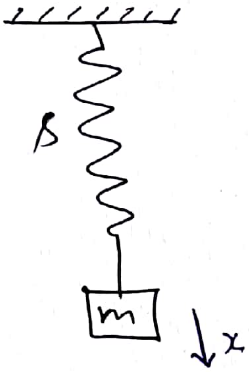


$\vec{c} = ? \rightarrow (\text{Damping force})_{max}$

$\vec{D} = ? \rightarrow (\text{Spring force})_{max}$

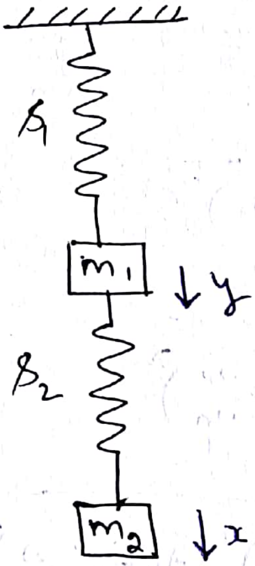
Degree of freedom of vibration system

1.



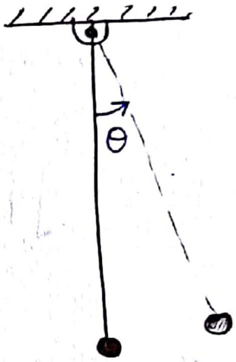
DOF = 1

2.



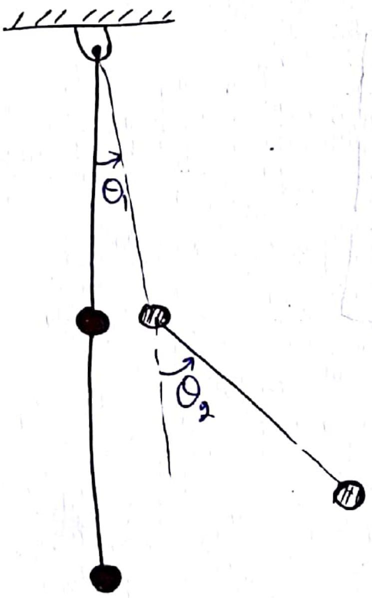
DOF = 2

3.



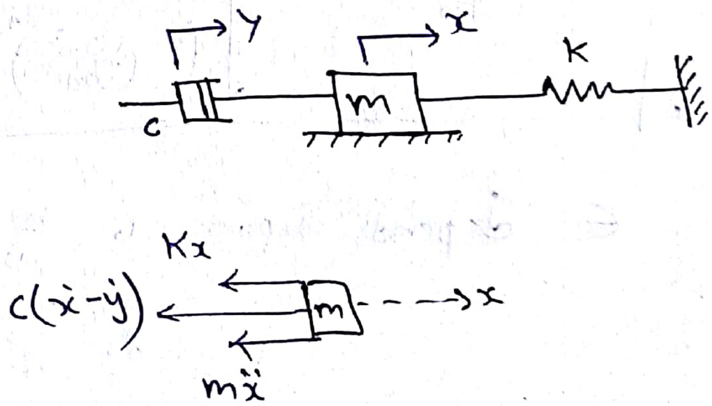
D.O.F. = 1

4.



D.O.F. = 2

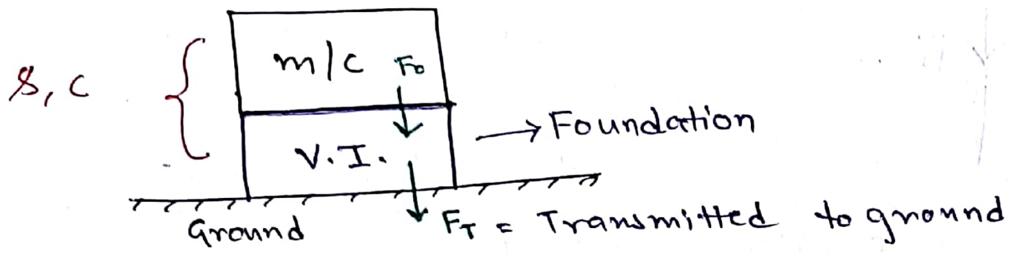
Ques:



Vibration Isolation

$\Downarrow$

How to isolate the ground from the vibration of running m/c.



$F_T <<<<< F_0$

Transmissibility $\epsilon = \frac{F_T}{F_0}$

$$F_T = \sqrt{(sA)^2 + (c\omega A)^2}$$

$$F_T = sA\sqrt{1 + \left(\frac{2\xi\omega}{\omega_n}\right)^2}$$

$$F_0 = sA\sqrt{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(\frac{2\xi\omega}{\omega_n}\right)^2}$$

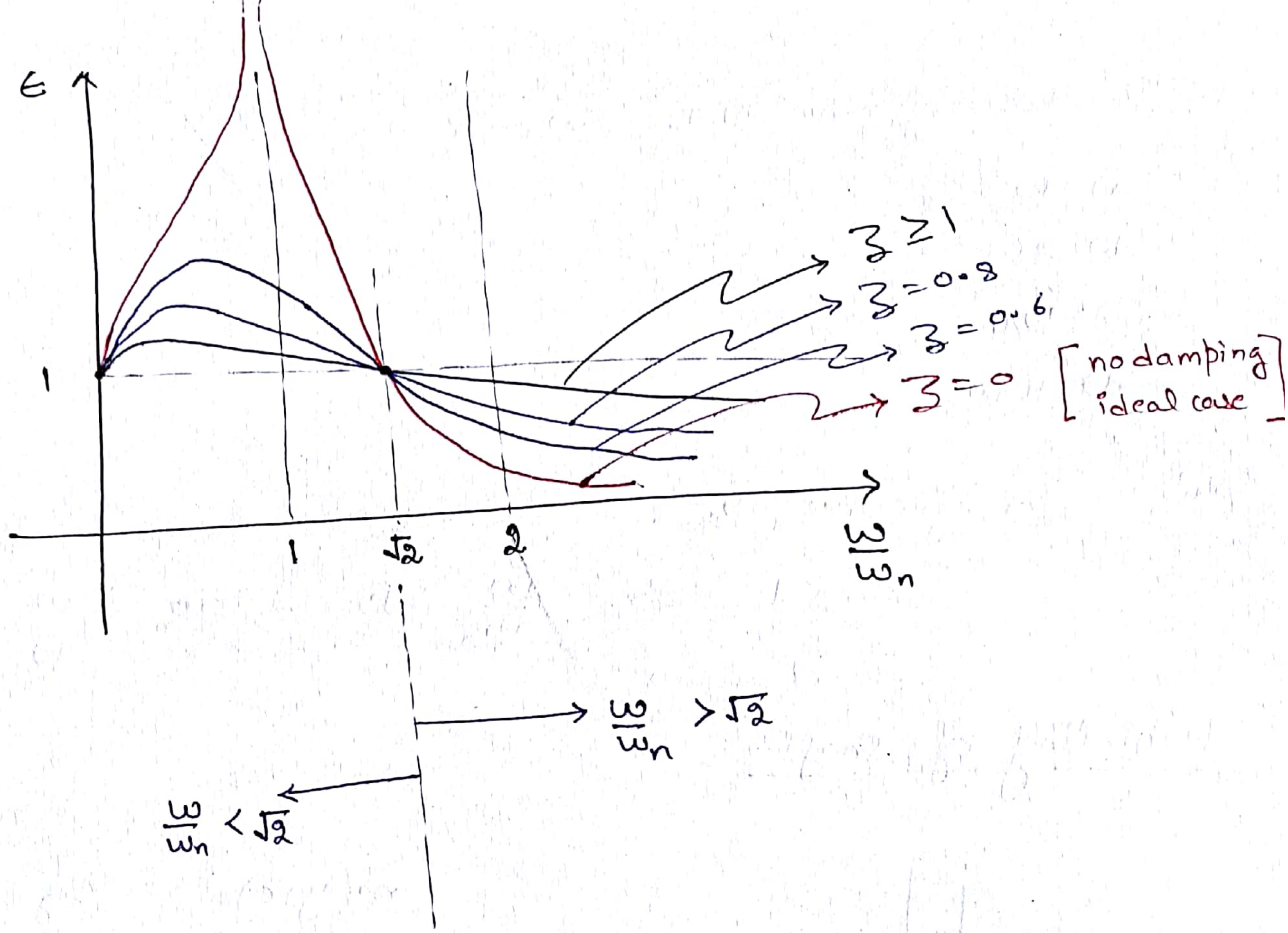
$$\boxed{\epsilon = \frac{F_T}{F_0} = \frac{\sqrt{1 + \left(\frac{2\xi\omega}{\omega_n}\right)^2}}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(\frac{2\xi\omega}{\omega_n}\right)^2}}}$$

ϵ depends upon
1. $\frac{\omega}{\omega_n}$
2. ξ

If $\frac{\omega}{\omega_n} = 0 \Rightarrow \epsilon = 1$

$\frac{\omega}{\omega_n} = \sqrt{2} \Rightarrow \epsilon = 1$

For all values of ξ



1. $\uparrow$ underdamping

$\Rightarrow \zeta \downarrow$

$\epsilon \uparrow$ if $\frac{\omega}{\omega_n} < \sqrt{2}$

$\epsilon \downarrow$ if $\frac{\omega}{\omega_n} > \sqrt{2}$

ϵ will remain same if $\frac{\omega}{\omega_n} = \sqrt{2}$

2. Vibration isolation will be effective when $\epsilon < 1$

$\Rightarrow \frac{\omega}{\omega_n} > \sqrt{2}$

3. In effective V.I. zone

$\frac{\omega}{\omega_n} > \sqrt{2} \Rightarrow \epsilon < 1$

$\Rightarrow$ No damping is the Best $(\epsilon \longrightarrow 0)$

$\Rightarrow$ Damping is harmful

* Rubber is best for Vibration isolation

Ques: For effective V.I. $\omega_n = ?$

a) ω

b) 2ω

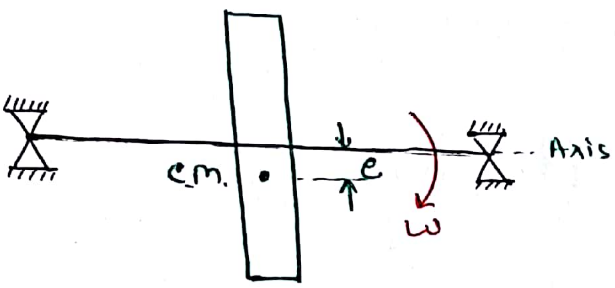
c) $\omega/4$

d) 10ω

Solu^n for effective V.I.

$$G < 1 \Rightarrow \frac{\omega}{\omega_n} > \sqrt{2} \Rightarrow \omega_n < \frac{\omega}{\sqrt{2}}$$

Whirling of shaft

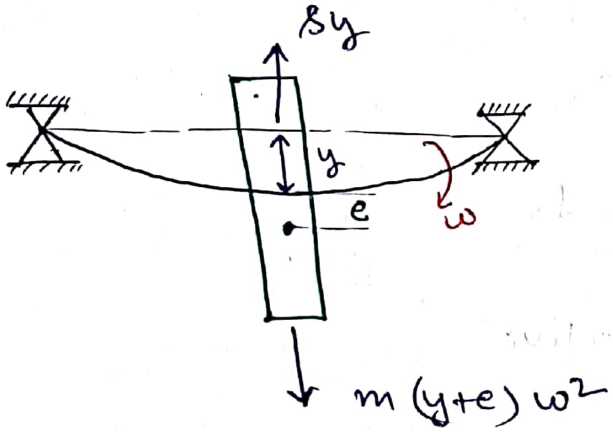


$$m(y+e)\omega^2 = sy$$

$$my\omega^2 + me\omega^2 = sy$$

$$me\omega^2 = sy - my\omega^2$$

$$\cancel{me\omega^2} = \cancel{my\omega^2}\left(\frac{s}{m\omega^2} - 1\right)$$



$$\boxed{y = \frac{e}{\left(\frac{\omega_n}{\omega}\right)^2 - 1}}$$

In some system, running life is very less

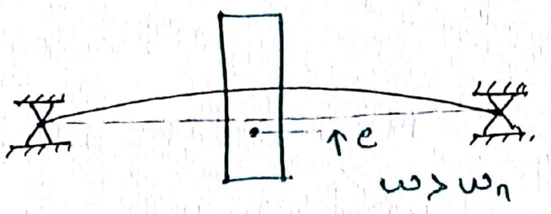
⇓

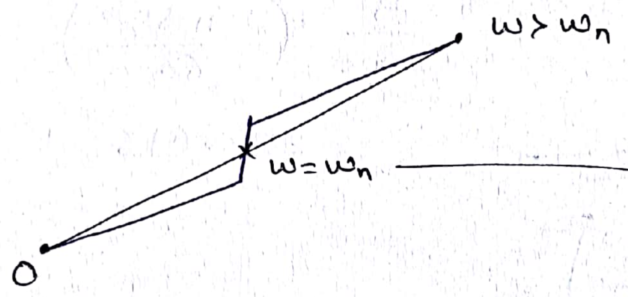
How to increase running life

⇓

To increase Running life $\omega > \omega_n$

shaft bending in opposite dirn





$\omega = \omega_n$ → Resonance

⇓

Death of shaft

⇓

(whirling)

Fundamental / Primary whirling speed $= \sqrt{\frac{s}{m}}$ rad/s

↓

Critical speed

Note :- Secondary whirling speed

⇓

only in Horizontal shaft

⇓

Almost half of the Primary whirling speed.

⇓

here also magnitude (Amplitude) is high

⇓

Reason: Due to self wt. of shaft.

WB
Q 56

$m = 17\,kg$

$s = 1000\ N/m$

$\omega_n = \sqrt{\frac{1000}{17}}\ rad/s = 7.67$

$\frac{\omega}{\omega_n} = 6.826$

$\zeta = 0.20$

$m_{ecc} = 2\,kg$

$r = \frac{75}{2000}\ m$

$N = 500\ rpm$

$\omega = \frac{2\pi\,500}{60} = 52.36\ rad/s$

$F_0 = 2 \times \frac{75}{2000} \times \left(\frac{2\pi\,500}{60}\right)^2 = 205.616$

(i) $A = \dfrac{F_0/s}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2}}$

$= \dfrac{205.616/1000}{\underbrace{\sqrt{\underbrace{\left(1-(6.826)^2\right)^2}_{2078.838} + (2\times 0.2\times 6.826)^2}}_{45.6759}} = 4.5\ mm$

09 313467 612

9:30 pm – 10:30 pm

(ii) $G = \dfrac{F_T}{F_0} = \dfrac{\sqrt{1+\left(\frac{2\zeta\omega}{\omega_n}\right)^2}}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2}} = \dfrac{2.9077}{45.6759}$

$= 0.06366$

$F_T = G \times F_0 = 13.089\ N$

TOM-2

Something about Theory of m/c

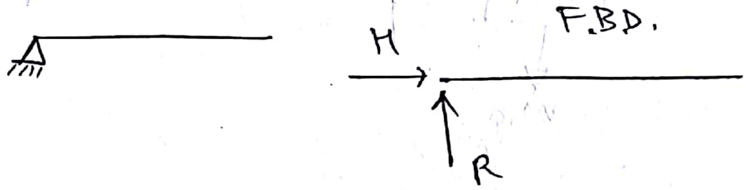
1. If body is in rest and static equilibrium

$\Sigma H = 0$

$\Sigma V = 0$

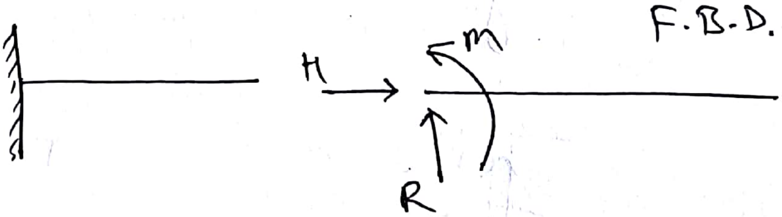
$\Sigma M = 0$

a) Hinged joint

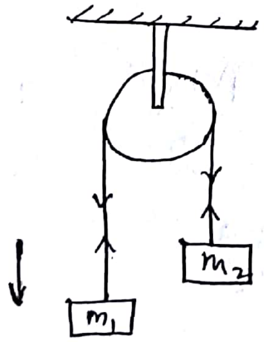


b) Roller joint

c) Fixed



2. If body is moving (translatory motion)
if we will apply a force m_A in opposite direction to the body will come to rest. This force is called Pseudo force

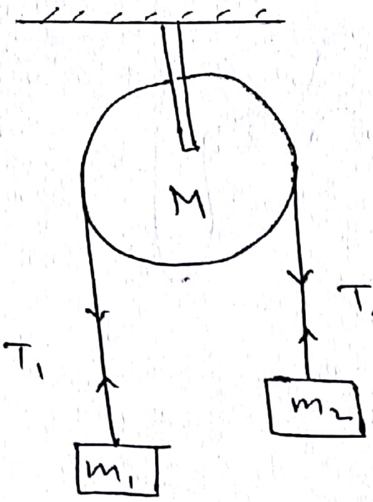


$$a = \frac{(m_1 - m_2)g}{m_1 + m_2}$$

$$T = m_1(g - a) = m_1\left(g - \frac{m_1 g + m_2 g}{m_1 + m_2}\right)$$

$$T = \frac{2 m_1 m_2 g}{m_1 + m_2}$$

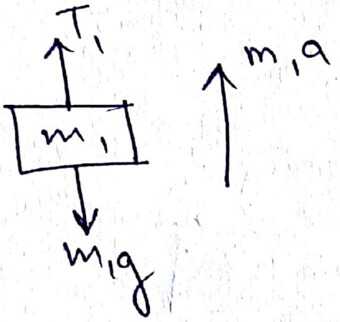
Ques:



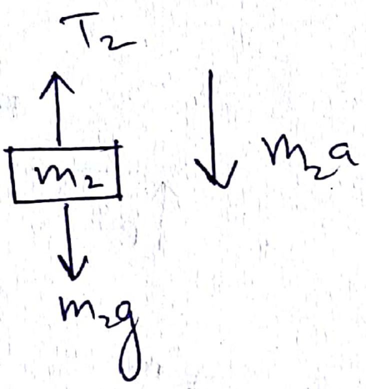
find T_1, T_2, a

$m_1 > m_2$

Solu^n

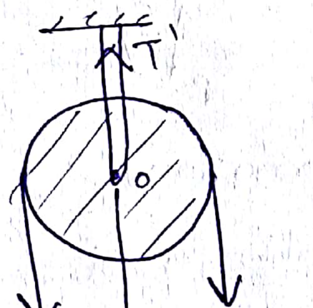


$$T_1 + m_1 a = m_1 g$$



$$T_2 = m_2 a + m_2 g$$

Let solid disc.



$$T' = T_1 + T_2 + Mg$$

$$\Sigma M_o = 0$$

$$T_1 r - T_2 r = I\alpha$$

$$T_1 \cancel{r} - T_2 \cancel{r} = \frac{M r^{\cancel{2}}}{2} \times \frac{a}{\cancel{r}}$$

$$T_1 - T_2 = \frac{Ma}{2}$$

$a = r\alpha$

$\alpha = \frac{a}{r}$

$T_1 \cancel{r} - T_2 \cancel{r} = \frac{M r^{\cancel{2}}}{2} \times \frac{a}{\cancel{r}}$

$$m_1 g - m_1 a - (m_2 a + m_2 g) = \frac{Ma}{2}$$

$$m_1 g - m_1 a - m_2 a - m_2 g = \frac{Ma}{g}$$

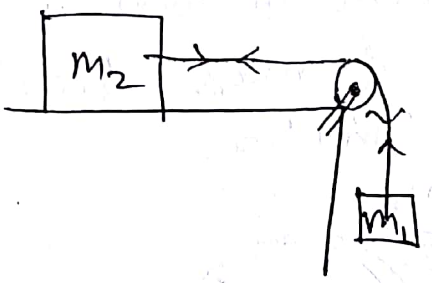
$$(m_1 - m_2) g = \left(m_1 + m_2 + \frac{M}{2}\right) a$$

$$\Rightarrow a = \frac{(m_1 - m_2) g}{\left(m_1 + m_2 + \frac{M}{2}\right)}$$

$$T_1 = m_1 (g - a)$$

$$T_2 = m_2 (g + a)$$

Ans:



$$a = \frac{m_1 g}{m_1 + m_2}$$

$$T = m_1 (g - a)$$

$$= m_1 g \left(1 - \frac{m_1}{m_1 + m_2}\right)$$

$$= \frac{m_1 m_2 g}{m_1 + m_2}$$

Translation	Rotation
1. displacement $= x$	1. displacement $= \theta$
2. Velocity $= v = \frac{dx}{dt}$	2. angular velocity $= \omega = \frac{d\theta}{dt}$
3. acceleration $= a = \frac{dv}{dt} = \frac{d^3x}{dt^2}$	3. angular acceleration (α) $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$

4. Force = ma

$= m\frac{dv}{dt} = m\frac{d^2x}{dt^2}$

5. W.D. = $\vec{F}.d\vec{x}$

6. Power = $\vec{F}.\vec{v}$

7. KE = $\frac{1}{2}mv^2$

8. $v = u + at$

9. $s = ut + \frac{1}{2}at^2$

10. $v^2 - u^2 = 2as$

4. Torque = $T = I\alpha$

5. W.D. = $Td\theta$

6. Power = $T\omega$

7. KE = $\frac{1}{2}I\omega^2$

8. $\omega_2 = \omega_1 + \alpha t$

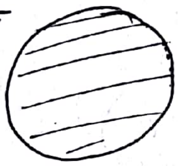
9. $\theta = \omega_1 t + \frac{1}{2}\alpha t^2$

10 ⇒ $\omega_2^2 - \omega_1^2 = 2\alpha\theta$

Something about moment of Inertia (2nd ~~area~~ mass moment)

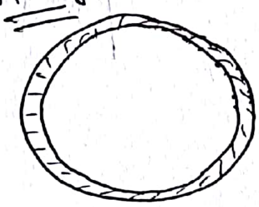
Moment of inertia depends on mass as well as distribution of mass, more the distance of mass from center, more the moment of inertia.

1) disc



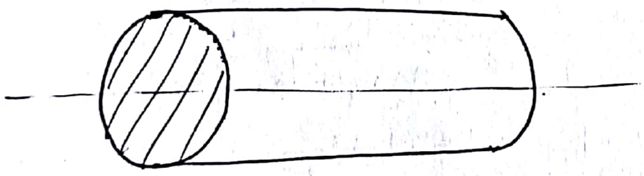
⇒ mR^2

2) Ring



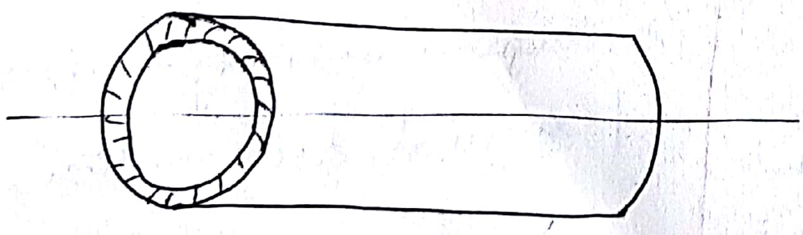
⇒ $\frac{mR^2}{2}$

3) Solid cylinder



$I_{axis} = \frac{MR^2}{2}$

4) Hollow cylinder



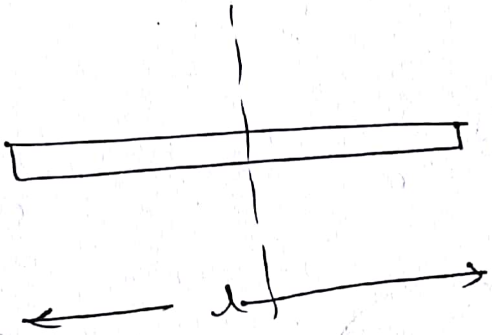
$I_{axis} = MR^2$

5) Solid sphere

$\rightarrow I = \frac{2}{5} mR^2$

6) Hollow sphere $\Rightarrow I = \frac{2}{3} mR^2$

7)

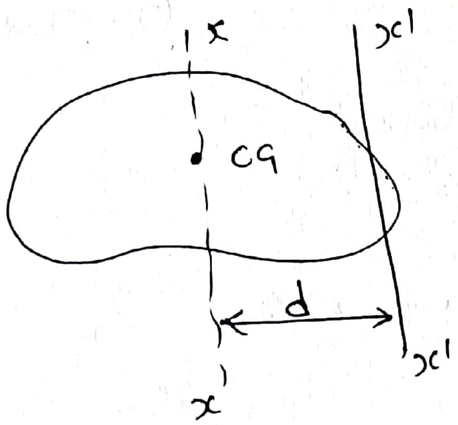


$I_{axis} = \frac{ml^2}{12}$

2nd Area moment of Inertia

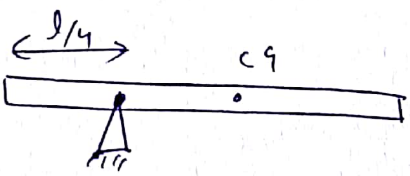
$I = Ar^2$

Parallel axis theorem



$$\boxed{I_{x'x'} = I_{xx} + md^2}$$

ex:-

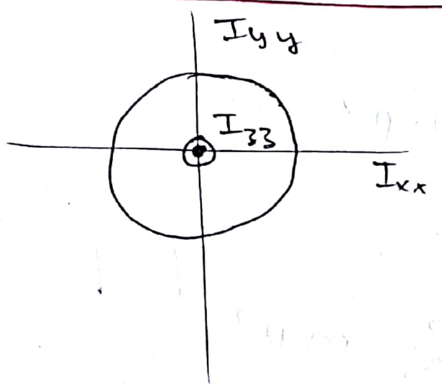


$I_{cg} = \frac{ml^2}{12}$

$I_{hinge} = \frac{ml^2}{12} + \frac{ml^2}{16}$

$= \frac{7}{48} ml^2$

Lar Axis Theorem



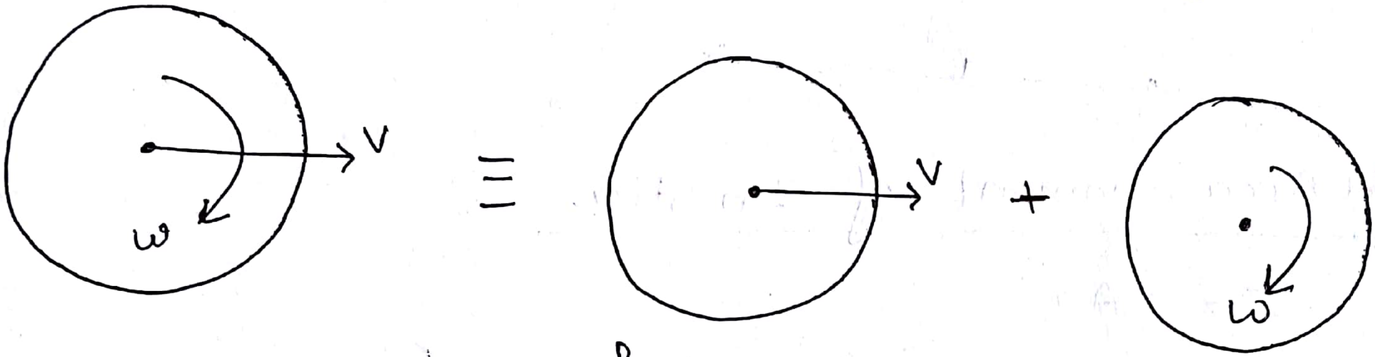
$$\boxed{I_{zz} = I_{xx} + I_{yy}}$$

$$= \frac{\pi d^4}{64} + \frac{\pi d^4}{64}$$

$$= \frac{\pi d^4}{32}$$

Rolling

It is a mixture of translatory and rotatory motion.



Pure translation + Pure rotation

$(F = ma)$ $(T = I\alpha)$

1) Pure rolling

$$\boxed{v = r\omega}$$

$$\frac{dv}{dt} = r\frac{d\omega}{dt} \Rightarrow a = r\alpha$$

2) Rolling with sliping

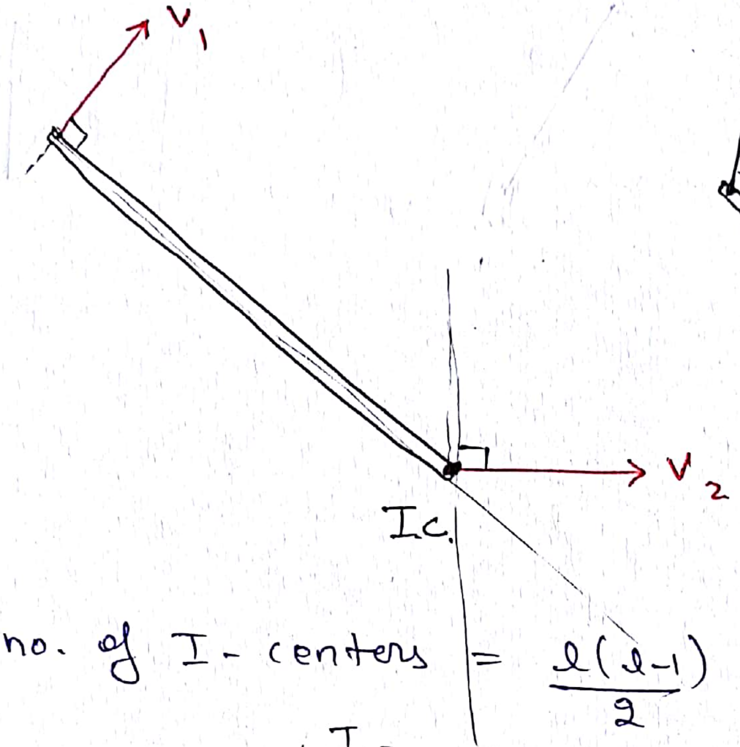
$$\boxed{v > r\omega}$$

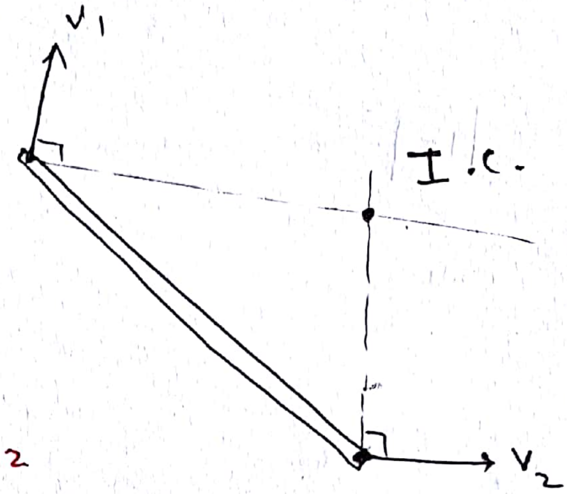
3) Rolling with skiding

$$\boxed{v < r\omega}$$

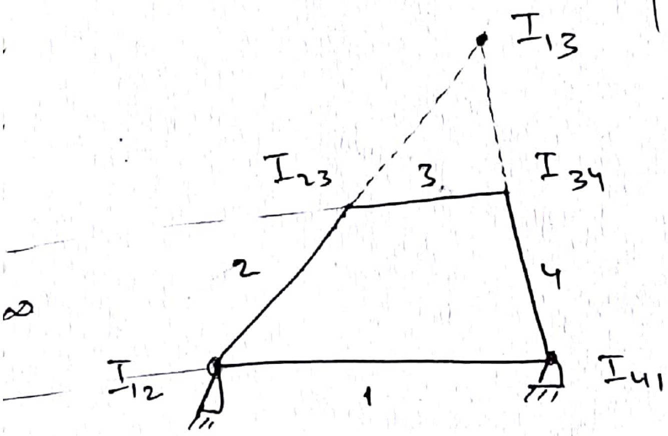
Instantaneous center

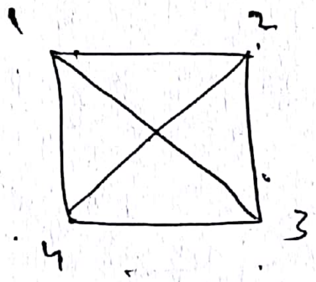
The Point about which Pure rotation occur is called I-center



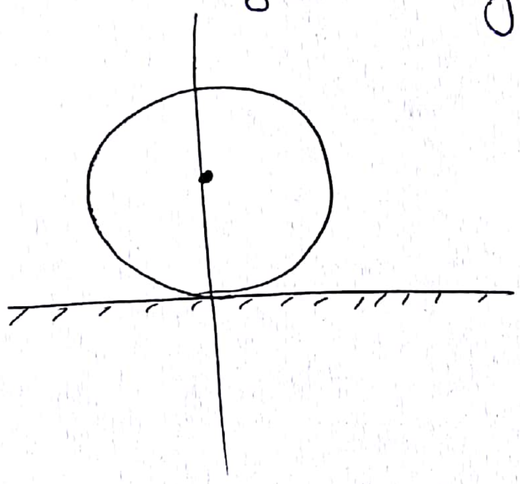


no. of I - centers $= \dfrac{l(l-1)}{2}$; $l =$ no. of links





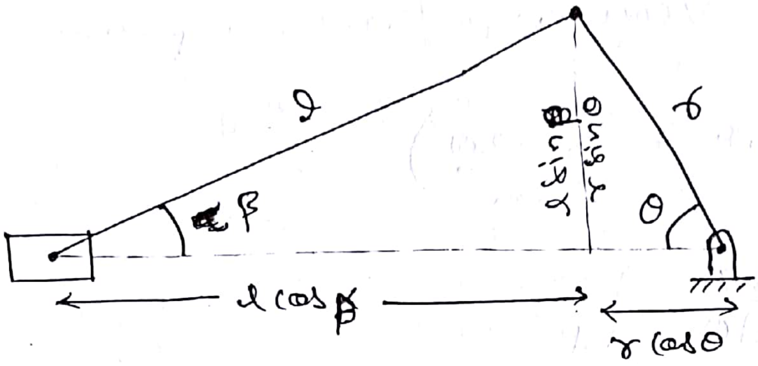
- In case of skiding or sliding in Rolling



Anywhere on normal

CHAPTER - I

SLIDER CRANK MECHANISM



$$l \sin\beta = r \sin\theta \Rightarrow \sin\beta = \frac{\sin\theta}{n} \qquad \left[\because n = \frac{l}{r}\right]$$

x = Piston displacement

$$x = l + r - (l\cos\beta + r\cos\theta)$$

$$= l + r - l\cos\beta - r\cos\theta$$

$$= \left(\frac{l}{r} + 1 - n\sqrt{1 - \frac{\sin^2\theta}{n^2}} - \cos\theta\right) r$$

$$\boxed{x = r\left[1 - \cos\theta + n - \sqrt{n^2 - \sin^2\theta}\right]}$$

Piston velocity (V)

$$x = r\left[1 - \cos\theta + n - \sqrt{n^2 - \sin^2\theta}\right]$$

$$V = \frac{dx}{dt} = \frac{dx}{d\theta} \cdot \frac{d\theta}{dt} = \omega \frac{dx}{d\theta}$$

$$V = r\omega\left[0 + \sin\theta + 0 + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right]$$

$$\boxed{V = r\omega\left[\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right]}$$

Piston acceleration (a)

$$v = r\omega\left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right)$$

for $\frac{l}{r} = n \approx$ very large in comparision to $\sin\theta$

$$v = r\omega\left(\sin\theta + \frac{\sin 2\theta}{2n}\right)$$

$$a = \frac{dv}{dt} = \frac{dv}{d\theta} \times \frac{d\theta}{dt}$$

$$\boxed{a = r\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right)}$$

Angular velocity of connecting rod.

$$l \sin\beta = r \sin\theta$$

$$l \cos\beta \times \frac{d\beta}{dt} = r \cos\theta \frac{d\theta}{dt}$$

$$\omega_{CR} = \frac{d\beta}{dt} = \frac{\omega \cos\theta}{n \cos\beta} = \frac{\omega \cos\theta}{n\sqrt{1 - \sin^2\beta}}$$

$$\Rightarrow \omega_{CR} = \frac{\omega \cos\theta}{n \times \sqrt{1 - \frac{\sin^2\theta}{n^2}}}$$

$$\Rightarrow \boxed{\omega_{CR} = \frac{\omega \cos\theta}{\sqrt{n^2 - \sin^2\theta}}}$$

for $n >> \sin\theta$

$$\boxed{\omega_{CR} = \frac{\omega \cos\theta}{n}}$$

Angular Acceleration of connecting rod (α_{CR})

$$\alpha_{C.R.} = \frac{d\omega_{CR}}{dt} = \frac{d\omega_{CR}}{d\theta} \times \underbrace{\frac{d\theta}{dt}}_{\omega}$$

$$\omega_{CR} = \frac{\omega \cos\theta}{\sqrt{n^2 - \sin^2\theta}}$$

$$\alpha_{CR} = \omega^2 \frac{d}{d\theta}\left(\frac{\cos\theta}{\sqrt{n^2 - \sin^2\theta}}\right)$$

$$= \omega^2 \left[\frac{-\sin\theta \times \sqrt{n^2 - \sin^2\theta} - \frac{\cos\theta \times (-1) \times 2\sin\theta\cos\theta}{2 \times \sqrt{n^2 - \sin^2\theta}}}{n^2 - \sin^2\theta}\right]$$

$$= -\omega^2 \left[\frac{\sin\theta\,(n^2 - \sin^2\theta) - \cos^2\theta \sin\theta}{2 \times \left(\sqrt{n^2 - \sin^2\theta}\right)^3}\right]$$

$$= -\omega^2 \left[\frac{\sin\theta}{2 \times [n^2 - \sin^2\theta]^{3/2}} \times \left[n^2 - \sin^2\theta - \cos^2\theta\right]\right]$$

$$\alpha_{CR} = \frac{-\omega^2 \sin\theta\,(n^2 - 1)}{2 \times [n^2 - \sin^2\theta]^{3/2}}$$

or

$$\alpha_{CR} = \frac{d\omega_{CR}}{dt} = \frac{d\omega_{CR}}{d\theta} \cdot \frac{d\theta}{dt} = \frac{-\omega^2 \sin\theta}{n}$$

$$\boxed{\alpha_{CR} = \frac{-\omega^2 \sin\theta}{n}}$$

Force on Piston

$$a = r\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

$$F_{inertia} = ma$$

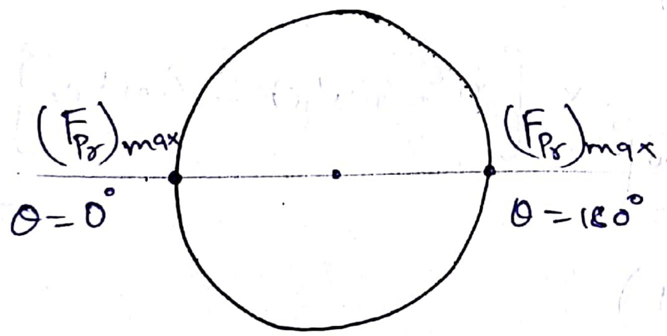
$$F_i = mr\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

$$\boxed{F_i = \underbrace{mr\omega^2\cos\theta}_{\text{Primary force}} + \underbrace{mr\omega^2\frac{\cos 2\theta}{n}}_{\text{2ndary force}}}$$

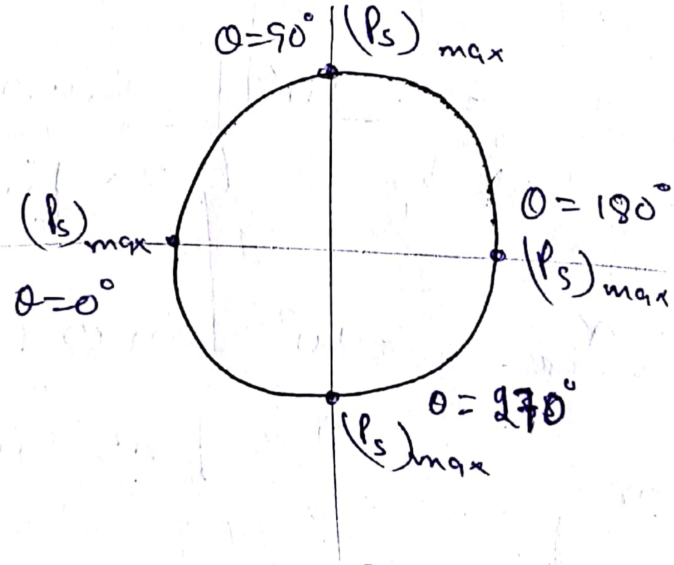
Primary force $= mr\omega^2 \cos\theta$

Secondary force $= mr\omega^2 \frac{\cos 2\theta}{n}$

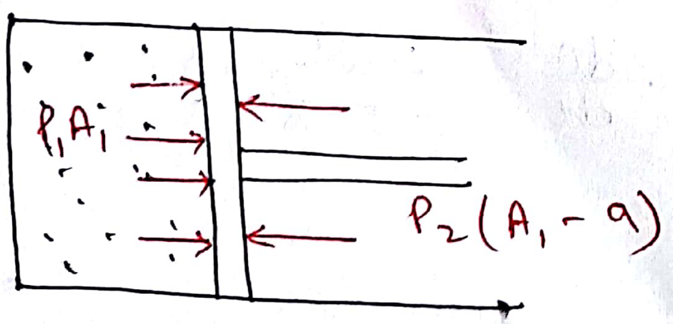
Primary force (F_{Pr})



Secondary force (F_s)



Pressure force on Piston (F_P)



$A_1 = \frac{\pi}{4} D^2$

$a = \frac{\pi}{4} d^2$

D = dia of Piston

d = dia of Plunger

$F_1 = P_1 A$

$F_2 = P_2 (A - a)$

$F_p = F_1 - F_2$

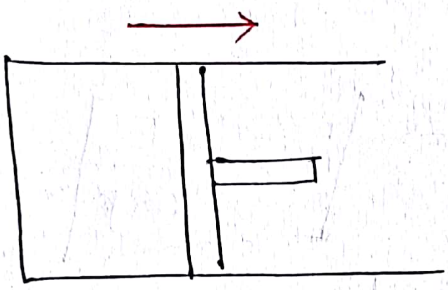
F_p = Pressure force

F_i = inertia force

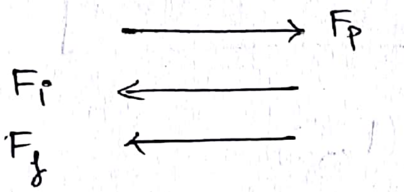
F_g = gravity force

F_f = friction force

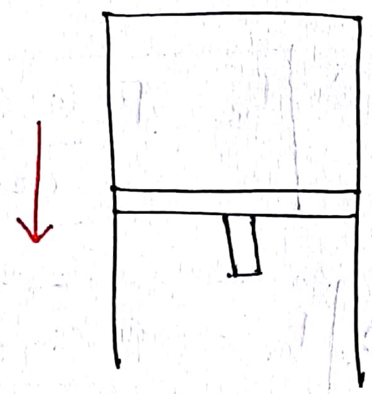
Case-1 Horizontal Cylinder

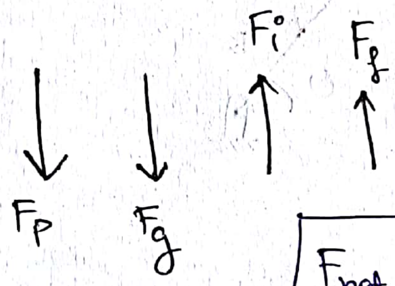


$$\boxed{F_{net} = F_p - F_i - F_f}$$



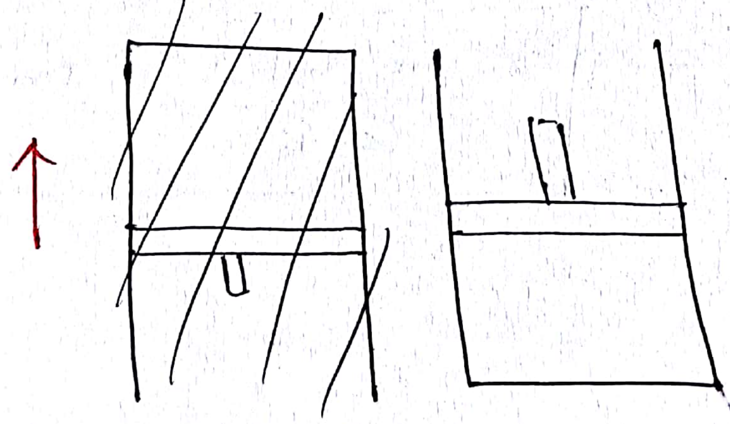
Case-2 Vertical cylinder (Piston moving down)

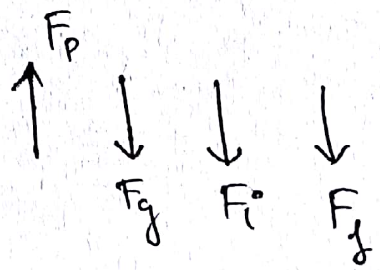




$$\boxed{F_{net} = F_p + F_g - F_i - F_f}$$

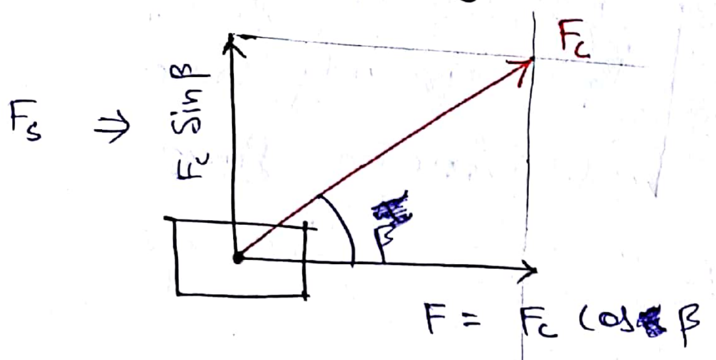
Case-3 Vertical cylinder (Piston moving up)





$$\boxed{F_{net} = F_p - F_g - F_i - F_f}$$

Force on connecting rod (F_c)



$F_{net} = F$

$F_s =$ Side thrust

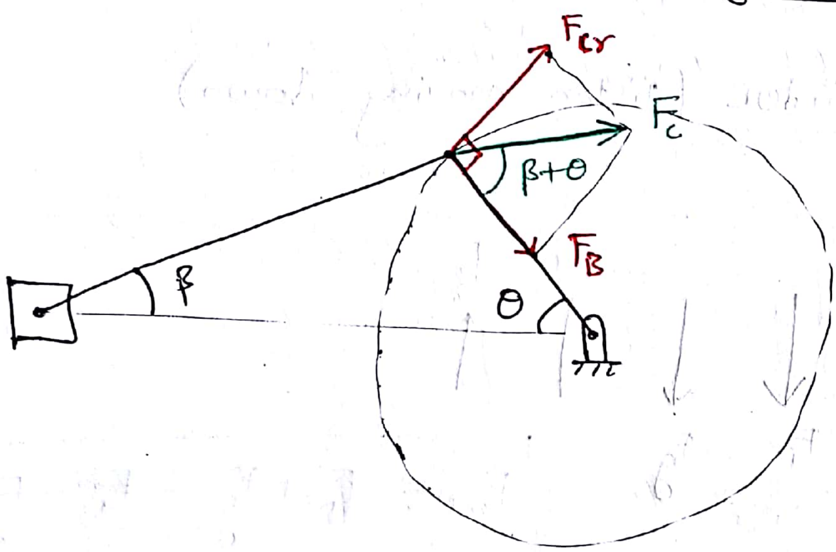
$$\boxed{F_s = F_c \sin\beta}$$

$$F = F_c \cos\beta$$

$$\Rightarrow \boxed{F_c = \frac{F}{\cos\beta}}$$

$$\Rightarrow \boxed{F_s = F \tan\beta}$$

Bearing thrust (F_B), Crank effort (F_{cr})



$$F_c \cos(\beta+\theta) = F_B$$

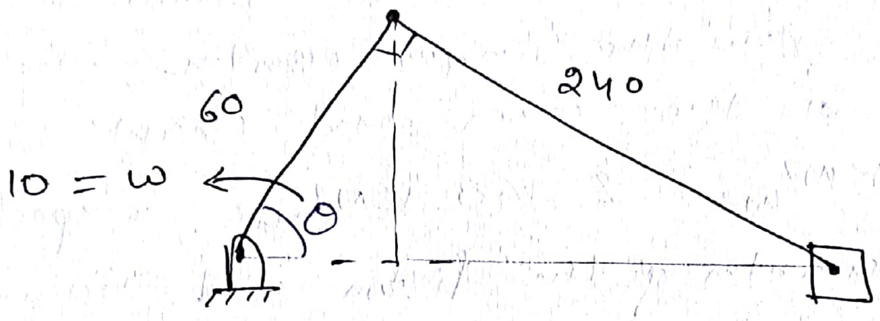
$$\Rightarrow \boxed{F_B = F\frac{\cos(\theta+\beta)}{\cos\beta}}$$

$$\Rightarrow \boxed{F_{cr} = F\frac{\sin(\theta+\beta)}{\cos\beta}}$$

Turning moment (Tm)

$$Tm = F_{cr} \times r = \boxed{F.r.\frac{\sin(\theta+\beta)}{\cos\beta} = Tm}$$

Ques: A slider crank mechanism with crank radius 60 mm & connecting rod length = 240 mm as shown in figure. (ω = 10 rad/sec ccw) the speed in m/s ________ of slider



Solu^n

$$\tan\theta = \frac{240}{60} = 4$$

$$V = r\omega\left(\sin\theta + \frac{\sin 2\theta}{2n}\right)$$

$$= .06 \times 10 \times \left(\frac{4}{\sqrt{17}} + \frac{\cancel{2}\times}{\cancel{2\times 4}} \frac{\cancel{4}}{\sqrt{17}} \times \frac{1}{\sqrt{17}}\right)$$

~~= 1.19 m/s~~

$= 0.617$ m/s

Ques: In a certain slider crank mechanism, Crank & connecting rod are equal. If the crank rotates with a uniform angular speed of 14 rad/s and crank length is 300 mm. The max. acceleration of the slider in m/s^2 is

Solu^n

$$v = r\omega\left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right]$$

$$a = r\omega^2\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

at $\theta = 0$ $\quad a_{max} = 2r\omega^2 = 117.6\ m/s^2$

Ques: A horizontal gas engine running at 210 rpm has a bore of 220 mm and a stroke of 440 mm. The connecting rod is 924 mm long and reciprocating parts weight is 20 kg. When the crank has turned an angle of 30° from the inner center, the gas pressure on the cover and crank side are 500 KN/m2 & 60 KN/m2 respectively. Determine (Diameter of the piston rod = 40mm)

1) Thrust ~~moment~~ on Bearing
2) Thrust moment on the Crank Shaft
3) Acceleration of fly wheel which has mass 8 kg and radius of gyration of 600 mm & Power of engine is 22 KW.

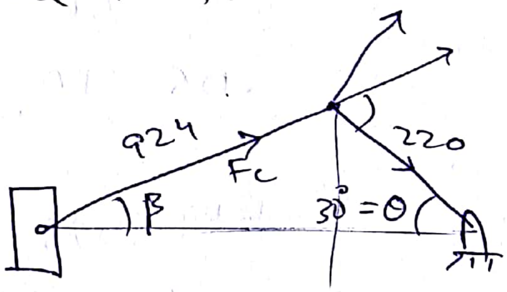
Soln:

$N = 210$ rpm

$D = 220$ mm

$r = 220$ mm

$l = 924$ mm $\Big] \Rightarrow n =$



$m = 20$ kg

$\theta = 30°$

$P_1 = 500\ \frac{KN}{m^2}$

$P_2 = 60\ KN/m^2$

40 mm

$a = \frac{\pi}{4} d^2 =$

1) $F_B = F \frac{\cos(\theta + \beta)}{(\cos\beta)}$

$220 \times \frac{1}{2} = 924 \times \sin\beta$

$\boxed{\beta = 6.837°}$

$F = F_p - F_i$

16.8 (KN) $F_p = F_1 - F_2 = P_1 A_1 - P_2 A_2$

$= 500 \times 10^3 \left(\frac{\pi}{4} \times (.22)^2\right)$

$- 60 \times 10^3 \left(\frac{\pi}{4}\left[(.22)^2 - (.04)^2\right]\right.$

$$F_i = mr\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

$$= 20 \times .22 \times \left(\frac{2\pi \times 210}{60}\right)^2\left(\frac{\sqrt{3}}{2} + \frac{220}{2 \times 924}\right)$$

$$= 2.096 \text{ KN}$$

$$F = 16.8 - 2.096 = 14.7038 \text{ KN}$$

1) $F_B = 14.7038 \times \dfrac{\cos(30 + 6.837)}{\cos(6.837)}$

$$= \underline{11.85 \text{ KN}}$$

2) $F_{cr} = F \dfrac{\sin(\theta + \beta)}{(\cos \beta)}$

$$= \underline{8.878 \text{ KN}}$$

$$TM = F_{cr} \times r$$
$$= \underline{1.953 \text{ KNm}}$$

3) $I = mk^2$

$$= 8 \times .6 \times .6 = 2.88 \text{ kg m}^2$$

$$T = I\alpha$$

$$\alpha = \cancel{\frac{1.953 \times 10^3}{2.88}} = \cancel{678.125 \text{ rad/s}^2}$$

$$P = T\omega \Rightarrow T = \frac{P \times 60}{2\pi \times 210} = 1000 \text{ Nm}$$

$$T_{excess} = 1953 - 1000 = 953 \text{ Nm} = I\alpha$$

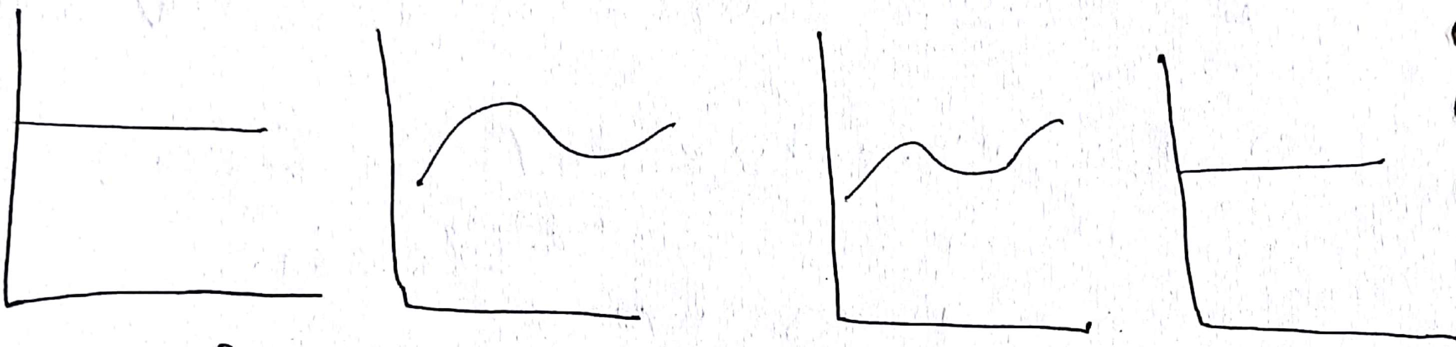
$$\alpha = \underline{330.9 \text{ rad/s}^2}$$

CHAPTER-2

FLYWHEEL

Case-1 input = const.
output = fluctuating

Case-2 Input = fluctuating
output = const



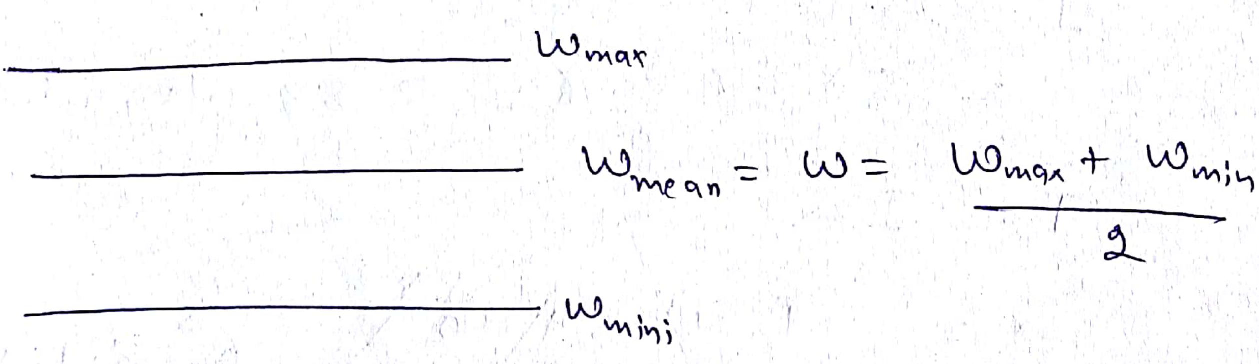
eg:- Punching m/c

eg:- all engines

⇓

flywheel needed

- A flywheel is used to control the variation in speed during each cycle of an engine.



$$\text{Change in Energy} = \frac{1}{2} I \left(\omega_{max}^2 - \omega_{min}^2\right)$$

$$= \frac{1}{2} I \left(\omega_{max} - \omega_{min}\right)\left(\omega_{max} + \omega_{min}\right)$$

$$= I \left(\omega_{max} - \omega_{min}\right) \omega$$

$$\Delta E_{max} = I(\omega_{max} - \omega_{min})\,\omega \times \frac{\omega}{\omega}$$

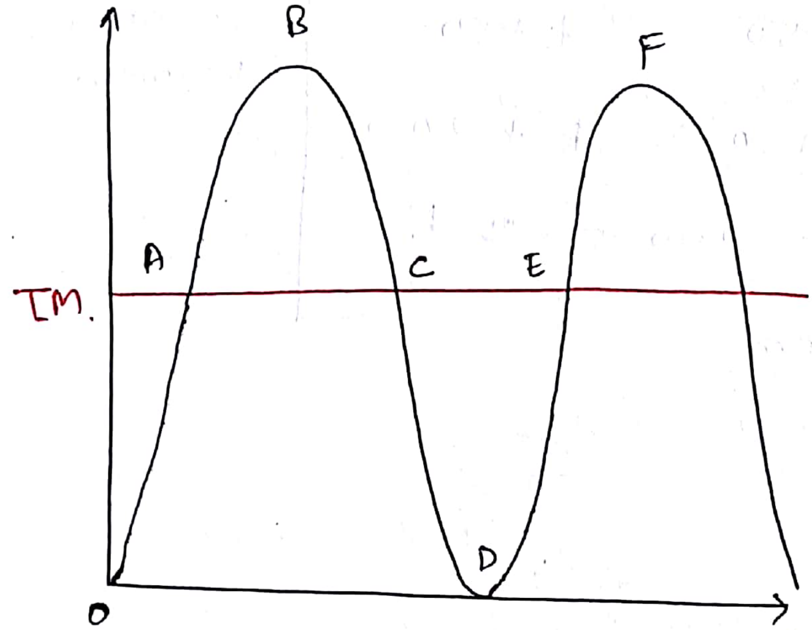
$$\boxed{\Delta E_{max} = I\omega^2 C_s}$$

$$\left[C_s = \frac{\omega_{max} - \omega_{min}}{\omega} \right]$$

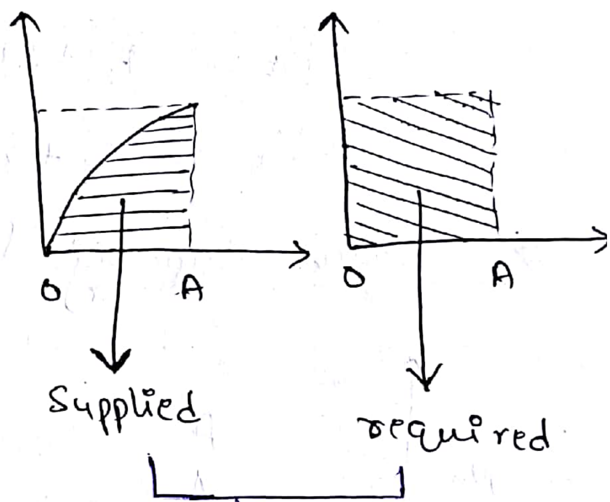
$$\Delta E_{max} = \frac{1}{2} I(\omega_{max}^2 - \omega_{min}^2) = I\omega^2 C_s$$

I = moment of inertia of flywheel

C_s = Coefficient of speed fluctuation

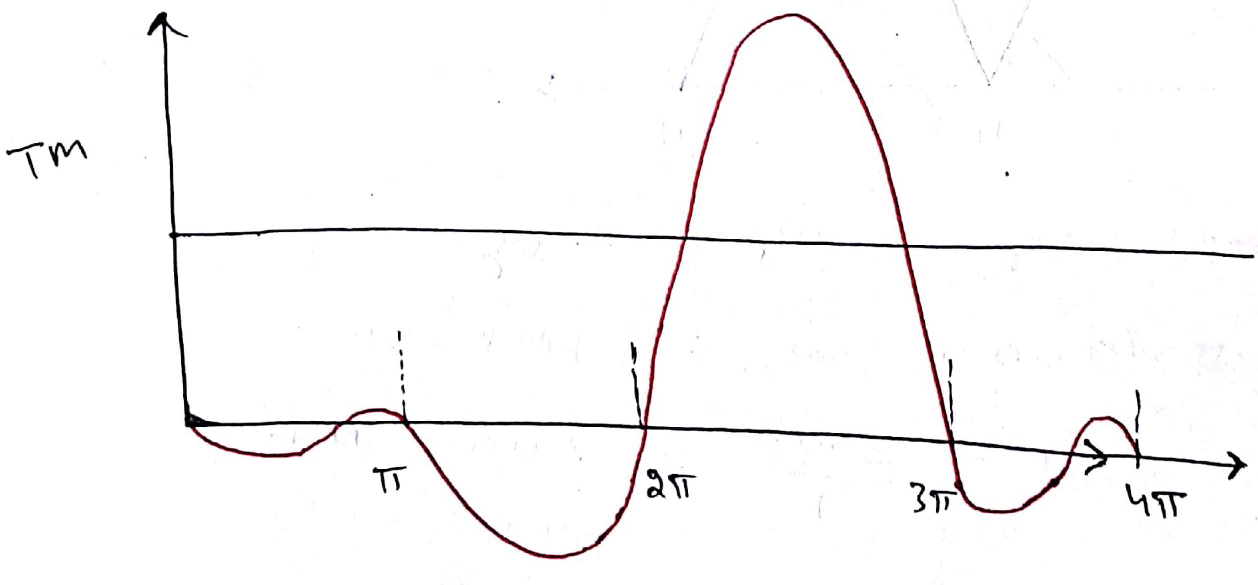


Crank angle

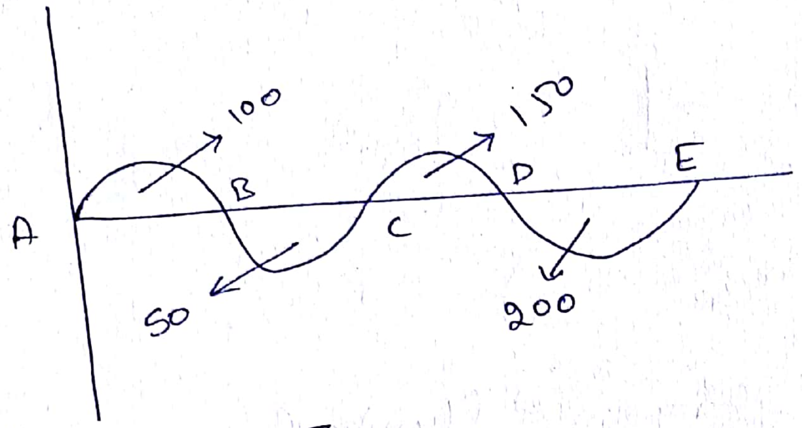


In region OA
flywheel deaccelerate

⇒ <u>Turning moment diagram of four stroke engine</u>



⇒ First type problem



Let, $E_A = E$

then, $E_B = E + 100$

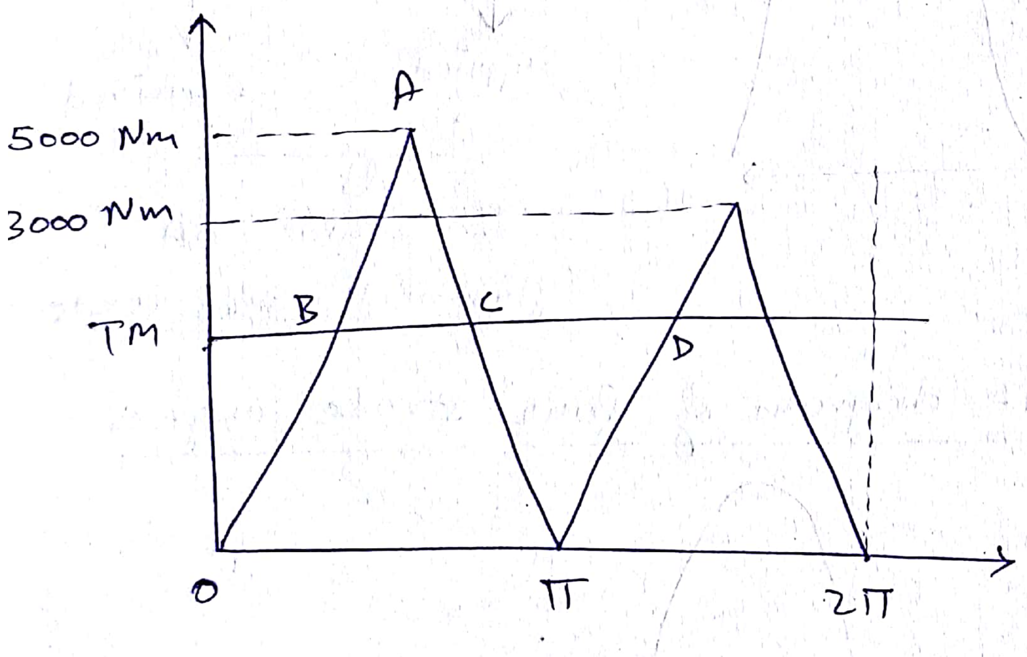
$E_C = E + 100 - 50 = E + 50$

$E_D = E + 50 + 150 = E + 200$

$E_E = E + 200 - 200 = E$

units should be taken problem

⇒ Second type of Problem



Input energy = output energy

$$\frac{1}{2} \times \cancel{\pi} \times (5000 + 3000) = TM \times 2\cancel{\pi}$$

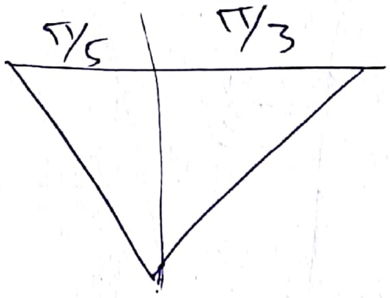
$$TM = \frac{8000}{4} = 2000 \text{ Nm}$$

- Power = $(T_m \cdot \omega)$
- $\frac{\text{work done}}{\text{cycle}} = T_m \times \theta_{cycle}$
- Change in energy fluctuation

$$BC = \pi \times \frac{3000}{5000} = \frac{3\pi}{5}$$

$$\Delta E_{max} = \frac{1}{2} \times \frac{3\pi}{5} \times 3000 = \quad = I\omega^2 C_s$$

<u>Rough</u>



$$\pi - \frac{3\pi}{5} = \frac{2\pi}{5}$$

$$\pi \times \frac{1000}{3000} = \frac{\pi}{3}$$

$$\pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

$$\frac{8}{15}\pi$$

$$\boxed{\frac{1}{2} \times \frac{8}{15}\pi \times 2000}$$

⟹ <u>Third type problem</u>

1) $T = a + b\sin(3\theta)$

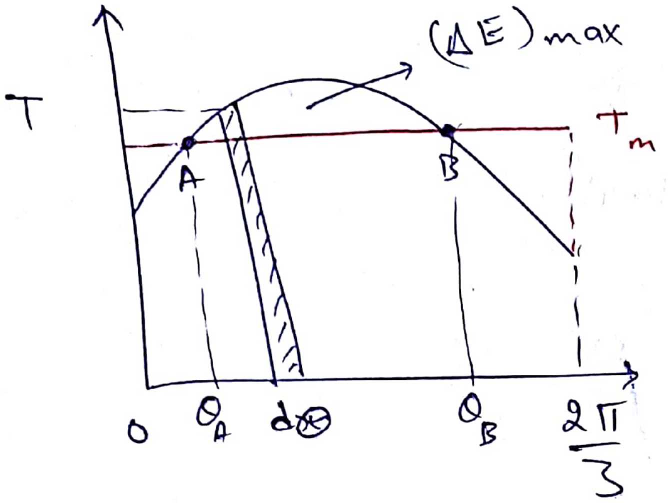
Time period $= \frac{2\pi}{3}$

2) $T = a + b\sin 2\theta + \cos\theta$

$\sin 2\theta \downarrow \pi$, $\cos\theta \downarrow 2\pi$

Time period = LCM = 2π

⇒ Type-4 Problem :



Input energy

$$= \int_0^{2\pi/3} T\, d\theta$$

output energy

$$= T_m \times \frac{2\pi}{3}$$

Input energy = Output energy

$$\int_0^{2\pi/3} T\, d\theta = T_m \times \frac{2\pi}{3}$$

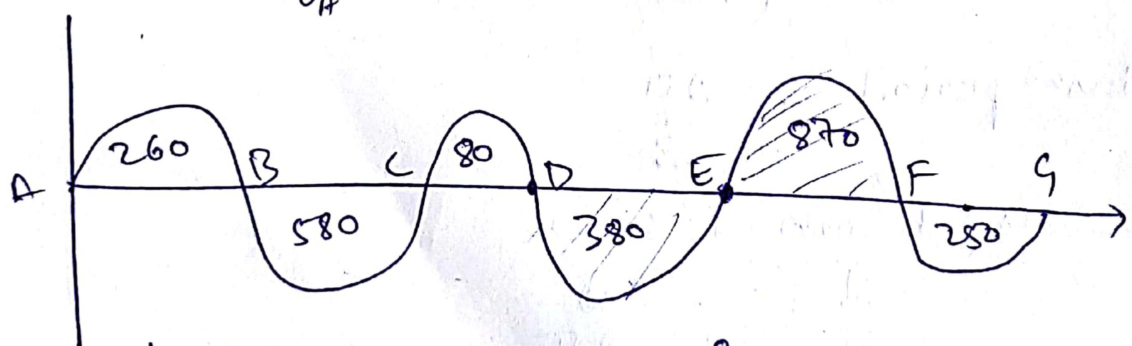
To get θ_A & θ_B

$$T_m = T \qquad [T = f(\theta)]$$

$$\Delta E_{max} = \int_{\theta_A}^{\theta_B} (T - T_m)\, d\theta$$

$$\Delta E_{max} = \int_{\theta_A}^{\theta_B} T\, d\theta - T_m(\theta_B - \theta_A) = I\omega^2 C_s$$

Ques:



horizontal scale = 3°

Vertical scale = 500 Nm

$N = 1600$ rpm

radius of gyration = 2.1 m

Find C_s

ΔE_{max}

Soln

$E_A = E$

$E_B = E + 260$

$E_C = E - 320$

$E_D = E - 240$

$E_E = E - 620$

$E_F = E + 250$

$E_G = E$

$(\Delta E)_{max} = 620$

$\cancel{620} = I \omega^2 C_s$

870

$1\,mm^2 = \left(\frac{\pi}{180} \times \frac{3}{60}\right) \times 500$

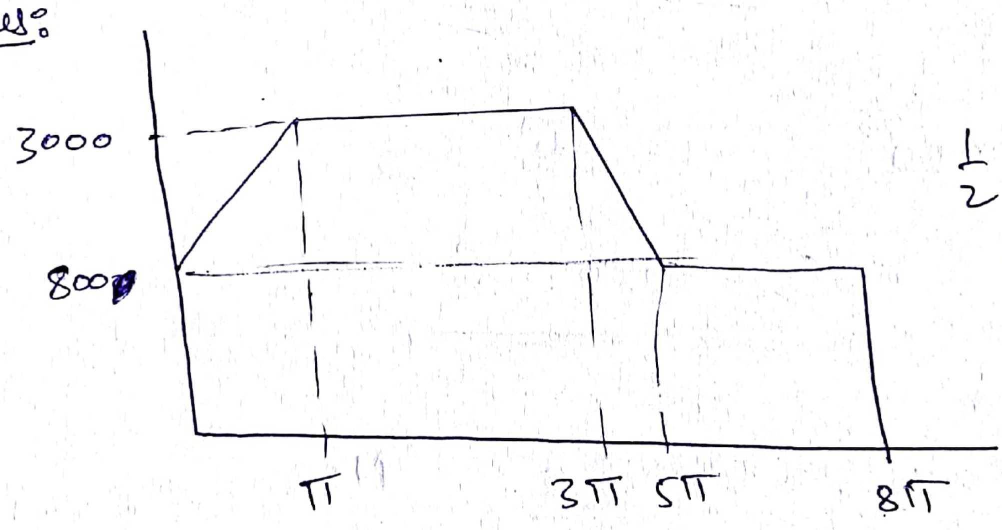
$= 26.18$

$$\frac{870}{620} \times 16231.56 = 1 \times (2.1)^2 \times \left(\frac{2\pi \times 1600}{60}\right)^2 C_s$$

$$\Rightarrow C_s = 0.1311 \times \frac{870}{620}$$

$$= 0.1839$$

Ques:



$\frac{1}{2}(2\pi + 5\pi) \times 220$

962.5

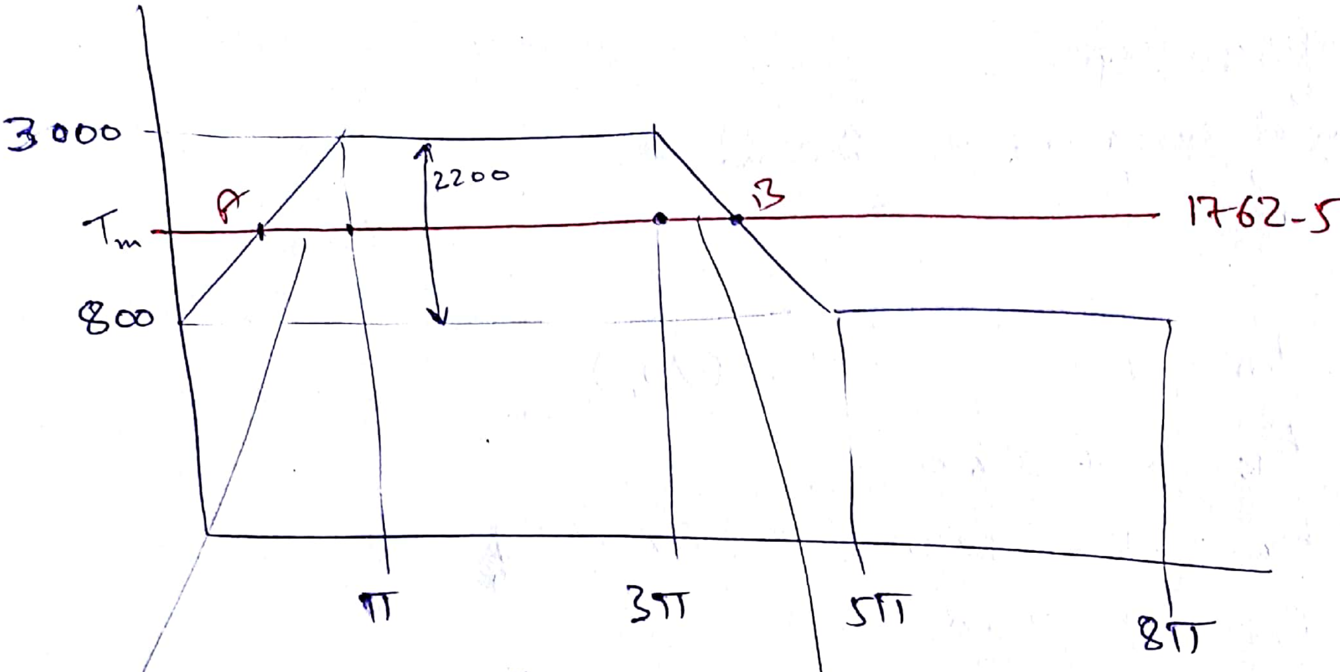
$T_m = 1762.5$ N-m

$N = 250$ rpm

Power = $T_m \omega$ = 46.14 KW

WD = 44.296 KW

$\Delta E_{max} =$



$$\left(\pi \times \frac{1237.5}{2200}\right) \qquad 2\pi \times \frac{1237.5}{2200}$$

K = .5 m
m = 1800 kg

$\Rightarrow 5.3014 + 2\pi$

$AB = 11.5846$

$$\Delta E_{max} = \frac{1}{2}(2\pi + 11.5846) \times 1237.5$$

$$= 11.055 \text{ KJ}$$

$$11.055 \times 10^3 = I \omega^2 C_s$$

$$\boxed{C_s = 0.0358}$$

Ques

$T = 800 + 180 \sin 3\theta$ N-m

θ = crank angle

$$\omega = \frac{2\pi \times 400}{60} \text{ rad/s}$$

The flywheel & other rotating parts attached to the engine have a mass of 350 kg at a radius of gyration of 220 mm.

find
1) T_m
2) ΔE
3) C_s

Soln

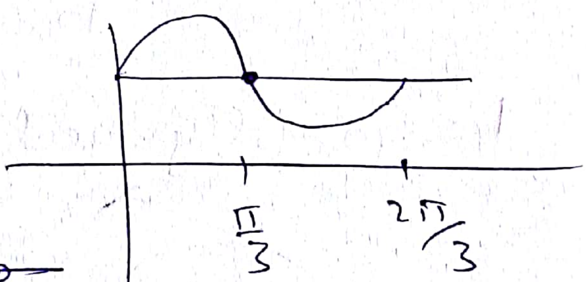
$T = 800 + 180 \sin 3\theta$ Time period $= \frac{2\pi}{3}$

$$WD = \int_0^{2\pi/3} \left(800 + 180 \sin 3\theta\right) d\theta$$

$$= 800\left(\frac{2\pi}{3} - 0\right) + 180 \times \frac{(-)}{3}\left[\cancel{\cos 2\pi} - \cancel{\cos 0}\right]$$

$$= 800 \times \frac{2\pi}{3} = 1675.5 \text{ J}$$

1) $\boxed{T_m = 800 \text{ Nm}}$



2) $T = T_{mean}$

$\cancel{800} + 180 \sin 3\theta = \cancel{800}$

$\sin 3\theta = 0$

$3\theta = 0, \pi$

$\boxed{\theta = 0, \pi/3}$

$$\Delta E_{max} = \int_0^{\pi/3} 180 \sin 3\theta \, d\theta$$

$$= 180 \times \left(\frac{-1}{3}\right)\left[\cos \pi - \cos 0\right]$$

$+2$

-1 $+1$

$\boxed{\Delta E_{max} = \frac{360}{3} \text{ J}} = \underline{120 \text{ J}}$

$360 = I\omega^2 C_s$

$\Rightarrow C_s = \frac{360}{250 \times (.22)^2 \times \omega}$

~~0.004~~

$\cancel{C_s = \frac{0.0169}{3}}$

$\cancel{= 5.63 \times 10^{-3}}$

Ques: A machine tool requires a torque of ~~100+20~~ $100 + 20 \sin\theta$ Nm where θ is crank angle. A flywheel with a movement of inertia of 20 Kgm² is attached. The max acceleration of the flywheel is.

Solⁿ $T = I\alpha$

$$\cancel{T_m} = \frac{1}{2\pi} \times \int_0^{2\pi} 100 + 20 \sin\theta$$

$T_m = 100$ N-m

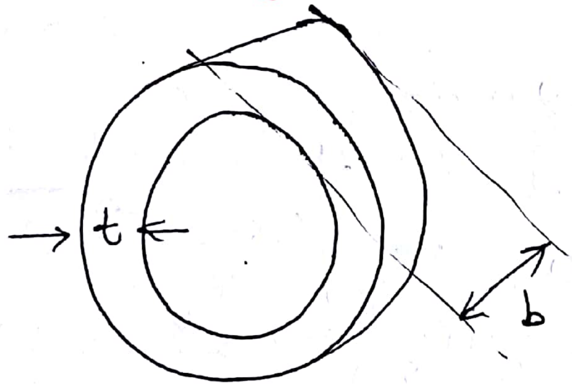
$T_{max} - T_m = I\alpha$

$120 - 100 = 20\alpha \Rightarrow \alpha = 1 \text{ rad/s}^2$

Design of flywheel

ρ = density

mass = $(\pi D t b)\rho$

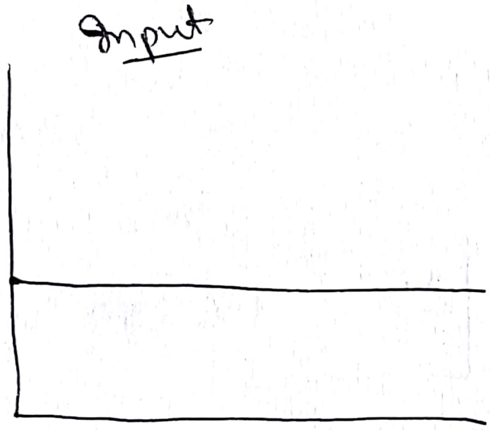


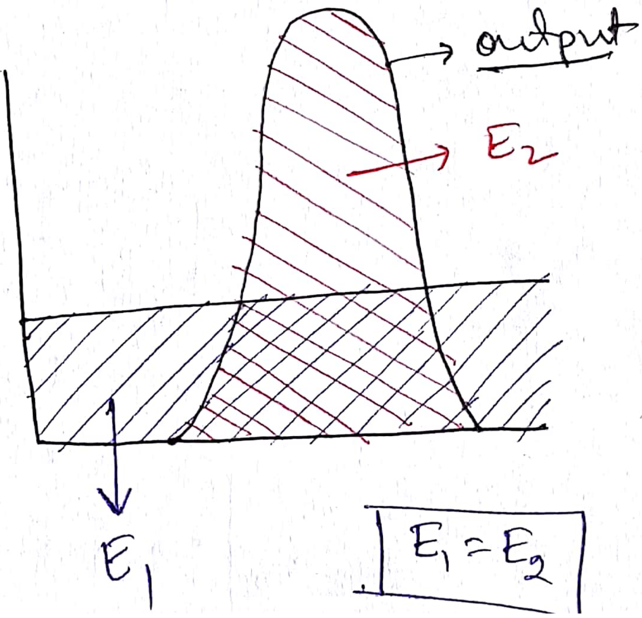
Hoop stress or centrifugal stress = ρv^2

here $v = r\omega = r \times \frac{2\pi N}{60} = \frac{\pi D N}{60}$

$$\boxed{\sigma_h = \rho v^2}$$

2nd Type Questions [Punching machine]





$\boxed{E_1 = E_2}$

Ques: A punching machine 38 mm hole in 32 mm thick plate requires 7 Nm of energy per square mm of sheared area and punches one whole in every 10 sec. Calculate the Power of motor required if mean speed of flywheel is 25 rad/s. The punch has a stroke of 100 mm. Find the moment of inertia of flywheel required if total fluctuation of speed is not exceeding 3% of mean speed.

Solu^n

$$\frac{\text{Energy}}{\text{Cycle}} = \pi \times 38 \times 32 \times 7 \text{ Nm}$$

$$= 26741.236 \text{ Nm}$$

$$\text{Power} = \frac{E}{10} = 2.674 \text{ KW}$$

$$C_s = 0.03$$

$$(\Delta E)_{max} = 26741.236 \times \left(1 - \frac{32}{2 \times 100}\right)$$

$$= 22.462 \text{ KJ}$$

$I = 1198 \text{ kgm}^2$

Quest: A punching press is driven by constant torque electric motor. The press is provided with a flywheel that rotates at maximum speed of 225 rpm. The radius of gyration is 0.5 m. The press punches 720 holes per hour. Each punching operation takes 2 second and requires 15 KNm of energy.

1) find power of motor.

2) maximum mass of flywheel if speed of same is not fall below 200 rpm.

Solu

Soluⁿ

$K = 0.5\text{ m}$ no. of cycle = 5 sec/cycle

~~N = 2 cycle/sec, N_1 = N_2 = rpm~~

$$\text{Power} = \frac{\text{Energy/cycle}}{\text{time/cycle}}$$

$$= \frac{15000}{5} = 3000\text{W} = 3\text{KW}$$

~~total energy/cycle = 3×5 = 15 KJ~~

~~Energy used during cycle = 3×2 = 6 KJ~~

$$(\Delta E_{max}) = 15000 - 6000 = 9000\text{ J}$$

$$9000 = \frac{1}{2} I \times \left(\frac{2\pi}{60}\right)^2 \left(N_{max}^2 - N_{min}^2\right)$$

$$I = 154.485 \text{ Kg m}^2 = mK^2$$

$$\boxed{m = 617.94\text{ kg}}$$

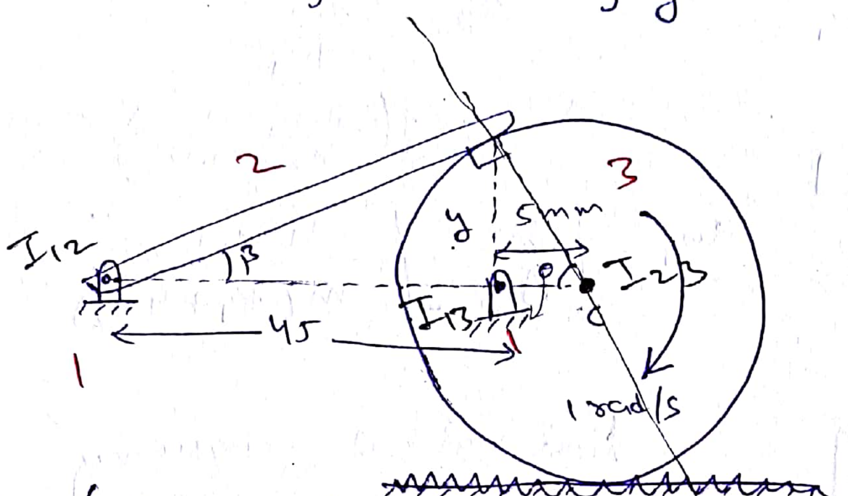
CHAPTER – 3

GYROSCOPIC COUPLE

[only GATE]

Ques: find angular velocity of bar.

$v = r\omega$



$\frac{d\beta}{dt} = ?$

$\tan\beta = \frac{y}{45}$

$\sec^2\beta\left(\frac{d\beta}{dt}\right) = \frac{1}{45}\frac{dy}{dt}$

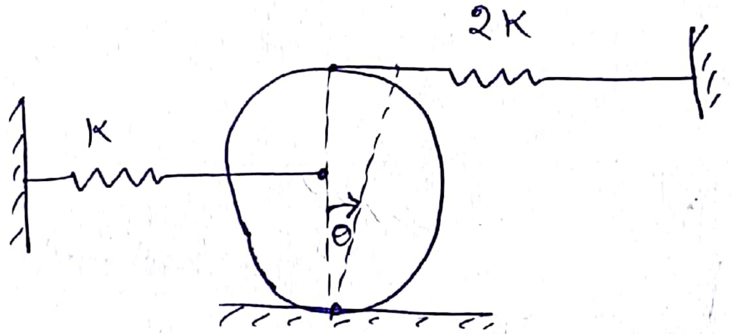
$y^2 + 25 = r^2$

$$\omega_2 (I_{23} I_{12}) = \omega_3 (I_{23} I_{13})$$

$$\omega_2 (50) = 1 \times 5$$

$$\omega_2 = \frac{5}{50} = 0.1 \text{ rad/s}$$

Ques:



find natural frequ

$\frac{mR^2}{2} + mR^2$

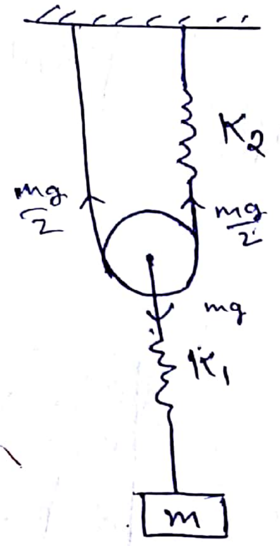
$\frac{3}{2} mr^2$

$$I\ddot{\theta} + Kr^2\theta + 2K(2r\theta)\times 2r = 0$$

$$\frac{3}{2}mr^2\ddot{\theta} + (Kr^2 + 8Kr^2)\theta = 0$$

$$\omega_n = \sqrt{\frac{3\,9K\cancel{r^2}}{\frac{3}{2}m\cancel{r^2}}} \Rightarrow \omega_n = \sqrt{\frac{6K}{m}}$$

Ques:



find natural frequency

$$F = kx \Rightarrow \frac{x}{4} = \frac{mg}{2 \times K_2}$$

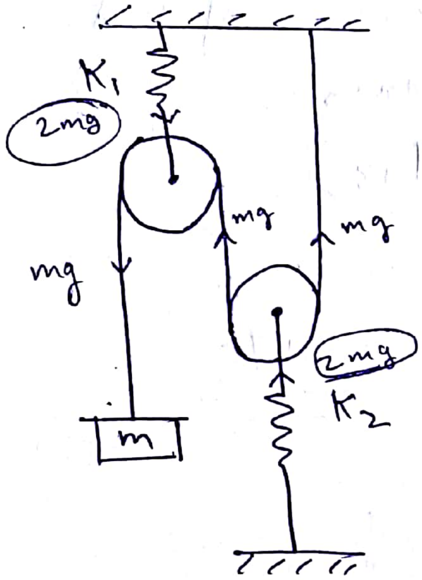
$$\Delta_1 = \frac{mg}{4K_2}$$

$$\Delta_2 = \frac{mg}{K_1}$$

$$\Delta = mg\left(\frac{1}{K_1} + \frac{1}{4K_2}\right) = mg\left(\frac{4K_2 + K_1}{4K_1K_2}\right)$$

$$\omega_n = \sqrt{\frac{g}{\Delta}} = \sqrt{\frac{4K_1K_2}{m(K_1 + 4K_2)}}$$

Ques:



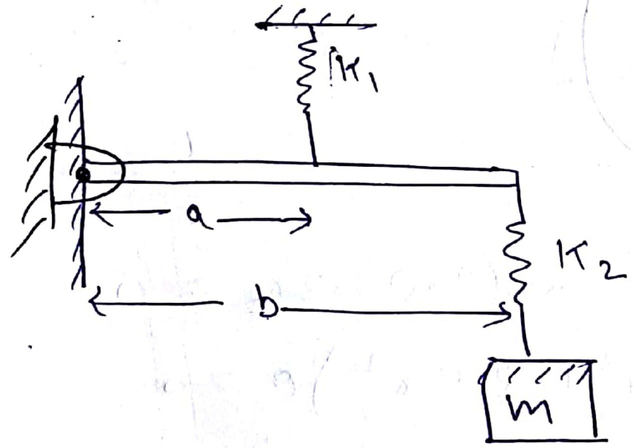
find natural frequency

$$\uparrow \frac{2mg}{K_2}$$

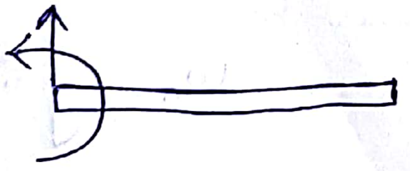
$$\Delta = \frac{4mg}{K_2} + \frac{4mg}{K_1} = 4mg\left(\frac{K_1 + K_2}{K_1K_2}\right)$$

$$\omega_n = \sqrt{\frac{K_1K_2}{4m(K_1 + K_2)}}$$

Ques:

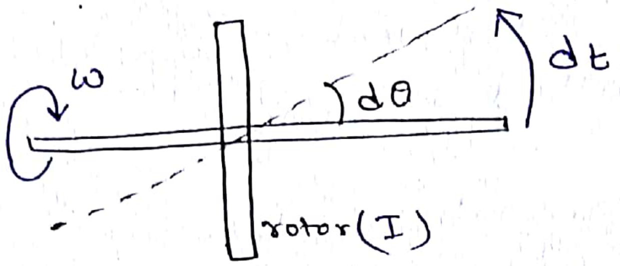


find natural frequency



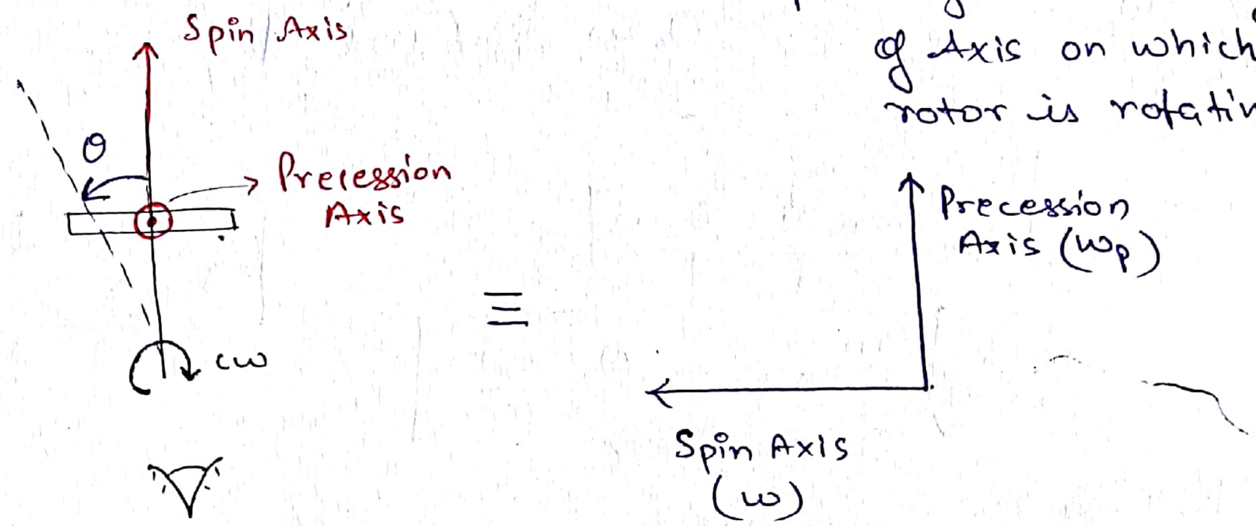
Gyroscopic couple

gyroscopic couple $= L \frac{d\theta}{dt}$

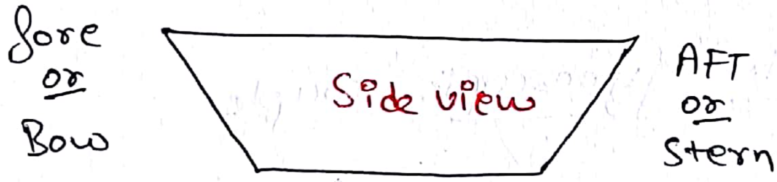


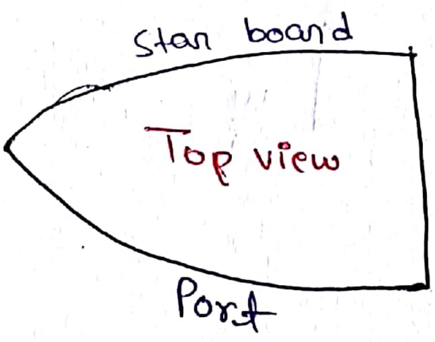
$$\Rightarrow \boxed{C_g = I \omega \omega_p}$$

C_g = gyroscopic couple
ω = angular velocity of rotor
ω_p = angular velocity of Axis on which rotor is rotating

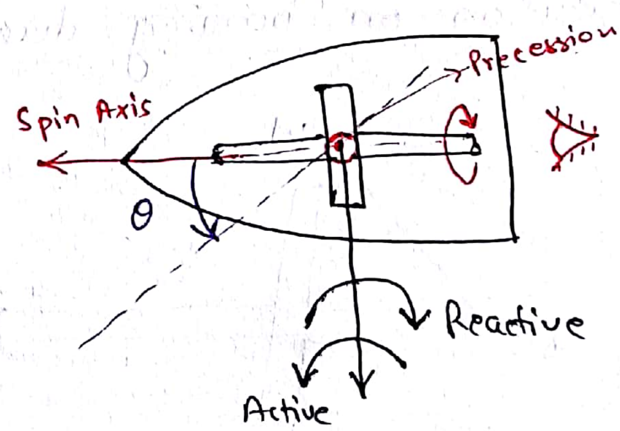


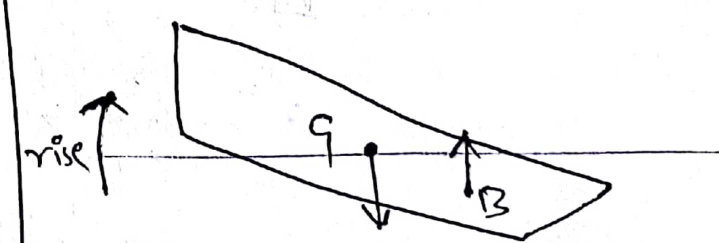
Ship





To find Direction of C_g





Steering :- moving left or right during forward motion, is known as steering.

Pitching :- up & down movement from horizontal plane is called pitching.

Assume simple harmonic motion

$$\theta = \theta_0 \cos(\omega t - \phi)$$

$$\omega_p = \frac{d\theta}{dt} = -\theta_0 \omega \sin(\omega t - \phi)$$

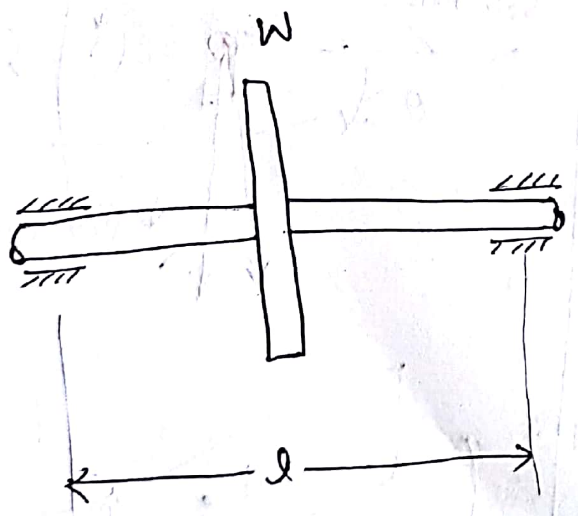
$$\boxed{(\omega_p)_{max} = \theta_0 \omega}$$

$$\boxed{(C_g)_{max} = I\omega(\omega_p)_{max}}$$

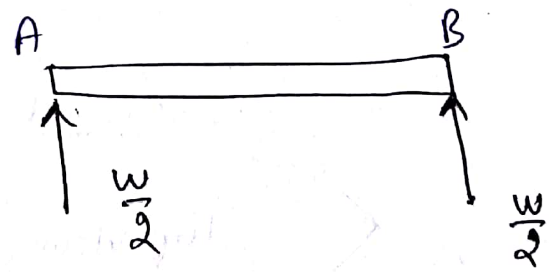
$$\boxed{(\omega_p)_{max} = \theta_0 \frac{2\pi}{T}}$$

Rolling : In rolling spin Axis become coniciding or parallel to precession axis so there will be no gyroscopic couple.

Reaction on bearing due to gyroscopic couple

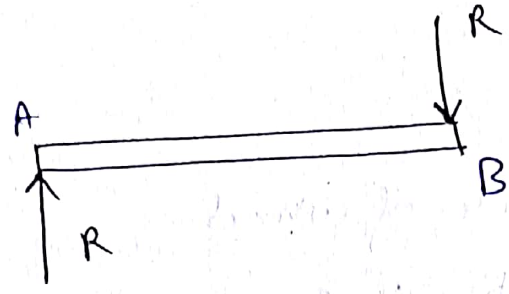


1) due to self wt. of shaft & rotor



2) due to gyroscopic couple

Let



$$Rl = I\omega\omega_p$$

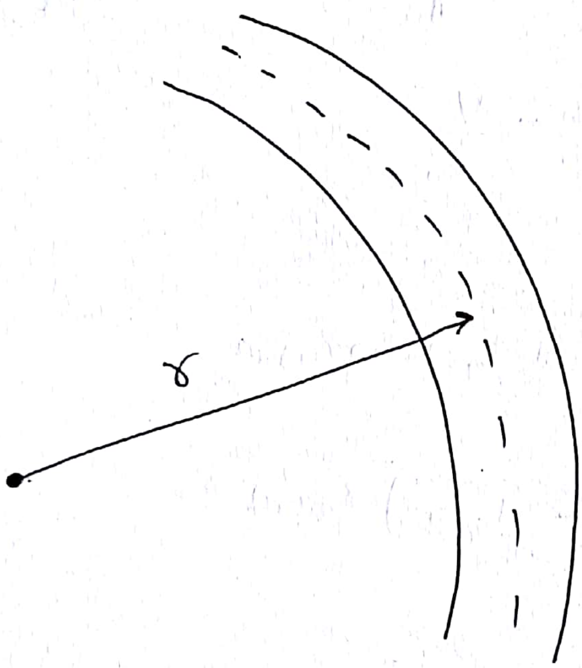
$$R = \frac{I\omega\omega_p}{l}$$

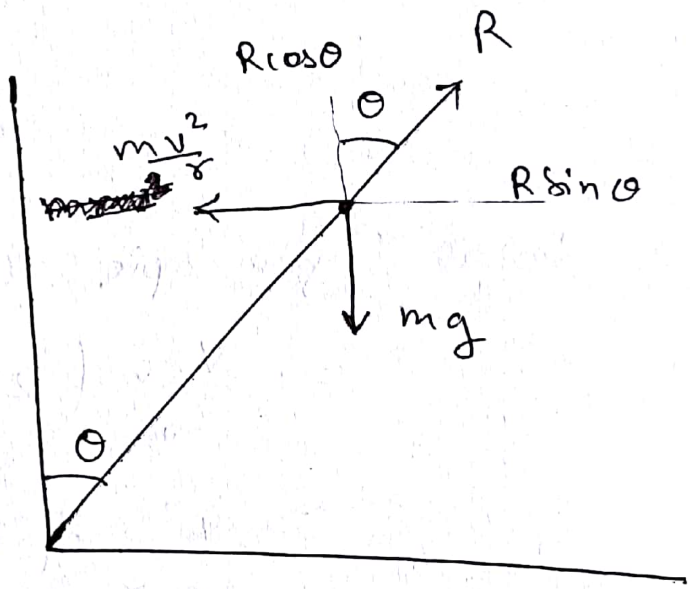
net reaction ⇒ A & B = 1 + 2

$$R_A = R + \frac{W}{2}$$

$$R_B = R - \frac{W}{2}$$

Gyroscopic couple in two wheeler





$$R\sin\theta = \frac{mv^2}{r}$$

$$\sin\theta = \frac{mv^2}{Rr}$$

$$R\cos\theta = mg$$

$$\cos\theta = \frac{mg}{R}$$

$$\Rightarrow \boxed{\tan\theta = \frac{v^2}{rg}}$$

r = radius of curvature

r_w = radius of wheel

G = gear ratio = $\frac{\omega_E}{\omega_w}$

I_w = moment of inertia of wheel

I_e = moment of inertia of engine

total angular momentum $= 2I_w\omega_w \pm I_e\omega_e$

$$= 2I_w\omega_w \pm GI_E\omega_w$$

$$= \omega_w(2I_w \pm GI_E)$$

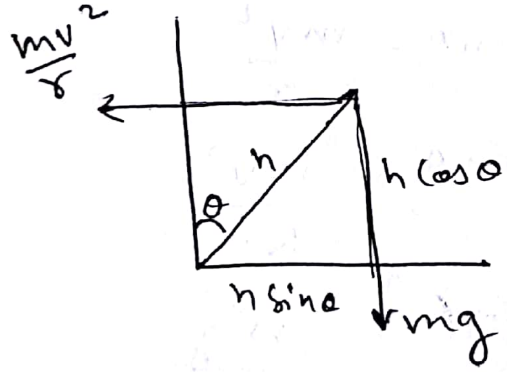
total angular momentum $= \frac{V}{r_w} \times (2I_w \pm GI_E)$

Let, $\omega_w = \frac{V}{r_w}$

total Gyroscopic couple

$$= \frac{V}{r_w} \times (2I_w \pm GI_E)\,\omega_p \cos\theta$$

$$C_g = \frac{V^2}{r_w r} \times (2I_w \pm GI_E)\cos\theta$$



<u>For stability</u>

$$\boxed{\frac{mv^2}{r}h\cos\theta + \frac{V^2}{r_w r}(2I_w + GI_e)\cos\theta = mgh\sin\theta}$$

Ques: A turbine of a ship has a mass of 3500 Kg and rotates at a speed of 2000 rpm. The rotor has a radius of gyration of 0.5 m and rotates in clockwise direction when viewed from the stern. Determine the magnitude of gyroscopic couple and its directio[n] for the following conditions.

1) when the ship runs at a speed of 6.2 m/s and steers to the left in a curve of 70 m radius

2) when the ship pitches 6° above or below the horizontal Position and the bow is lowered the pitching motion is simple harmonic motion with period 30 sec.

3) when the ship rolls and at certain instant it has velocity of 0.5 rad/s. clockwise.

Solu[n]

1) $m = 3500$ Kg $\qquad I = mk^2 =$

$N = 2000$ rpm

$K = 0.5$ m

rise from front

$v = r\omega \Rightarrow \omega_p = \frac{6.2}{70}$

$C_g = I\omega\omega_p$

$= 16231.56$ J

$= 16.231$ KJ

2) $(\omega_p)_{max} = \theta_0 \times \frac{2\pi}{T} = \frac{\pi}{180} \times 6 \times \frac{2\times\pi}{30}$

towards right
(when upward going) Bow

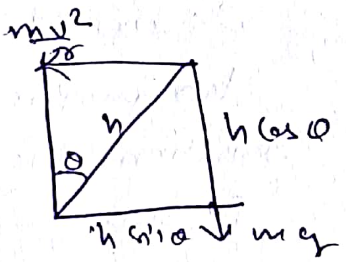
$C_g = I\omega(\omega_p)_{max} = 4019.33$ J

$= 4.019$ KJ

3) $C_g = 0$

⊗ towards left
(when going down) Bow

Ques: A motorcycle and its rider together weight 2000 N and their combined center of gravity is 550 mm above the ~~gravity~~ road when motorcycle is upright. Each wheel is of 580mm diameter and has a moment of intertia of $1\ kgm^2$. The moment of inertia of rotating parts of engine is $0.15\ kg\ m^2$. the engine rotates 5 times the speed of vehicle and in the same sense. Determine the angle of heel necessary when motorcycle is taking a turn over a track of 35m radius at speed of 60 Km/hr.



Solu^n

$$\boxed{60 \times \frac{5}{18} = \frac{50}{3}}$$

$$h \times mg \sin\theta = \frac{mv^2}{r} \times .55 \times \cos\theta + \frac{v^2}{r r_w} \times (2I_w + 5I_e)$$

$$.55 \times 2000 \times \sin\theta = \frac{2000}{9.81} \times \frac{2500}{9 \times 35} \times .55 \times \cos\theta + \frac{2500 \times 2}{9 \times 35 \times .58} \times (2 \times 1 + 5 \times 0.15)$$

$$1100 \sin\theta = 889.92 \cos\theta + 75.26 \cos\theta$$

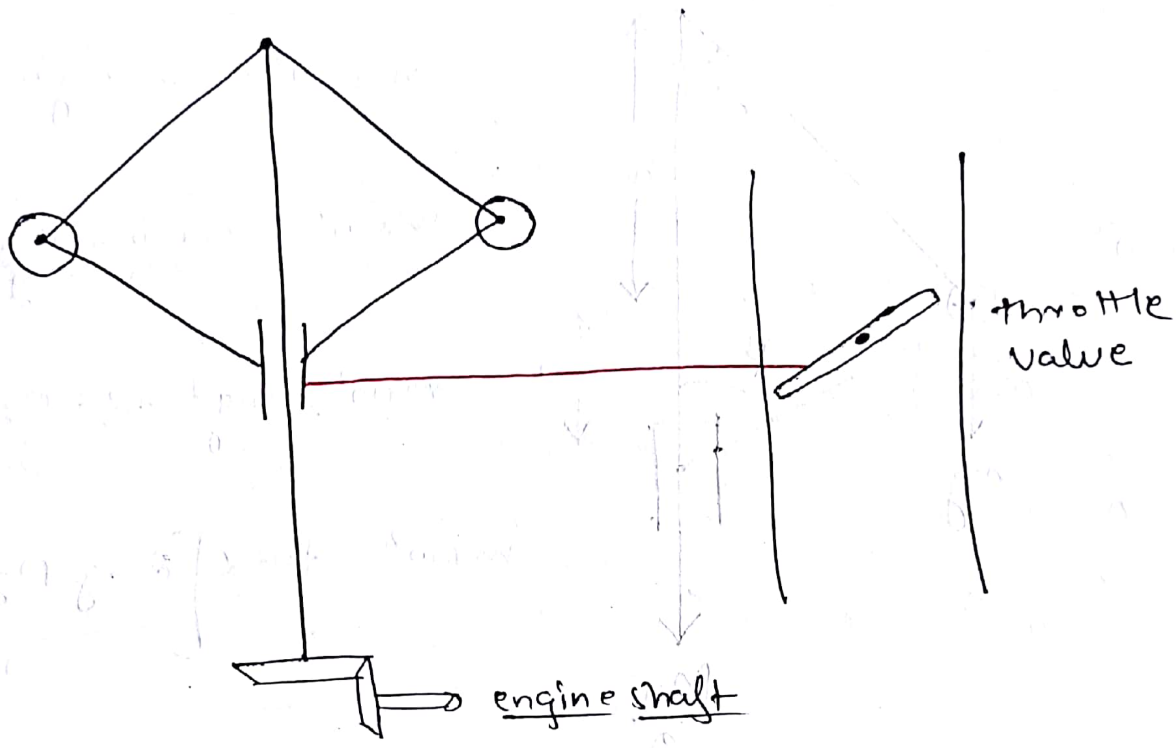
~~$\Rightarrow \theta \simeq 42°$~~

$$\Rightarrow \tan\theta = 0.87743$$

$$\boxed{\theta = 41.26°}$$

CHAPTER - 4

GOVERNER

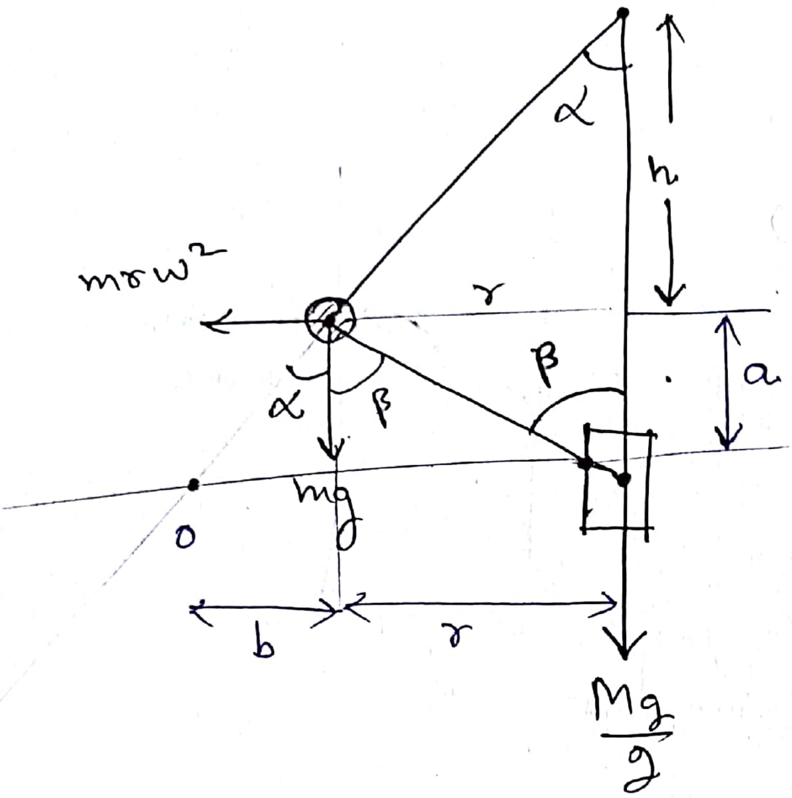


Governer controls fluctuation of speed of engine by controlling fuel supply.

Types of centrifugal Governer

- Watt governer
- loaded type
  - dead load
    - Porter governer
    - Proell governer
  - spring controlled
    - Hartnell
    - Hartung
    - wilson Hartnell
    - Pickering

1) Porter governer



$$\Sigma M_o = 0$$

$$mr\omega^2 a = mg\,b + \frac{Mg}{2}(b+r)$$

$$mr\omega^2 = mg\frac{b}{a} + \frac{Mg}{2}\left(\frac{b}{a}+\frac{r}{a}\right)$$

$$mr\omega^2 = mg\tan\alpha + \frac{Mg}{2}\left[\tan\alpha + \tan\beta\right]$$

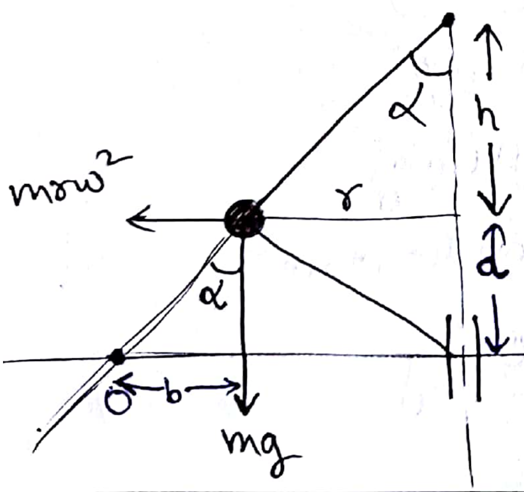
$$mr\omega^2 = \tan\alpha\left[\frac{2mg + Mg\left(1+\frac{\tan\beta}{\tan\alpha}\right)}{2}\right]$$

$$m\cancel{r}\,\omega^2 = \frac{\cancel{r}}{h}\left[\frac{2mg + Mg(1+K)}{2}\right]$$

$$\boxed{\omega^2 = \left(\frac{2mg + Mg(1+K)}{2mg}\right)\frac{g}{h}}$$

here $K = \frac{\tan\beta}{\tan\alpha}$

2) watt governer



$$\Sigma M_o = 0$$

$$\cancel{m}r\omega^2 a = \cancel{m}g\,b$$

$$r\omega^2 = g\frac{b}{a}$$

$$r\omega^2 = g\tan\alpha$$

$$\cancel{r}\omega^2 = g\frac{\cancel{r}}{h}$$

$$\boxed{\omega^2 = \frac{g}{h}}$$

$$\Rightarrow N^2 = \frac{895}{h}$$

Note : 1) Porter governer

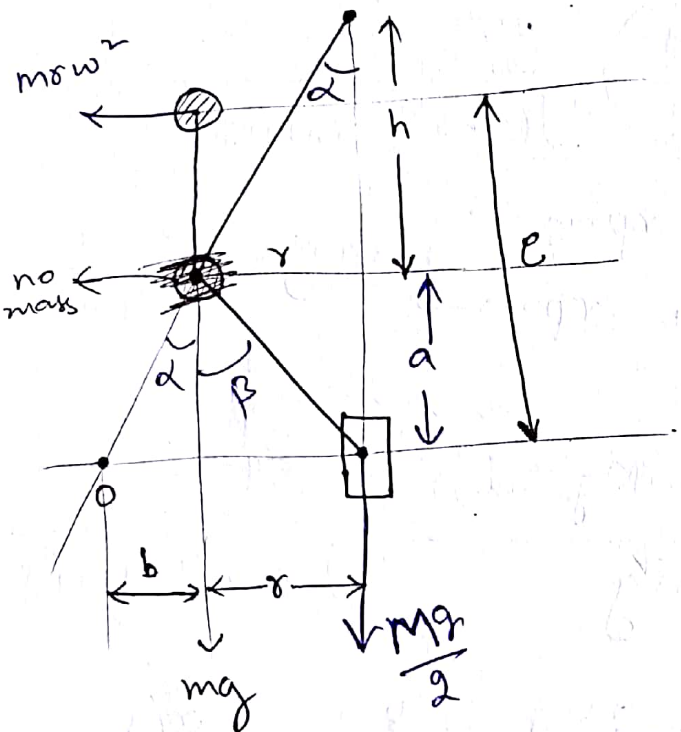
- $$N^2 = \left[\frac{2mg + Mg(1+K)}{2mg}\right] \frac{895}{h}$$ [without friction]

for $K = 1$

$$\boxed{N^2 = \left(\frac{m+M}{m}\right)\frac{895}{h}}$$

- $$N^2 = \left[\frac{2mg + (Mg \pm f)(1+K)}{2mg}\right]\frac{895}{h}$$ with friction

3 Proell Governer



$$\Sigma M_o = 0$$

$$mr\omega^2 e = mg\,b + \frac{Mg}{2}(b+r)$$

$$N^2 = \frac{a}{e}\left[\frac{2mg + (Mg \pm f)(1+K)}{2mg}\right]\frac{895}{h}$$

for $f = 0$ & $K = 1$

$$\boxed{N^2 = \frac{a}{e}\left(\frac{m+M}{m}\right)\frac{895}{h}}$$

Ques: Each arm of Porter governer is 200 mm long and is pivoted on the axis of governer. The radius of rotation of balls at the minimum and maximum speeds are 120 mm and 160 mm respectively. The mass of sleeve is 24 kg and each ball is 4 kg. Find the range of governer. Also determine the range of speed if friction at the sleeve is 18 N.

Soluⁿ 1) no friction

$$N^2 = \left(\frac{2mg + Mg(1+k)}{2mg} \right) \frac{895}{h}$$

$$N_1^2 = \left(\frac{4 + 24}{4} \right) \frac{895}{\sqrt{(200)^2 - (120)^2}}$$

$N_1 = 197.88$ rpm

$N_2 = 228.49$ rpm

rang = $\underline{30.61}$ rpm

2)

$$N_1^2 = \left(\frac{\cancel{2}mg + \cancel{2}(Mg \pm f)}{\cancel{2}mg} \right) \frac{895}{\sqrt{(.2)^2 - (.12)^2}}$$

$$= \frac{(4g + 24g - 18)}{4g} \times \frac{895}{\sqrt{(.2)^2 - (.12)^2}}$$

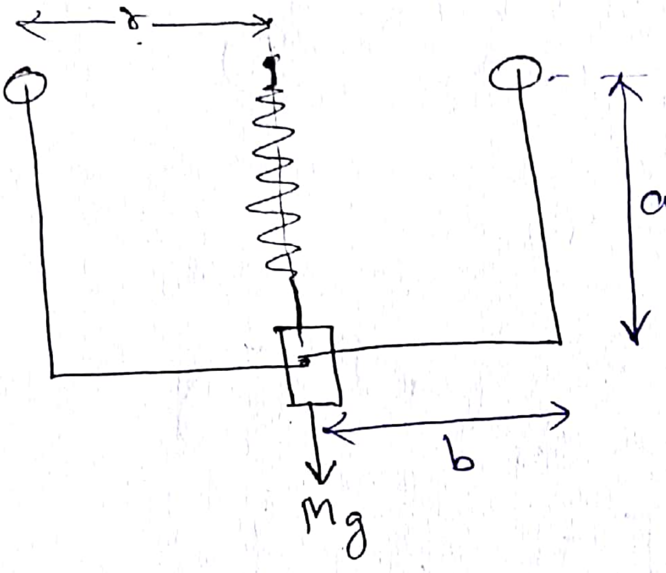
$N_1 = 191.28$ rpm

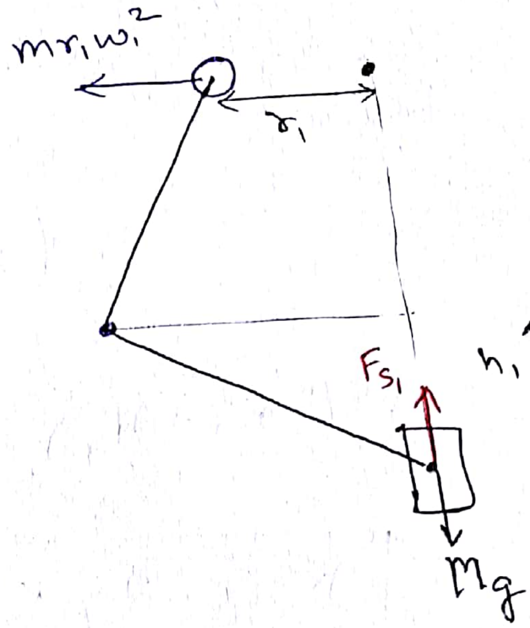
$N_2 = 235.86$ rpm

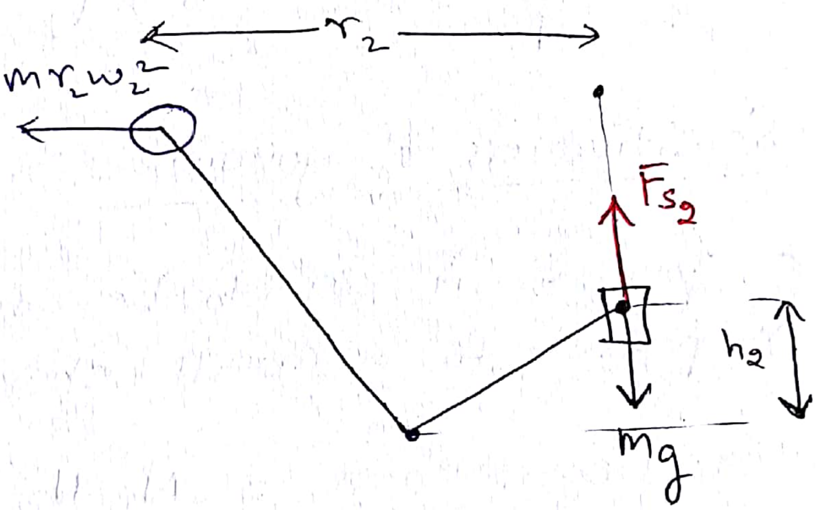
rang = $\underline{44.58}$ rpm

Spring loaded governers

1. Hartnell governer







F_{s_2}, F_{s_1} ⇒ spring force

$F_{c_1} = m r_1 w_1^2$

$F_{c_2} = m r_2 w_2^2$

M = mass of sleev

total lift = $h_1 + h_2$

$\Sigma M_o = 0$

$$F_{c_1} \times a + \frac{F_{s_1}}{2} \times b - \frac{Mg}{2} . b = 0 \quad \text{—— (1)}$$

$$F_{c_2} \times a + \frac{F_{s_2}}{2} . b - \frac{Mg}{2} . b = 0 \quad \text{—— (2)}$$

(1) - (2)

$$F_{s_2} - F_{s_1} = 2\left(\frac{a}{b}\right)(F_{c_1} - F_{c_2})$$

$$K(h_1 + h_2) = \frac{2a}{b}(F_{c_1} - F_{c_2})$$

$$\frac{h_1}{r-r_1} = \frac{b}{a} \quad ; \quad h_1 = \frac{b}{a}(r-r_1)$$

$$\frac{h_2}{r_2-r} = \frac{b}{a} \quad ; \quad h_2 = \frac{b}{a}(r_2-r)$$

$$\Rightarrow h_1+h_2 = \frac{b}{a}(r_2-r_1)$$

$$\Rightarrow K(h_1+h_2) = 2\frac{a}{b}(F_{c_1}-F_{c_2})$$

$$\boxed{K = 2\left(\frac{a}{b}\right)^2\left[\frac{F_{c_1}-F_{c_2}}{r_2-r_1}\right]}$$

Sensitiveness of governer

a governer is said to be sensitive when it readly response to a small change

- Sensitiveness $= \dfrac{N_1-N_2}{N} = \dfrac{N_1-N_2}{(N_1+N_2)/2} = \dfrac{2(N_1-N_2)}{(N_1+N_2)}$
- Sensitivity = It is defined for isolated

$$= \frac{N}{N_1-N_2}$$

- Isochronous governer sensitivity $= \infty$

$$\Rightarrow N_1 = N_2$$

Hunting

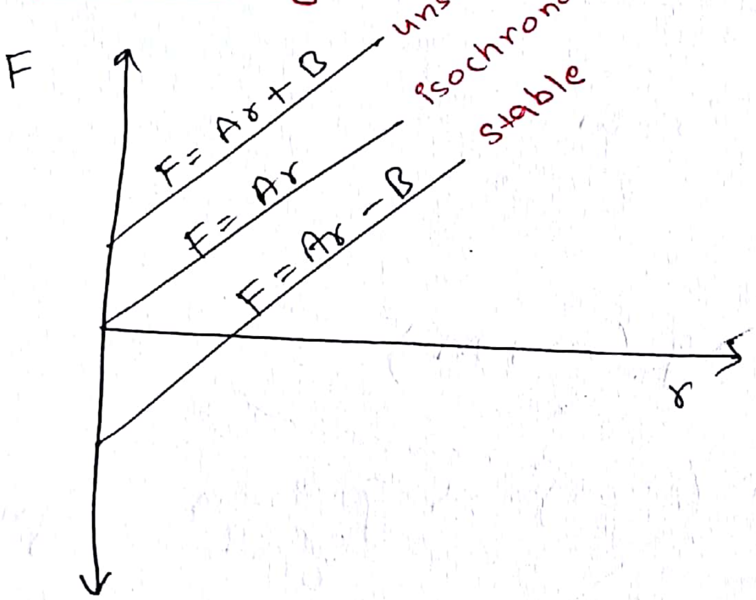
A governer is said to be hunting if the speed of the engine fluctuates continously above or belo[w] the mean speed..

Isochronous

- means range $= N_1 - N_2 = 0$
- for Porter governer
  $N_1 = N_2 \Rightarrow h_1 = h_2 \Rightarrow$ no meaning
- for Hartnell governer

$$N_1 = N_2 \Rightarrow \frac{r_2}{r_1} = \frac{Mg - F_{s2}}{Mg - F_{s1}}$$

Stability



Ques.:

$F = 15\ N \qquad r = 20\ mm$

$\quad\ \ 40\ N \qquad r = 60\ mm$

$F = Ar + B$

$\Rightarrow A = \frac{5}{8} = 0.625 \qquad B = +ve$

$B = 2.5 \qquad$ i.e unstable

Effort of governer

the effort of the governer is the mean force acting on the sleeve to raise or lower for a given change of speed at constant speed the governer is in Equilibrium and the resultant force acting on the sleeve is zero.

$$\left[\frac{2mg + Mg(1+K)}{2mg}\right]\frac{g}{\omega^2} = \left[\frac{2mg + (Mg+E)(1+K)}{2mg}\right]\frac{g}{(1+c)^2\omega^2}$$

$$\Rightarrow \boxed{\text{governer effort} = \frac{E}{2} = \frac{cg}{(1+K)}\left(2m + M(1+K)\right)}$$

$c = \%$ increase in ω

- for $K = 1$

$\Rightarrow$ governer effort $= cg(m+M)$

$\Rightarrow$ for watt governer $(M=0)$ $\Rightarrow$ governer effort $= cgm$

CHAPTER-5

BALANCING

- Static Balancing $\Rightarrow$ only forces are balanced

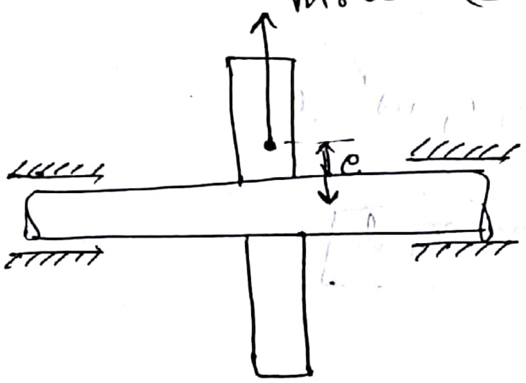
$$\Sigma F = 0$$

- Dynamic Balancing $\Rightarrow$ forces as well as moments are balanced.

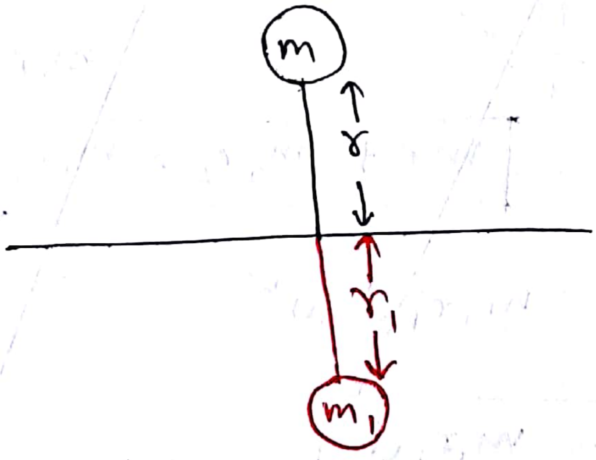
$$\Sigma F = 0$$
$$\Sigma M = 0$$

- Balancing involves redistribution of mass which may be carried out by addition ~~with~~ or removal of mass from to various machine members.

$mr\omega^2$ (centrifugal forces)



A) Balancing of single rotating mass by single mass in same plane

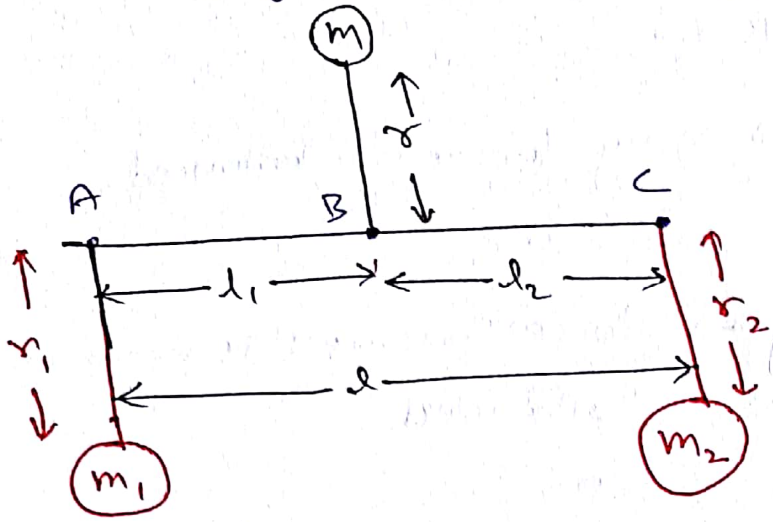


$$F = F_1$$

$$m r \cancel{\omega^2} = m_1 r_1 \cancel{\omega^2}$$

$$m r = m_1 r_1$$

B) Balancing of single mass by two mass



$$\Sigma F = 0 \Rightarrow m r \cancel{\omega^2} = m_1 r_1 \cancel{\omega^2} + m_2 r_2 \cancel{\omega^2}$$

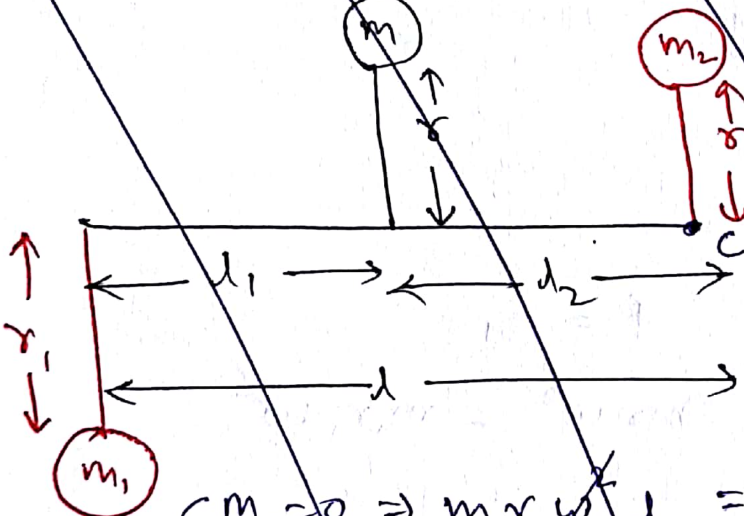
$$\boxed{m r = m_1 r_1 + m_2 r_2}$$

$$\Sigma M_c = 0$$

$$m r \cancel{\omega^2}\, l_2 = m_1 r_1 \cancel{\omega^2}\, l$$

$$\boxed{m r\, l_2 = m_1 r_1\, l}$$

c) Balancing of single mass by two mass



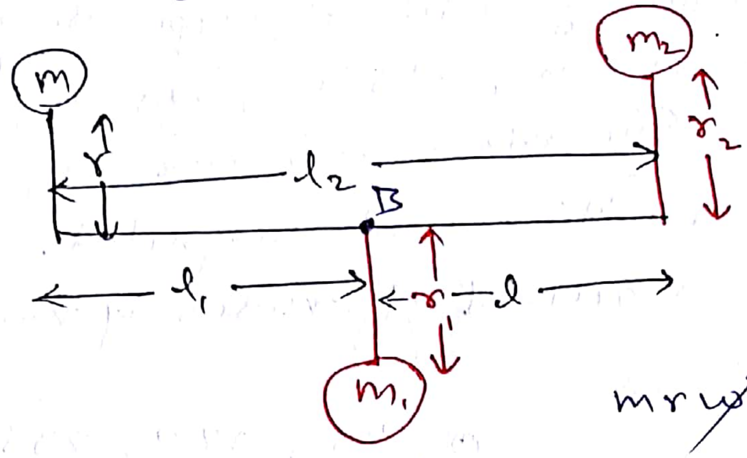
$$m r \cancel{\omega^2} + m_2 r_2 \cancel{\omega^2} = m_1 r_1 \cancel{\omega^2}$$

$$\boxed{m r + m_2 r_2 = m_1 r_1}$$

$$\Sigma M_c = 0 \Rightarrow m r \cancel{\omega^2}\, l_2 = m_1 r_1\, l\, \cancel{\omega^2}$$

$$\boxed{m r\, l_2 = m_1 r_1\, l}$$

D) Balancing of single mass by two masses



$$m r \cancel{\omega^2} + m_2 r_2 \cancel{\omega^2} = m_1 r_1 \cancel{\omega^2}$$

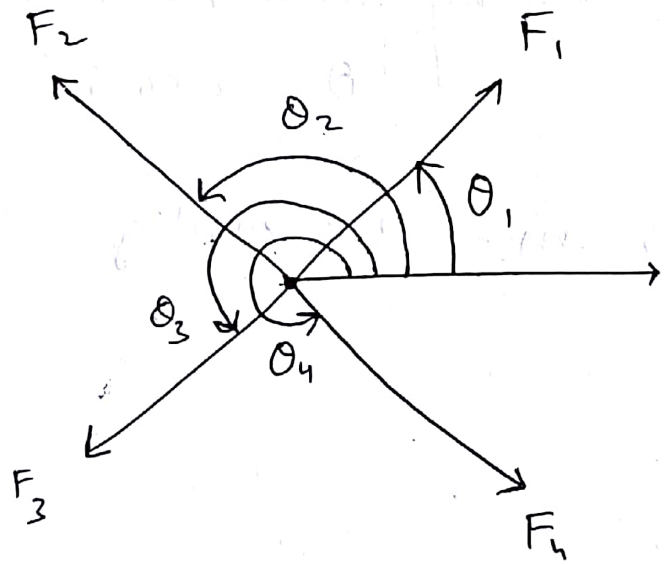
$$\boxed{m r + m_2 r_2 = m_1 r_1}$$

$$\Sigma M_B = 0$$

$$m r \cancel{\omega^2} l_1 = m_2 r_2 \cancel{\omega^2} l$$

$$\boxed{m r l_1 = m_2 r_2 l}$$

E) Different forces acting on same plane



$$\Sigma F_H = m_1 r_1 \omega^2 \cos\theta_1 + m_2 r_2 \omega^2 \cos\theta_2 + m_3 r_2 \omega^2 \cos\theta_3 + m_4 r_4 \omega^2 \cos\theta_4$$

$$\Sigma F_V = m_1 r_1 \omega^2 \sin\theta_1 + m_2 r_2 \omega^2 \sin\theta_2 + m_3 r_3 \omega^2 \sin\theta_3 + m_4 r_4 \omega^2 \sin\theta_4$$

Ques: four masses m_1, m_2, m_3, m_4 are 200 kg, 300 kg, 240 kg, and 260 kg. the corresponding radius of rotation are 0.2 m, .15 m, .25 m, .3 m and angle b/w successive masses are 45°, 75°, 135°. Find the position and magnitude of the Balancing mass required if radius of rotation is 0.2 m.

Solu:

0, 45, 120, 255

$200 \times .2 \times \cos \cancel{45}$

$+ 300 \times .15 \times \cos(\cancel{75+45})$

$240 \times .25 \times \cos(\cancel{135+45+75})$ $\Rightarrow$ ~~m cos θ = -377.52~~

$260 \times .3 \times \cos \cancel{\phi}(255)$

~~m sin θ = -570.886~~

$m \times .2 \times \cos\theta \quad = 0$

$\Rightarrow$ ~~tan θ = 1.5122~~

~~+180°~~

~~θ = 56.524~~

$m\cos\theta = -21.6319/.2$

$m\sin\theta = -8.439/.2$

~~kg 684.42~~ = m ~~m = 684.42 kg~~

~~θ = 236.52°~~

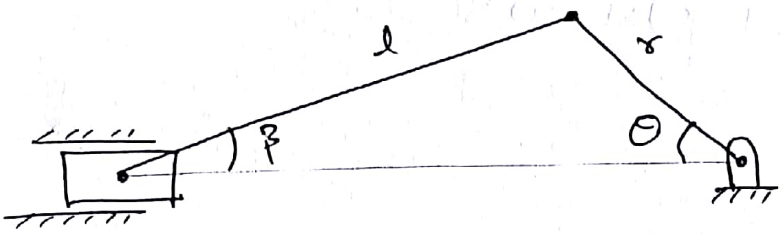
$\tan\theta = .39$

$\theta = 21.31° + 180°$

$= 201.31°$ $\Rightarrow$ $m = 23.21$ kg

E) Balancing of Different masses rotating in different plane.

Balancing of reciprocating system

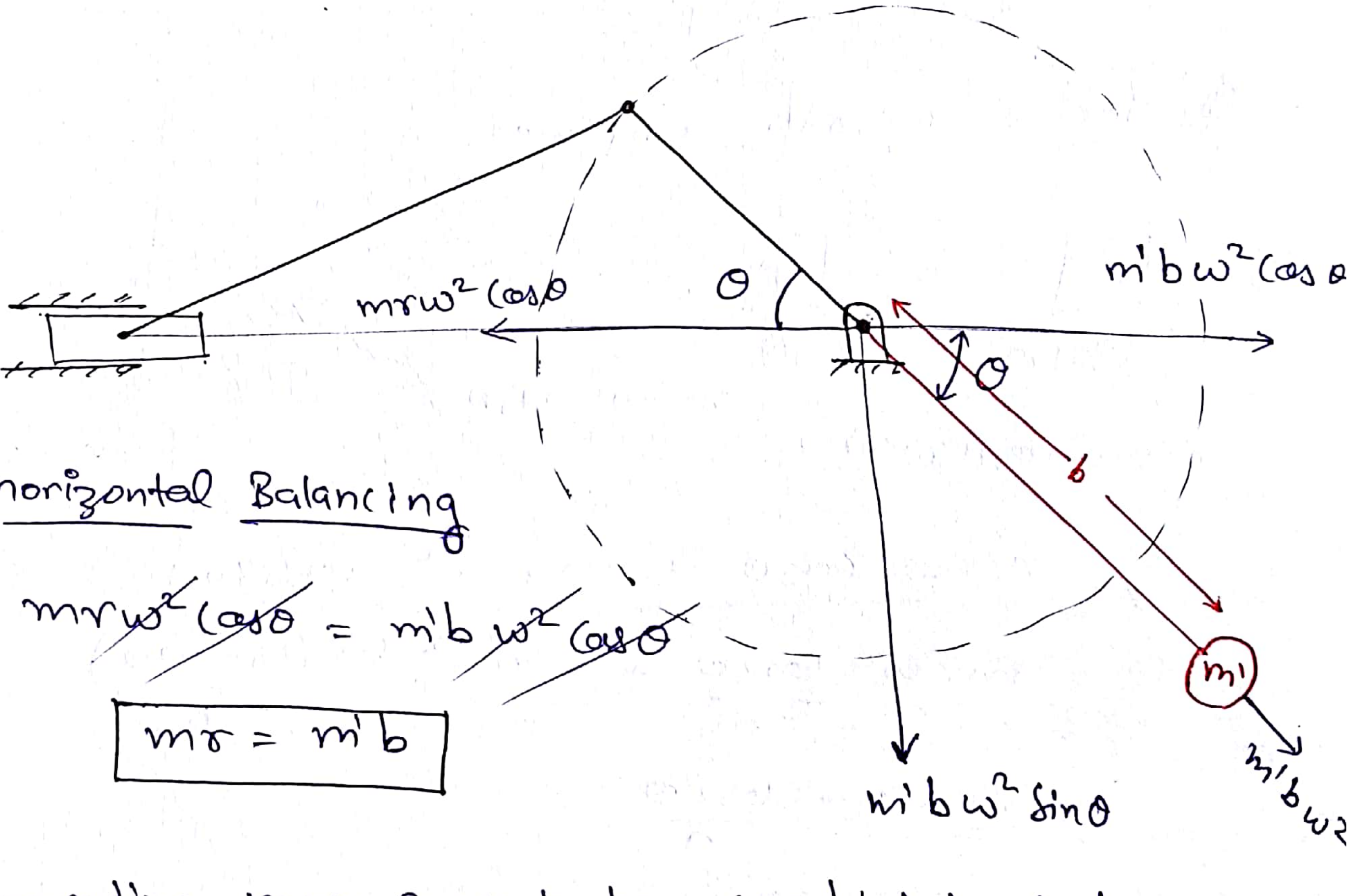


$n = \frac{l}{r}$

$$F_i = mr\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

Primary unbalanced force = $mr\omega^2 \cos\theta$

Secondary unbalanced force = $mr\omega^2 \frac{\cos 2\theta}{n}$



For horizontal Balancing

$$mr\cancel{\omega^2}\cancel{\cos\theta} = m'b\,\cancel{\omega^2}\cancel{\cos\theta}$$

$$\boxed{mr = m'b}$$

- Reciprocating mass cannot be completely balanced vertical unbalance always remains.

⇒ Horizontal unbalance force $= mr\omega^2 \cos\theta$

⇒ Vertical unbalance force $= mr\omega^2 \sin\theta$

Tractive force

$$F_{Tractive} = (1-c)mr\omega^2(\cos\theta - \sin\theta)$$

$$(F_T)_{max} = \pm\sqrt{2}(1-c)mr\omega^2$$

Swaying couple

$$M_c = (1-c)mr\omega^2 \cos\theta \frac{a}{2} - (1-c)mr\omega^2 \cos(90+\theta)\frac{a}{2}$$

$$= (1-c)mr\omega^2 \frac{a}{2}(\cos\theta + \sin\theta)$$

$$(M_c)_{max} = \pm\frac{(1-c)}{\sqrt{2}}mr\omega^2 . a$$

⇒ In line 2 cylinder engine

$180+\theta$ θ

$mr\omega^2\cos\theta$

$mr\omega^2\cos(180+\theta)$

$$F_p = mr\omega^2\cos\theta + mr\omega^2\cos(180+\theta) = 0$$

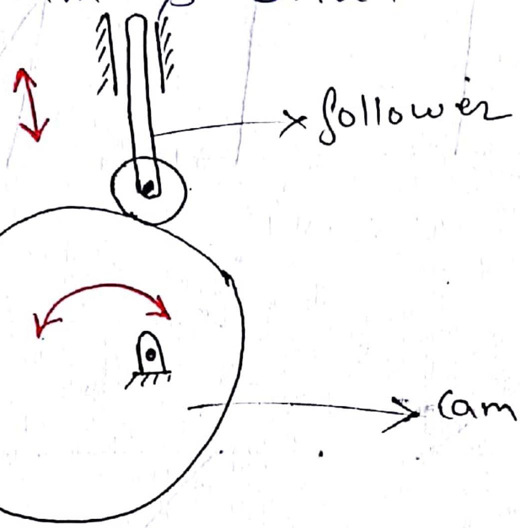
$$F_s = mr\omega^2\frac{\cos 2\theta}{n} + mr\omega^2\frac{\cos(360+2\theta)}{n}$$

$$= 2mr\omega^2\frac{\cos 2\theta}{n}$$

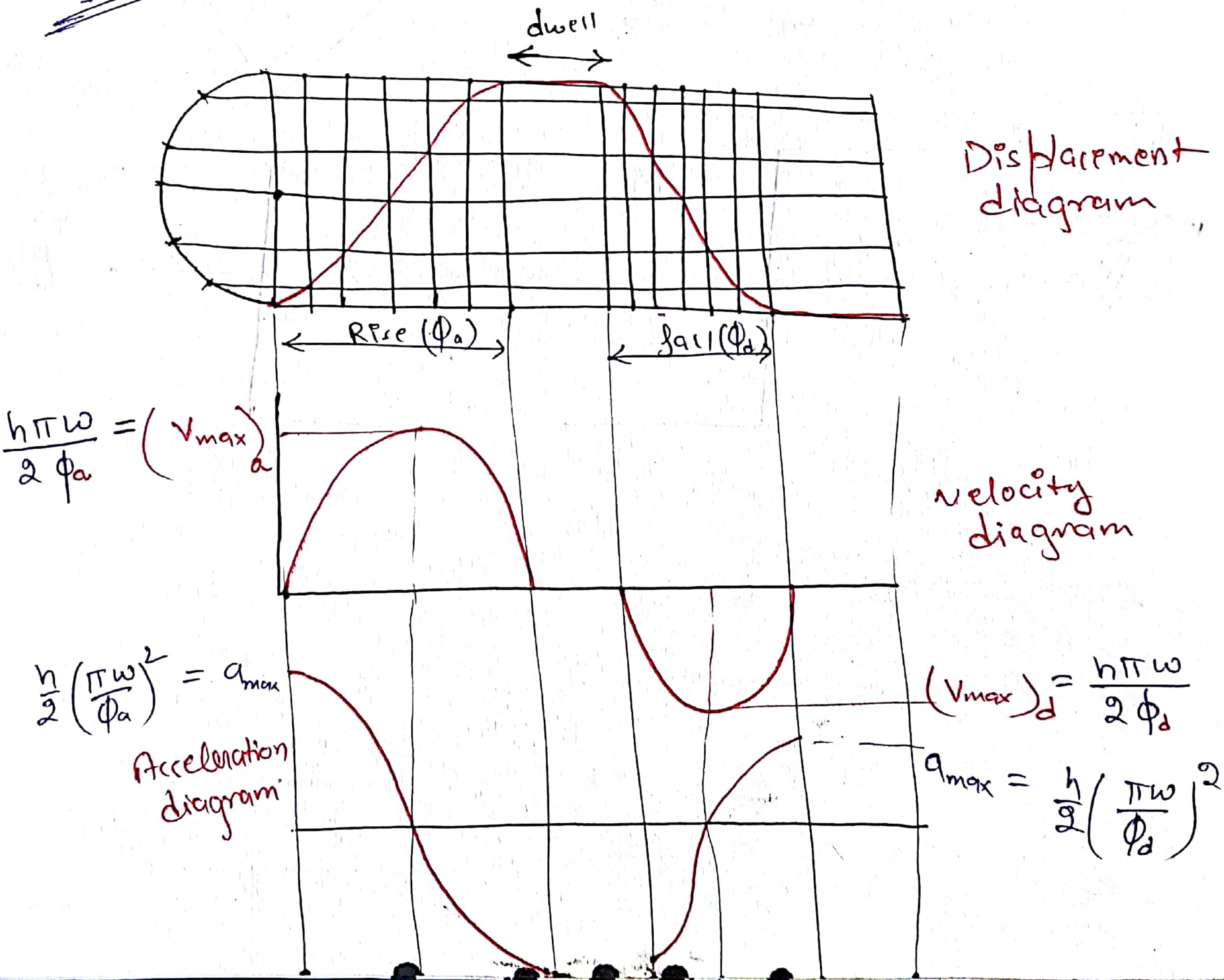
CHAPTER-6

CAM & FOLLOWERS

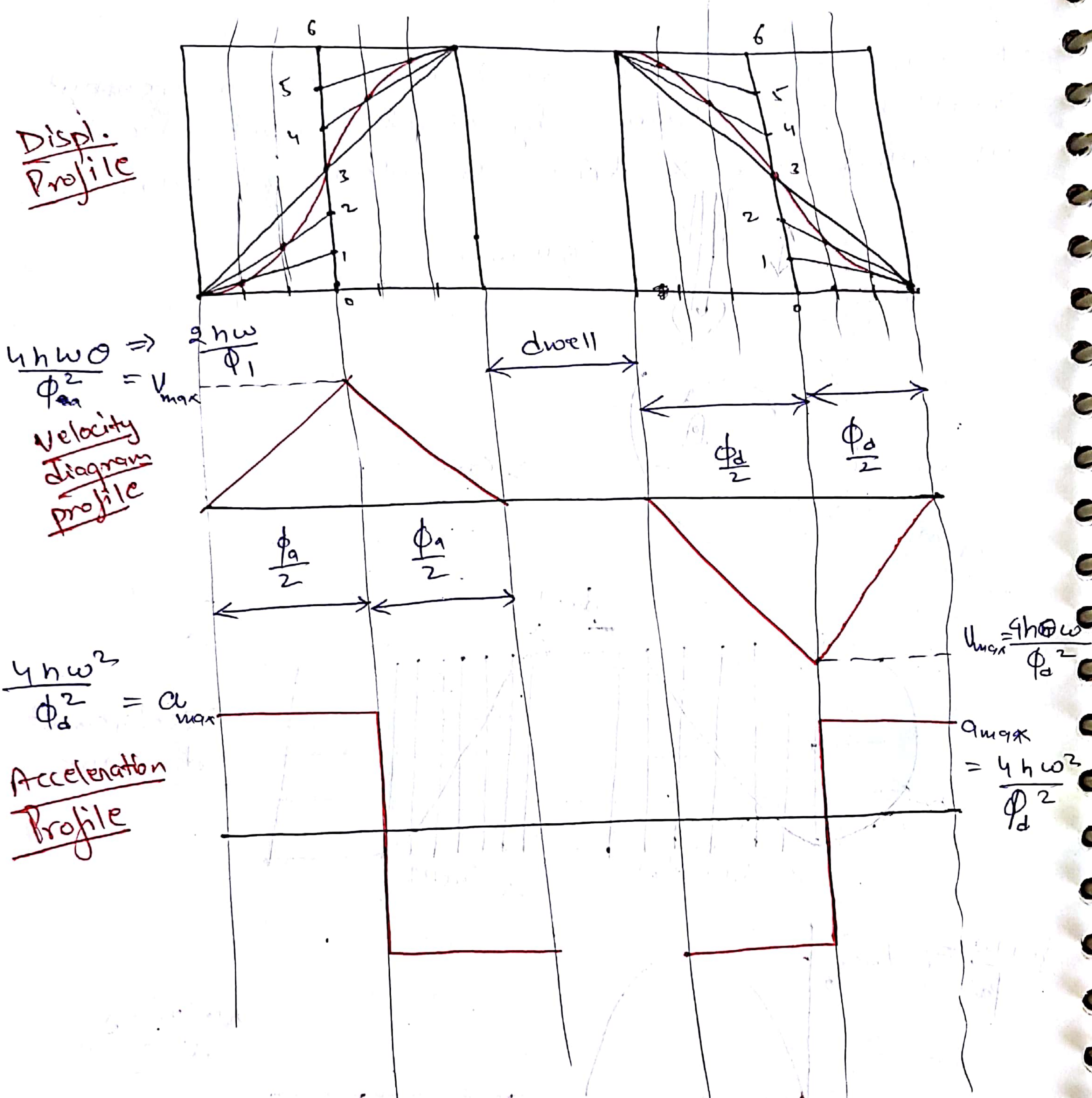
- A cam is a mechanical device used to transmit motion of a follower by direct contact. The driver is called cam & driven is called follower.



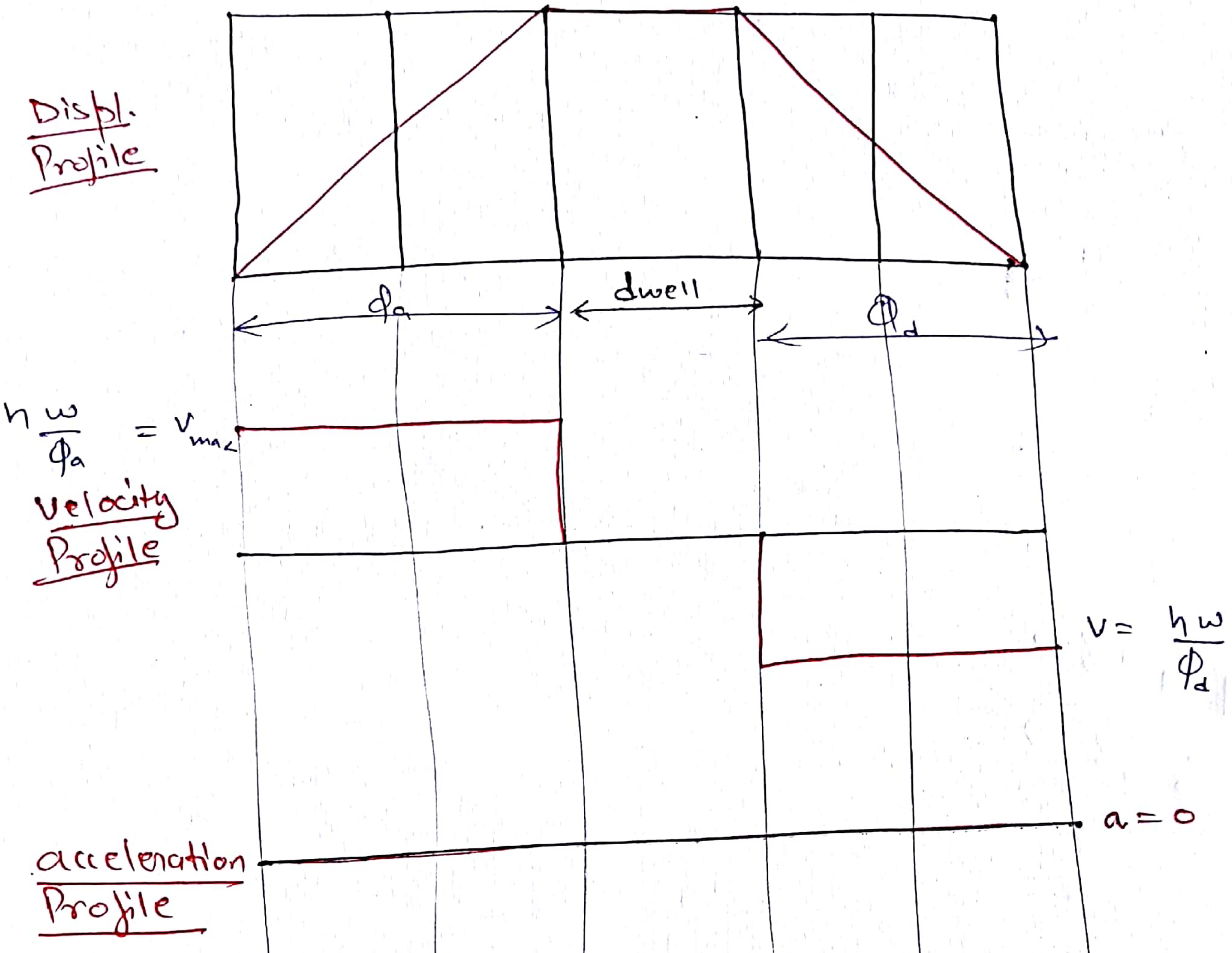
SHM



Follower motion with uniform acceleration & Retardation :



Follower motion with uniform velocity



follower motion ⇓	V_{max}	a_{max}
SHM	$\frac{\pi h}{2}\left(\frac{\omega}{\phi}\right)$	$\frac{\pi^2 h}{2}\left(\frac{\omega}{\phi}\right)^2$
uniform acceleration	$2h\left(\frac{\omega}{\phi}\right)$	$4h\left(\frac{\omega}{\phi}\right)^2$
uniform velocity	$h\left(\frac{\omega}{\phi}\right)$	0
Parabolic	$2h\left(\frac{\omega}{\phi}\right)$	$-4h\left(\frac{\omega}{\phi}\right)^2$
Cycloidal	$2h\left(\frac{\omega}{\phi}\right)$	$2h\pi\left(\frac{\omega}{\phi}\right)^2$